Two dimensional Super QCD on a lattice

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Lattice 2017

Outline …

- (Very) brief review of lattice constructions that preserve supersymmetry for pure SYM

- Generalizations to quiver gauge theories and super QCD

- Specific example: 2d super QCD with Nf flavors and Nc colors. Spontaneous SUSY breaking ?

- Numerical results

Exact lattice SUSY - general idea

- Change to twisted variables that expose a nilpotent supersymmetry $Q^2=0$

- Transformation will convert spinors to antisymmetric tensors and scalars to vectors

- Action $S=Q\,F(\text{fields})$ - invariant without invoking Leibniz

- Can discretize twisted theory while keeping gauge invariance and Q and without generating additional doublers

Ex: Q=8 SYM in D=3

A_a, B^I, Ψ^i **3D gauge field, 3 scalars, 2 Dirac fermions**

$$S = \int F_{ab}^2 + (DB^I)^2 + [B^I, B^J]^2 + \text{fermions}$$

twist:

$$SO_{\text{twist}}(3) = \text{diag}\left(SO_{\text{rot}}(3) \times SO_{\text{flavor}}(3)\right)$$

Twisted fields:

$$
\begin{aligned}
B^I &\to B_a \text{ vector} \\
A_a &\to A_a \text{ vector}
\end{aligned}
\qquad \mathcal{A}_a = A_a + iB_a
$$

$\Psi_\alpha^i \to \Psi_{i\alpha} \to (\eta, \psi_a, \chi_{ab}, \theta_{abc})$ antisymmetric integer spin

Twisted Q=8 SYM

$$S = \frac{1}{g^2} \mathcal{Q} \int \mathrm{Tr} \left[\chi_{ab} \mathcal{F}_{ab} + \eta [\overline{D}_a, \mathcal{D}_a] + \frac{1}{2} \eta d \right] + S_{\mathrm{closed}}$$

$$S_{\mathrm{closed}} = -\frac{1}{g^2} \int \mathrm{Tr}\, \theta_{abc} \overline{\mathcal{D}}_{[a} \chi_{ab]}$$

$$\mathcal{D}_a = \partial_a + \mathcal{A}_a \quad \overline{\mathcal{D}}_a = \partial_a + \overline{\mathcal{A}}_a \qquad [\mathcal{D}_a, \mathcal{D}_b] = \mathcal{F}_{ab}$$

SUSY:

$$
\begin{aligned}
\mathcal{Q}\mathcal{A}_a &= \psi_a \\
\mathcal{Q}\overline{\mathcal{A}}_a &= 0 \\
\mathcal{Q}\psi_a &= 0 \\
\mathcal{Q}\chi_{ab} &= -\overline{\mathcal{F}}_{ab} \\
\mathcal{Q}\eta &= d \\
\mathcal{Q}d &= 0
\end{aligned}
$$

Lattice

$\mathcal{U}_a(x, x+a)$: gauge transforms as $G(x)\mathcal{U}_a(x)G^\dagger(x+a)$

Similarly

$\psi_a(x, x+a)$

$\chi_{ab}(x+a+b, x)$

$\eta(x, x)$

χ_{ab}

$\mathcal{U}_b$

$\operatorname{Tr} \chi_{ab}\mathcal{D}_b\psi_a(x) \rightarrow$

ψ_a

Gauge invariant loop
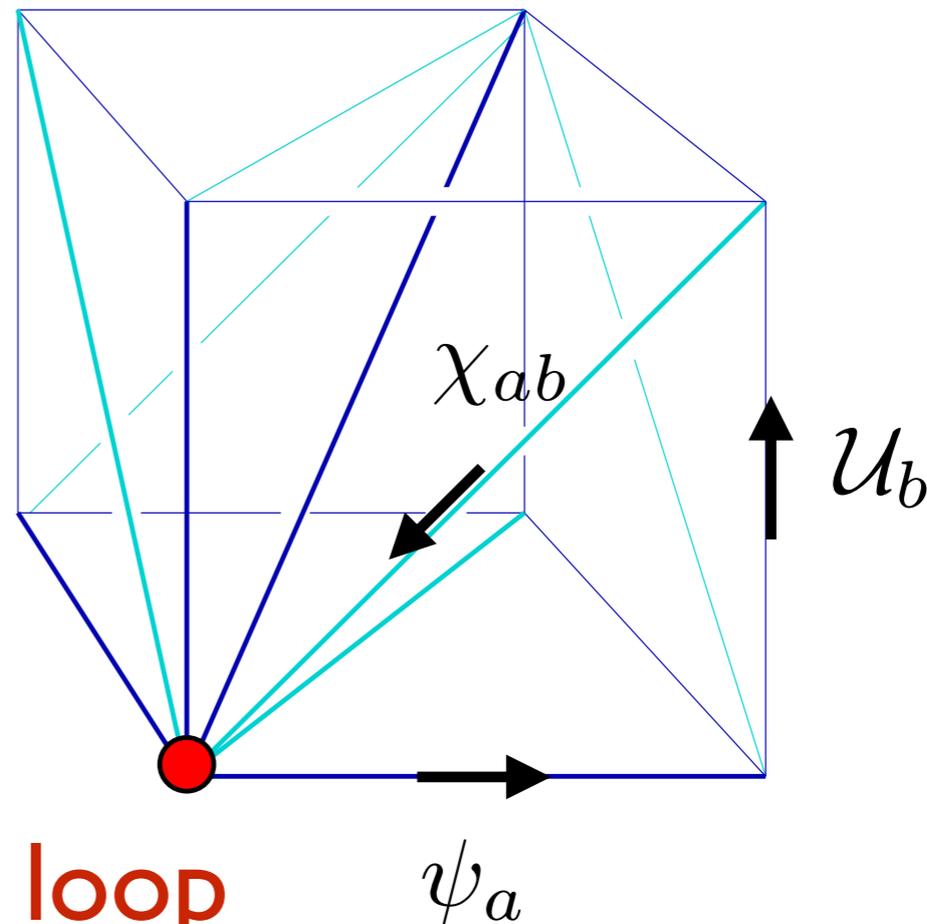
$$\operatorname{Tr}\left[\chi_{ab}(x+a+b, x)\psi_a(x, x+a)\mathcal{U}_b(x+a, x+a+b)\right]$$

Quiver gauge theories …

Consider typical term in SYM action e.g:

$$\mathrm{Tr}\left[G(x+a+b)\chi_{ab}(x)G(x)^\dagger G(x)\psi_a(x)G(x+a)^\dagger G(x+a)\mathcal{U}_b G(x+a+b)^\dagger\right]$$

$\downarrow$ Generalize gauge transformation

$$\mathrm{Tr}\left[H(x+a+b)\chi_{ab}(x)G(x)^\dagger G(x)\psi_a(x)G(x+a)^\dagger G(x+a)\mathcal{U}_b H(x+a+b)^\dagger\right]$$

Choose bi $-$ fundamental fields in some direction

b

SUSY preserved if superpartners have same bi-fundamental structure

$G \times H$ symmetries : $U(N_c) \times U(N_f)$

Super QCD from quivers

Consider single 2d slice and pbc $: \mathrm{x} \equiv \mathrm{x} + \hat{3}$

Take link fields pointing in say b=3 direction to be bi-fundamental

relabel $: \quad \chi_{3\mu} = \lambda_\mu \quad U_3 = \phi \quad \psi_3 = \lambda$

$$\chi_{3a}\mathcal{D}_3\psi_a \rightarrow \lambda_\mu(x)\left[\phi(x)\psi_\mu(x) - \psi_\mu(x)\phi(x+\mu)\right]$$

Yukawas

$$\chi_{a3}\mathcal{D}_a\psi_3 \rightarrow \lambda_\mu(x)\left[U_\mu(x)\lambda(x+\mu) - \lambda(x)U_\mu(x)\right]$$

kinetic terms

2d super QCD:

Adjoint Fields	Fundamental Fields
$\mathcal{Q}A_\mu = \psi_\mu$	$\mathcal{Q}\phi = \lambda$
$\mathcal{Q}\overline{A}_\mu = 0$	$\mathcal{Q}\overline{\phi} = 0$
$\mathcal{Q}\psi_\mu = 0$	$\mathcal{Q}\lambda = 0$
$\mathcal{Q}\chi_{\mu\nu} = -\overline{\mathcal{F}}_{\mu\nu}$	$\mathcal{Q}\lambda_\mu = -\overline{\mathcal{D}}_\mu\overline{\phi}$
$\mathcal{Q}\eta = d$	$\mathcal{Q}\lambda_{\mu\nu} = 0$

$$\lambda \equiv \lambda_c^f$$

$$N_c \times N_f \text{ matrix}$$

Finally send $g_H \to 0$ to get $U(N_f)$ global flavor symmetry

In general use D+1 dim SYM to build D dim super QCD…

SUSY breaking

Add term $\quad r\mathcal{Q}\sum_{x}\mathrm{Tr}\,\eta\quad$ U(1) FI term

Yields scalar potential:

$$V = \frac{1}{2}\sum_{a=1}^{Nf}\mathrm{Tr}\left(\phi_a\overline{\phi}_a - rI_{Nc}\right)^2$$

N_cxN_c matrix

Higgses theory

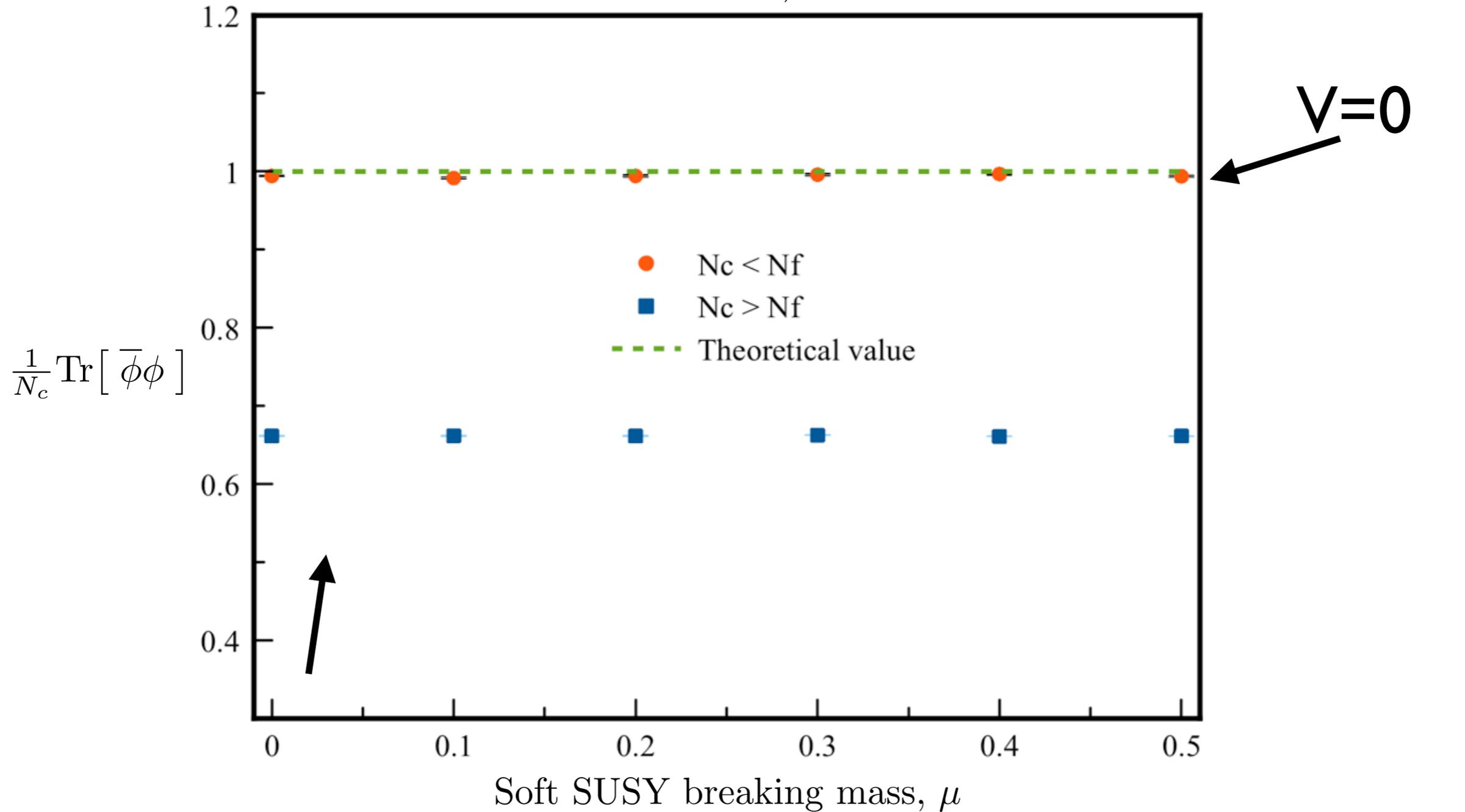Using SU(N_c) can diagonalize this matrix
Rank = N_f. If $N_f < N_c$
Cannot make V=0 — SUSY must break
spontaneously

Simulations

- Use RHMC. No sign problem. GPU accel.

- Add small scalar mass m to stabilize flat directions.

- Set r=1.0. lambda=0.1-2.0, m=0.05-0.6

- Examine Nc=2 Nf=3 and Nc=3 Nf=2 models corresponding to unbroken/broken SUSY

- Examined Ward identities revealing broken SUSY and correlation functions showing Goldstino

SUSY breaking I

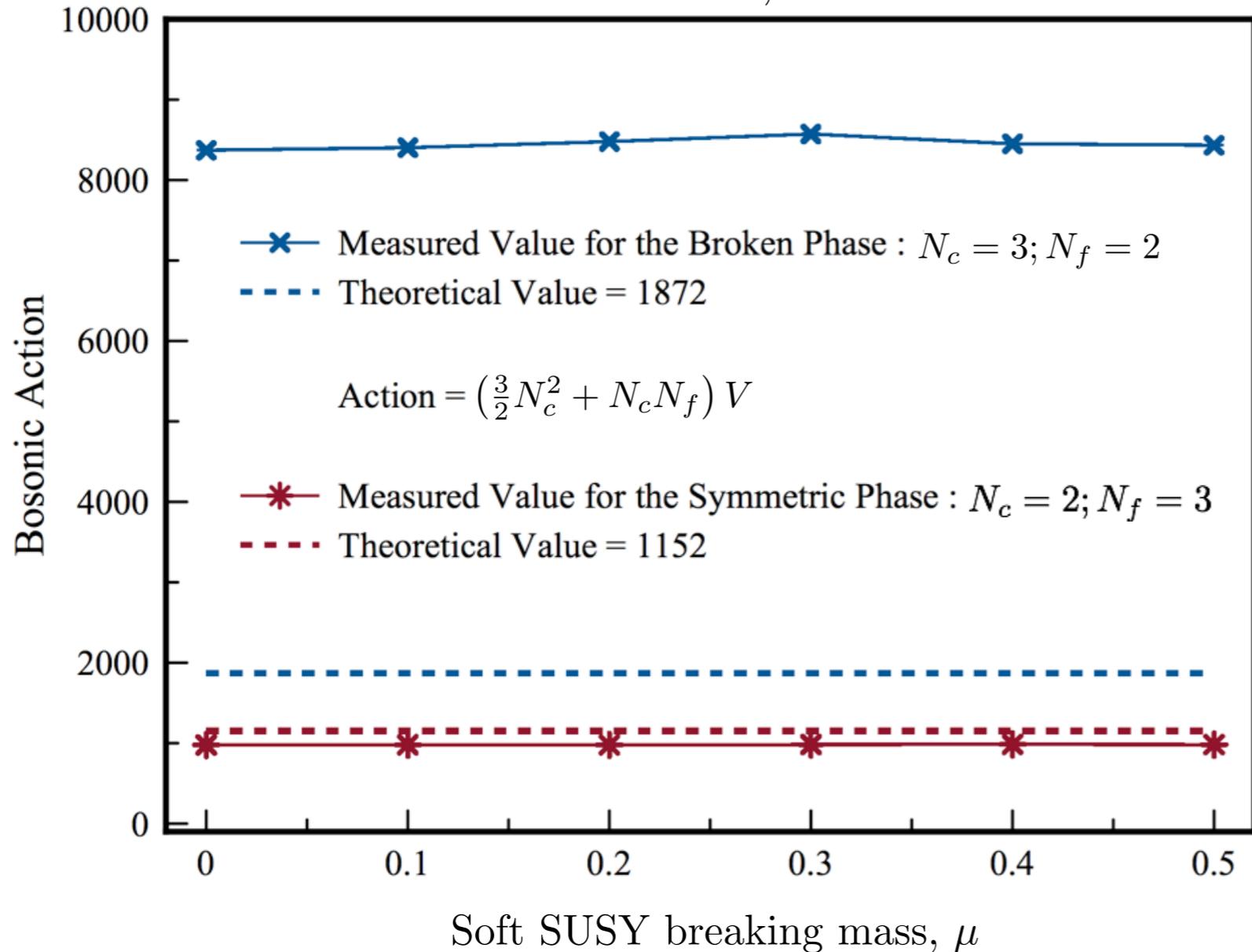16×6 lattice ; $\lambda = 1.0$

$\frac{1}{N_c}\mathrm{Tr}\big[\,\overline{\phi}\phi\,\big]$

- ● $Nc < Nf$
- ■ $Nc > Nf$
- - - Theoretical value

V=0

Soft SUSY breaking mass, μ

auxiliary field

$d = 1/N\mathrm{Tr}\,\phi\overline{\phi} - 1$

SUSY breaking II

16×6 lattice ; $\lambda = 1.0$

Bosonic Action (y-axis)

Soft SUSY breaking mass, μ (x-axis)

Legend:
- Measured Value for the Broken Phase : $N_c = 3; N_f = 2$
- Theoretical Value = 1872
- Action = $\left(\frac{3}{2}N_c^2 + N_c N_f\right) V$
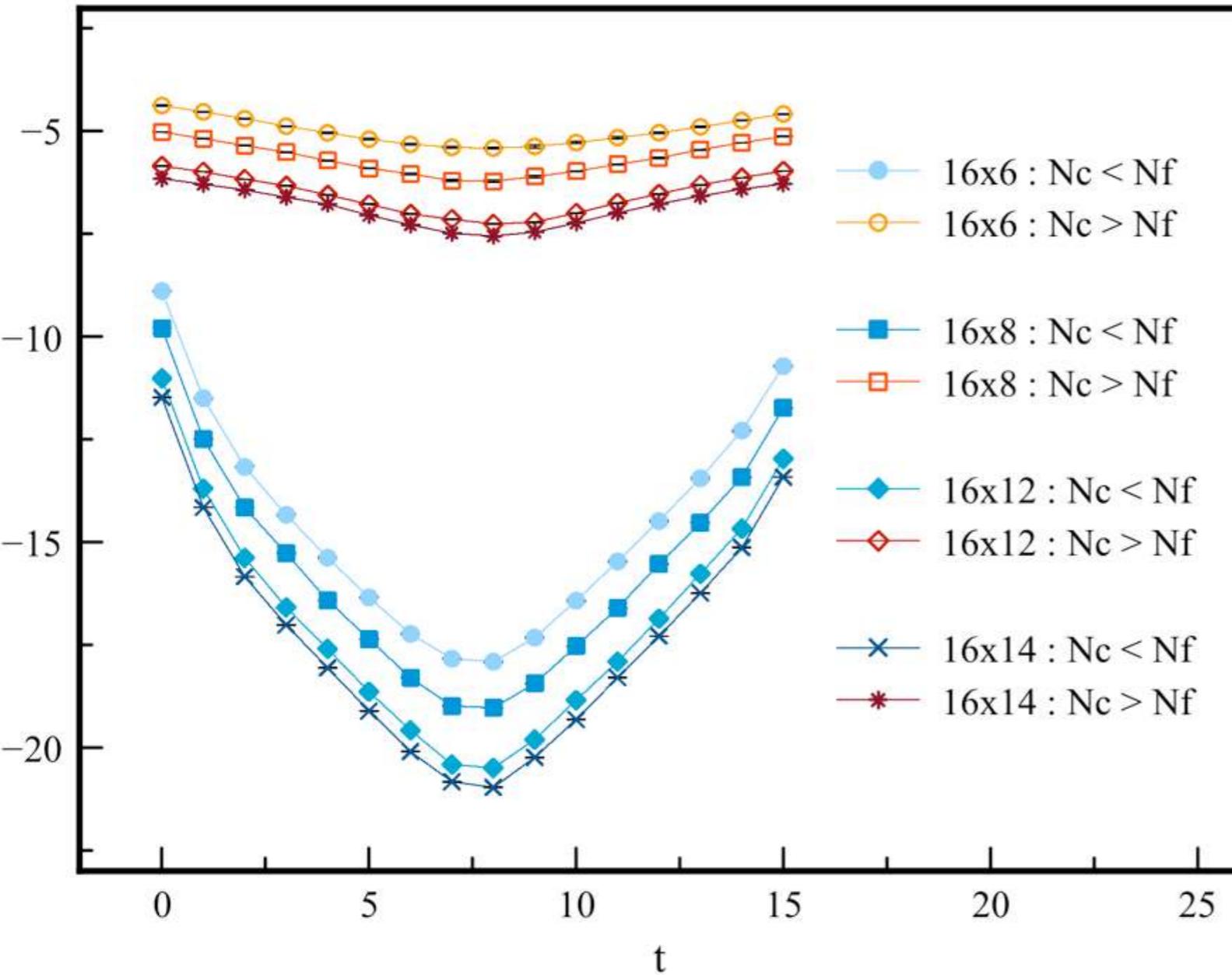- Measured Value for the Symmetric Phase : $N_c = 2; N_f = 3$
- Theoretical Value = 1152

Exact lattice SUSY implies Evac=0 unless SUSY breaks

$$\text{E}_{\text{vac}} = S_B - N_c V \left(\frac{3}{2}N_c + N_f\right)$$
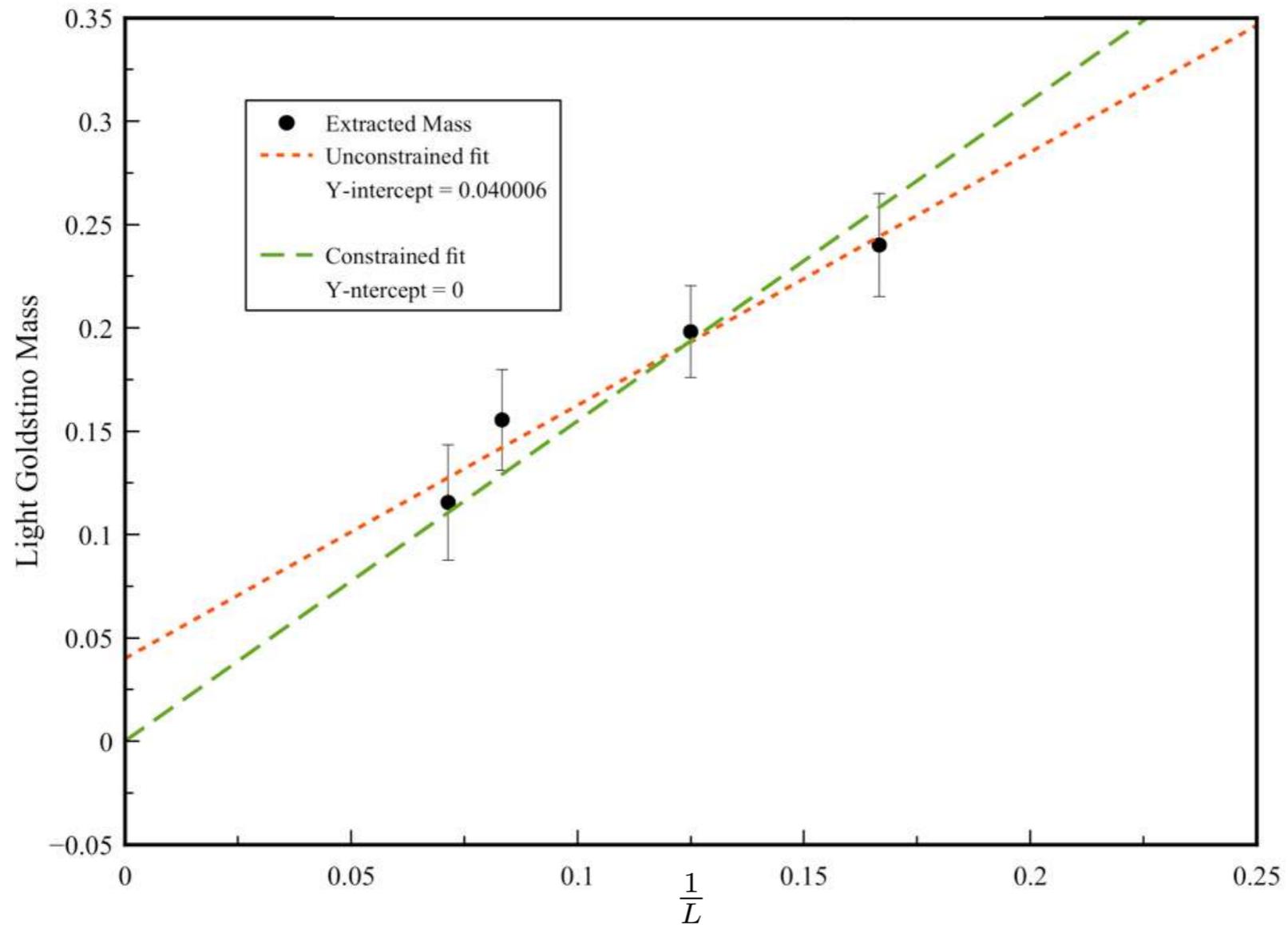
Goldstino

$$\lambda = 1.0 \; ; \mu = 0.3$$

$$< O(x)O'(y) >$$

$$O(x) = \text{Tr} \left(\sum_\mu \psi_\mu(x) \overline{\mathcal{U}}_\mu(x) d(x) \right)$$

$$O'(y) = \text{Tr} \left(\eta(y) d(y) \right)$$

Legend:
- 16x6 : Nc < Nf
- 16x6 : Nc > Nf
- 16x8 : Nc < Nf
- 16x8 : Nc > Nf
- 16x12 : Nc < Nf
- 16x12 : Nc > Nf
- 16x14 : Nc < Nf
- 16x14 : Nc > Nf

Goldstino mass goes to zero

$$\lambda = 1.0 \; ; \mu = 0.3$$

Summary

- Theories with exact lattice SUSY under active investigation - principally N=4 Yang-Mills in D=4

- Constructions can be generalized for quiver gauge theories containing fields in bi-fundamental representations.

- These can be used in combination with dimensional reduction to generate (Q preserving) lattice actions for super QCD (in D<4).

- Lattice studies may help us understand important non-perturbative features e.g. dynamical SUSY breaking

Thank you!