Nucleon axial coupling from Lattice QCD

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Granada, Spain

motivation

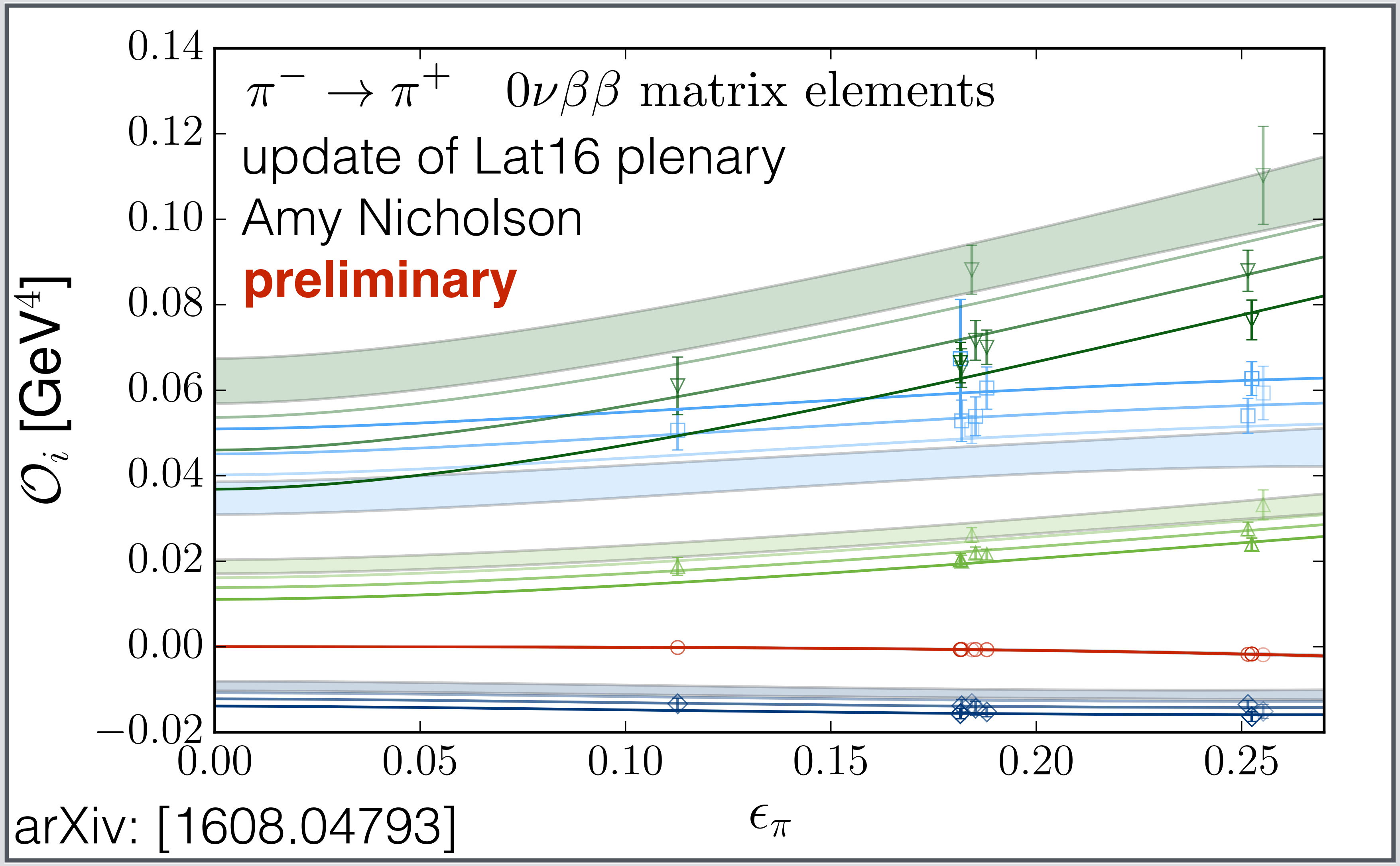
interest in understanding nuclear physics from QCD

precision nuclear physics experiments

- neutrinoless double beta decay
- neutrino scattering
- ultra-cold neutron decay
- elastic recoil from dark matter

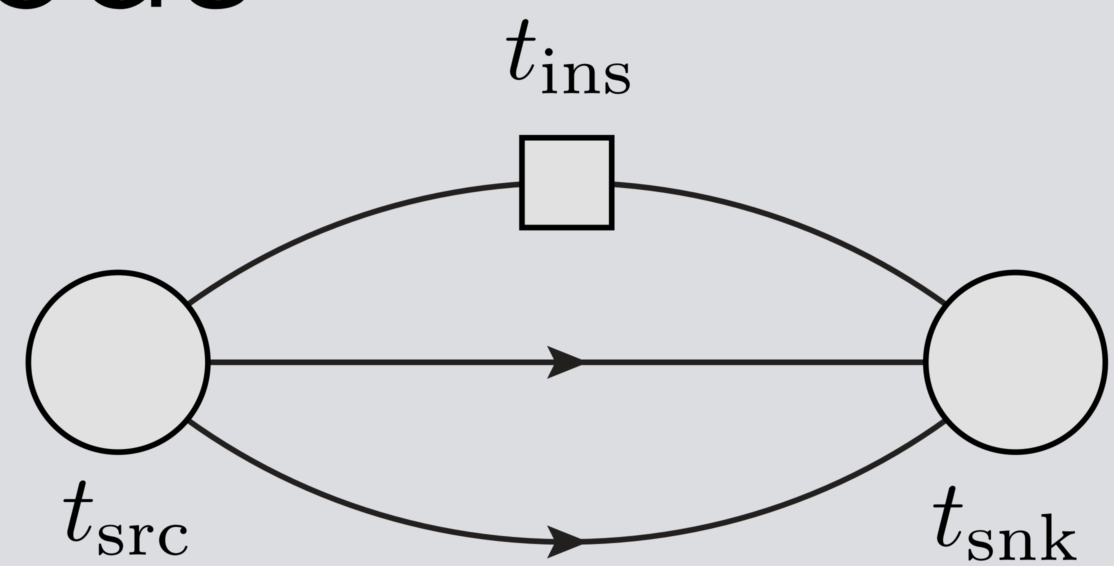
precise matrix elements for the precision frontier

nucleon axial coupling is a pre-eminent benchmark

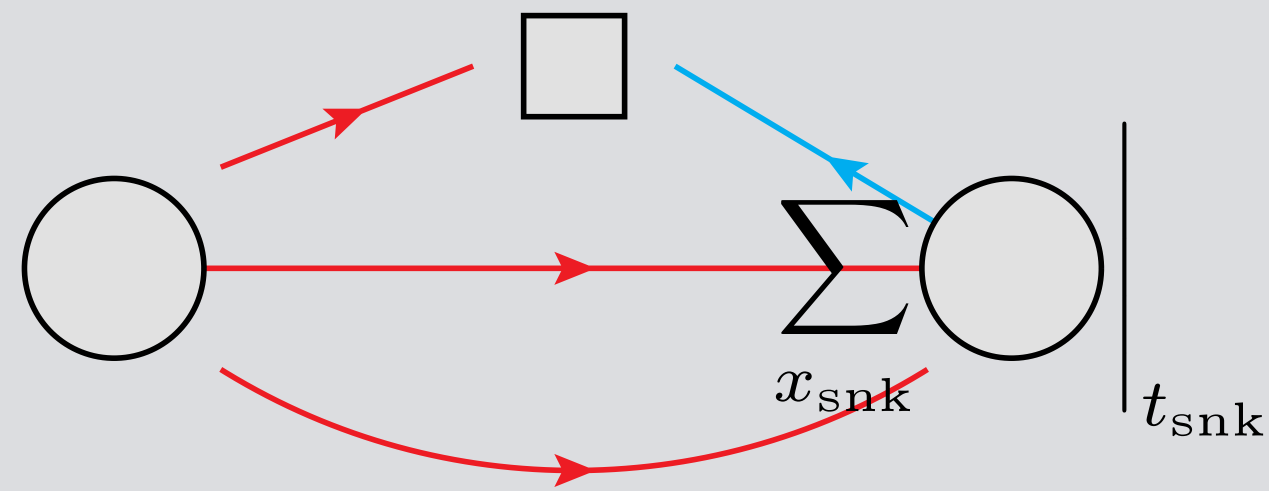


summary of methods

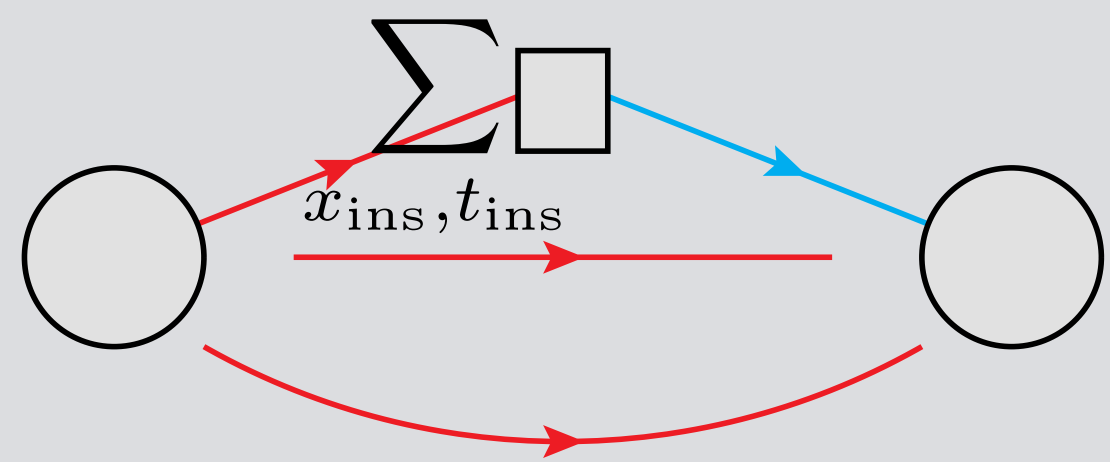
three-point correlator



fixed-sink method



Feynman-Hellmann method



the Feynman-Hellmann theorem

arXiv: [1612.06963]

definition

$$\frac{\partial E_\lambda}{\partial \lambda} = \left\langle \psi_\lambda \left| \frac{\partial \hat{H}_\lambda}{\partial \lambda} \right| \psi_\lambda \right\rangle \xrightarrow{\text{on the lattice}} \frac{\partial m_{\text{eff}}}{\partial \lambda} = \langle n | \mathcal{J} | n \rangle$$

derivation

$$m_{\text{eff}} = \frac{1}{\tau} \ln \left(\frac{C(t)}{C(t+\tau)} \right) \xrightarrow{t \to \infty} E_0$$

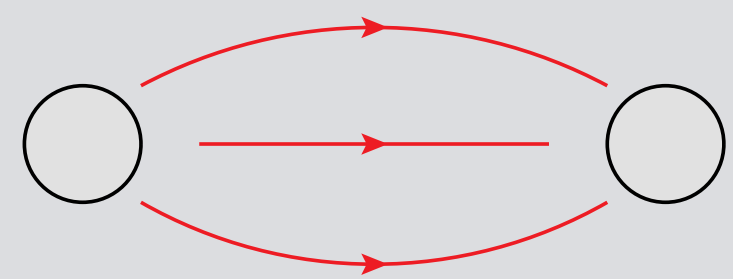
$$\left. \frac{\partial m_{\text{eff}}}{\partial \lambda} \right|_{\lambda=0} = \frac{1}{\tau} \left[\frac{\partial_\lambda C(t)}{C(t)} - \frac{\partial_\lambda C(t+\tau)}{C(t+\tau)} \right] \Bigg|_{\lambda=0}$$

$$\lim_{t \to \infty} \left. \frac{\partial_\lambda C(t)}{C(t)} \right|_{\lambda=0} = (t-1) g_A + \text{constant}$$

$$\lim_{t \to \infty} \left. \frac{\partial m_{\text{eff}}}{\partial \lambda} \right|_{\lambda=0} = g_A + \mathrm{O}\left(e^{-E_n t}\right)$$

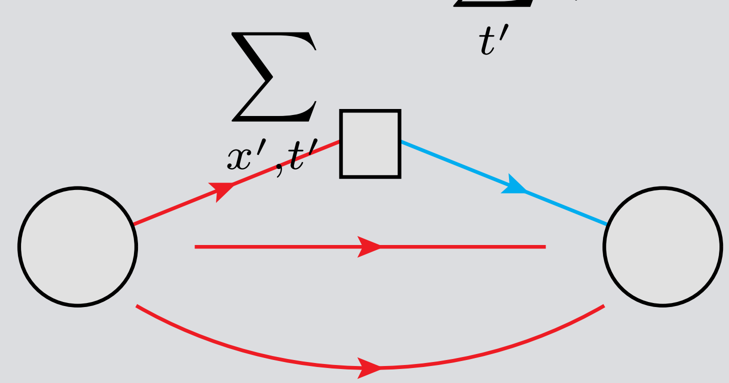
nucleon two-point correlation function

$$C(t) = \left\langle \mathcal{N}(t) \mathcal{N}^\dagger(0) \right\rangle$$



Feynman-Hellmann correlation function

$$\partial_\lambda C(t) = -\sum_{t'} \left\langle \mathcal{N}(t) \mathcal{J}(t') \mathcal{N}^\dagger(0) \right\rangle$$

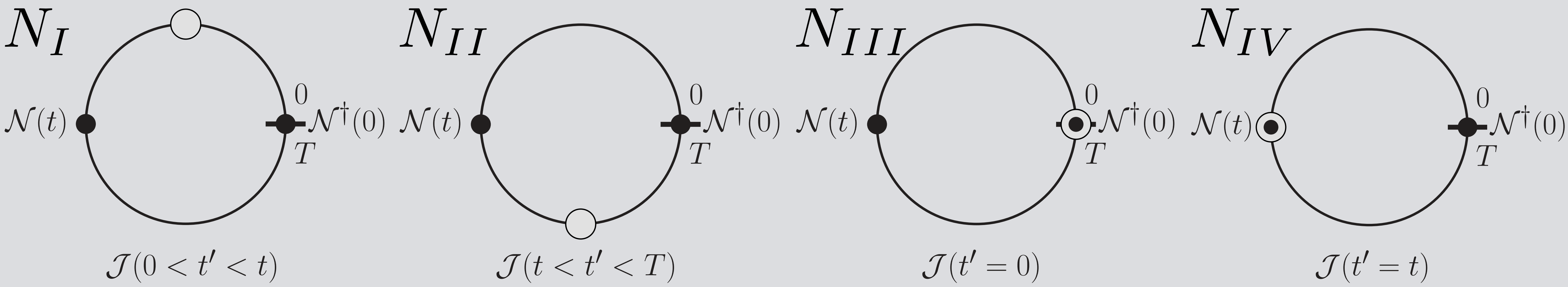


$t = t_{\text{snk}} - t_{\text{src}}$

$t' = t_{\text{ins}}$

$0 = t_{\text{src}}$

spectral decomposition



$$N_I(t) = (t-1)\sum_n \frac{Z_n J_{nn} Z_n^\dagger}{4E_n^2} e^{-E_n t} + \sum_{n,m\neq n} \frac{Z_n J_{nm} Z_m^\dagger}{4E_n E_m} \frac{e^{-E_n t} e^{\frac{\Delta_{nm}}{2}} - e^{-E_m t} e^{\frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{mn}}{2}} - e^{\frac{\Delta_{nm}}{2}}}$$

$$N_{II}(t) = \sum_n \frac{e^{-E_n t}}{2E_n} \left\{ Z_n Z_n^\dagger J_{\Omega\Omega} + \sum_j \frac{Z_n Z_{nj}^\dagger J_j^\dagger + J_j Z_{jn} Z_n^\dagger}{2E_j (e^{E_j} - 1)} \right\}$$

$$N_{III+IV}(t) = \sum_n \frac{e^{-E_n t}}{2E_n} \left\{ Z_n Z_{J:n}^\dagger + Z_{J:n} Z_n^\dagger \right\}$$

$$\Delta_{mn} \equiv E_m - E_n$$

reparameterize

$$N(t) = \sum_n \left[(t-1) z_n g_{nn} z_n^\dagger + d_n \right] e^{-E_n t} + \sum_{n\neq m} z_n g_{nm} z_m^\dagger \frac{e^{-E_n t} e^{\frac{\Delta_{nm}}{2}} - e^{-E_m t} e^{\frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{mn}}{2}} - e^{\frac{\Delta_{nm}}{2}}}$$

$$z_n \equiv \frac{Z_n}{\sqrt{2E_n}} \qquad g_{nm} \equiv \frac{J_{nm}}{\sqrt{4E_n E_m}} \qquad d_n \equiv Z_n Z_{J:n}^\dagger + \text{h.c.} + Z_n Z_n^\dagger J_{\Omega\Omega} + \sum_j \frac{Z_n Z_{nj}^\dagger J_j^\dagger + \text{h.c.}}{2E_j (e^{E_j} - 1)}$$

excited state systematics summary

Feynman-Hellmann correlation function

$$N(t) = \sum_n z_n g_{nn} z_n^\dagger (t-1) e^{-E_n t}$$

$$+ \sum_{n\neq m} z_n g_{nm} z_m^\dagger \frac{e^{-E_n t} e^{\frac{\Delta_{nm}}{2}} - e^{-E_m t} e^{\frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{mn}}{2}} - e^{\frac{\Delta_{nm}}{2}}}$$

$$+ d_n e^{-E_n t}$$

$$N(1) = \sum_n d_n e^{-E_n} \approx \mathrm{O}(d_0 e^{-E_0})$$

fixed-sink / fixed-insertion correlation functions

$$C^{3\mathrm{pt}}(t', t) = \sum_n |z_n|^2 g_{nn} e^{-E_n t}$$

$$+ \sum_{r\neq s} z_r g_{rs} z_s^\dagger e^{-(E_r - E_s)t'} e^{-E_s t}$$

$$t = t_{\mathrm{snk}} - t_{\mathrm{src}}$$

$$t' = t_{\mathrm{ins}} - t_{\mathrm{src}}$$

MILC ensembles & valence quarks

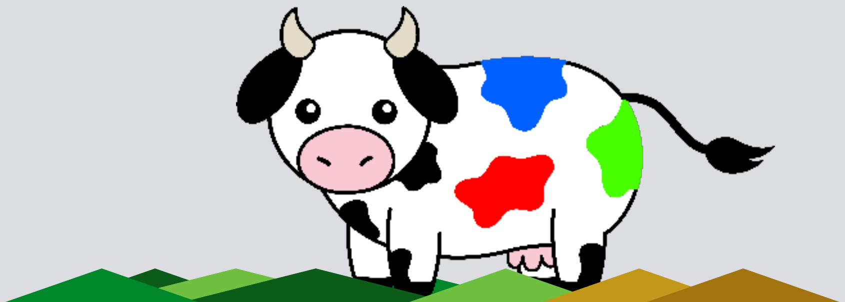
HISQ action
Errors starting at $O(\alpha_s a^2, a^4)$
-Weisz action
Errors starting at $O(\alpha_s^2 a^2, a^4)$

Mobius domain-wall
tune $m_{\text{res}} < 0.1 m_l$
Errors effectively start at $O(a^2, \alpha_s a^2)$

all ensembles are gradient-flow smeared with $t_{gf}/a^2 = 1$

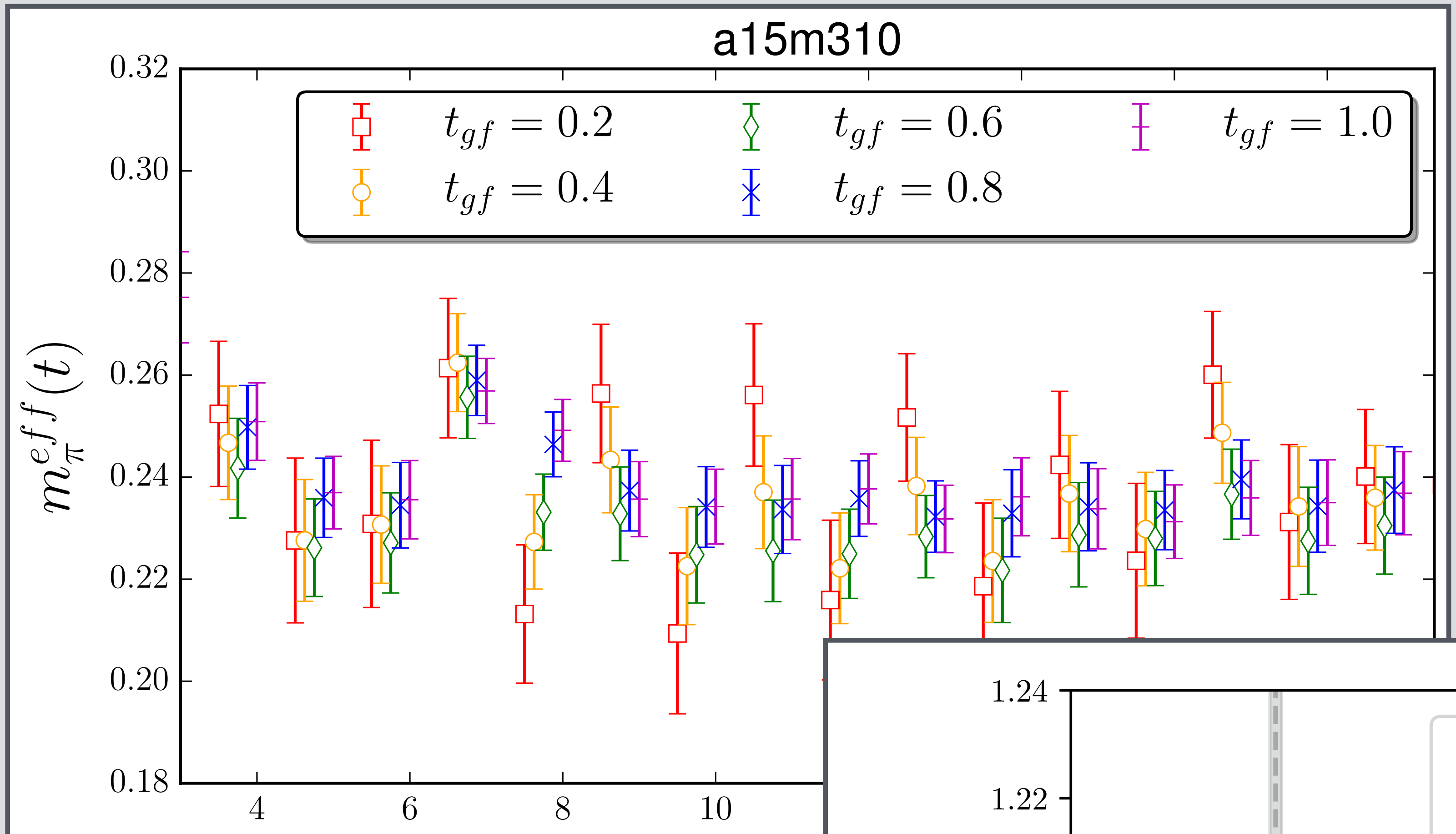
abbr.	a [fm]	m_l/m_s	volume	m_π [MeV]	$m_\pi L$	N_{cfg}	M_5	α	L_5	N_{src}
a15m310	0.15	0.2	$16^3 \times 48$	310	3.8	1960	1.3	2.0	12	24
a15m220	0.15	0.1	$24^3 \times 48$	220	4.0	1000	1.3	2.5	16	12
a15m130	0.15	0.036	$32^3 \times 48$	135	3.2	1000	1.3	3.5	24	5
a12m400	0.12	0.334	$24^3 \times 64$	400	5.8	1000	1.2	1.5	8	8
a12m350	0.12	0.255	$24^3 \times 64$	350	5.1	1000	1.2	1.5	8	8
a12m310	0.12	0.2	$24^3 \times 64$	310	4.5	1053	1.2	1.5	8	4
a12m220L	0.12	0.1	$40^3 \times 64$	220	5.4	1000	1.2	2.0	12	4
a12m220	0.12	0.1	$32^3 \times 64$	220	4.3	1000	1.2	2.0	12	4
a12m220S	0.12	0.1	$24^3 \times 64$	220	3.2	1000	1.2	2.0	12	4
a12m130	0.12	0.036	$48^3 \times 64$	135	3.9	1000	1.2	3.0	20	3
a09m310	0.09	0.2	$32^3 \times 96$	310	4.5	784	1.1	1.5	6	8
a09m220	0.09	0.1	$48^3 \times 96$	220	4.7	1001	1.1	1.5	8	6

additional HISQ ensembles generated @ LLNL

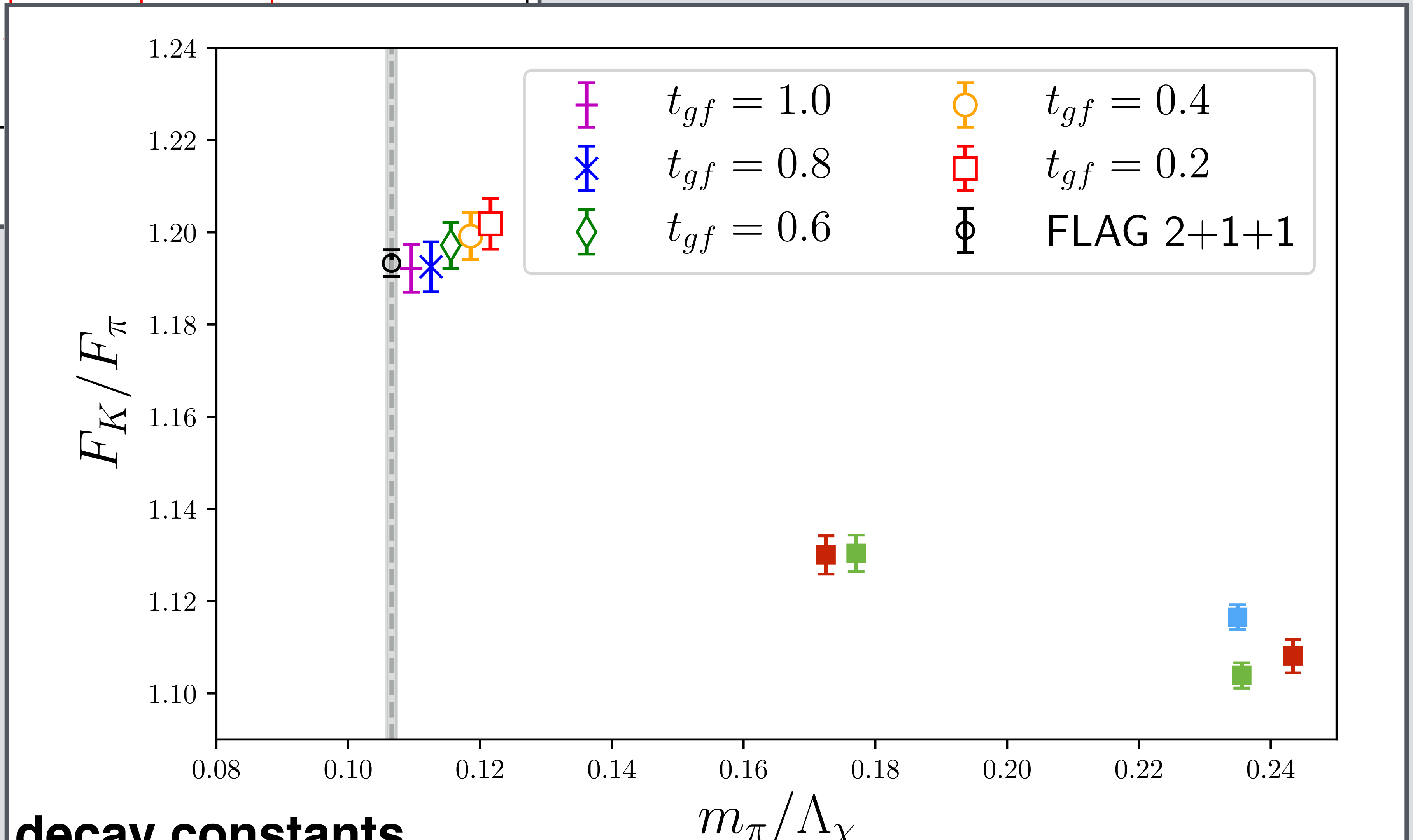


DWF on gradient-flowed HISQ

gradient-flow time (in)dependence



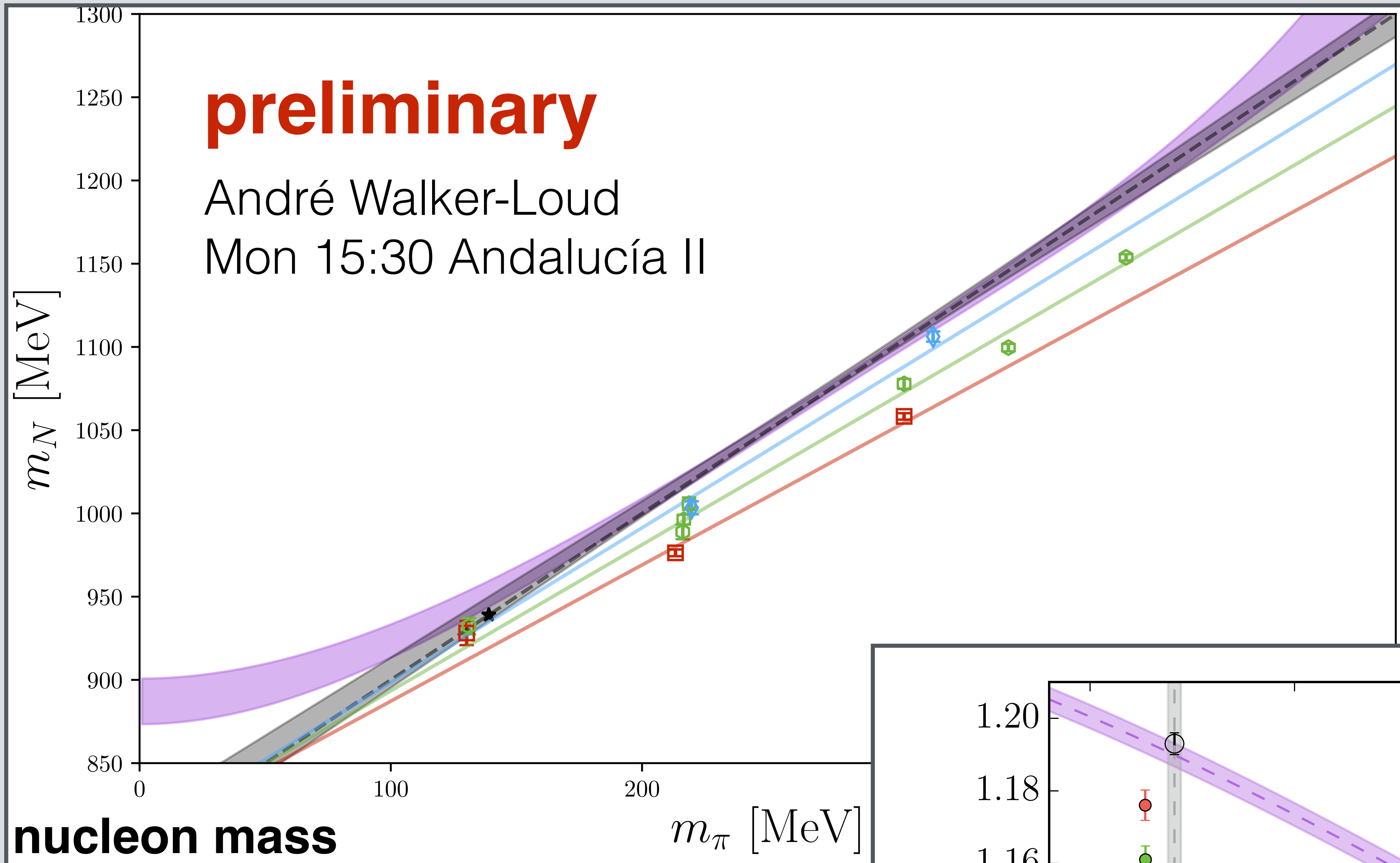
pion effective mass

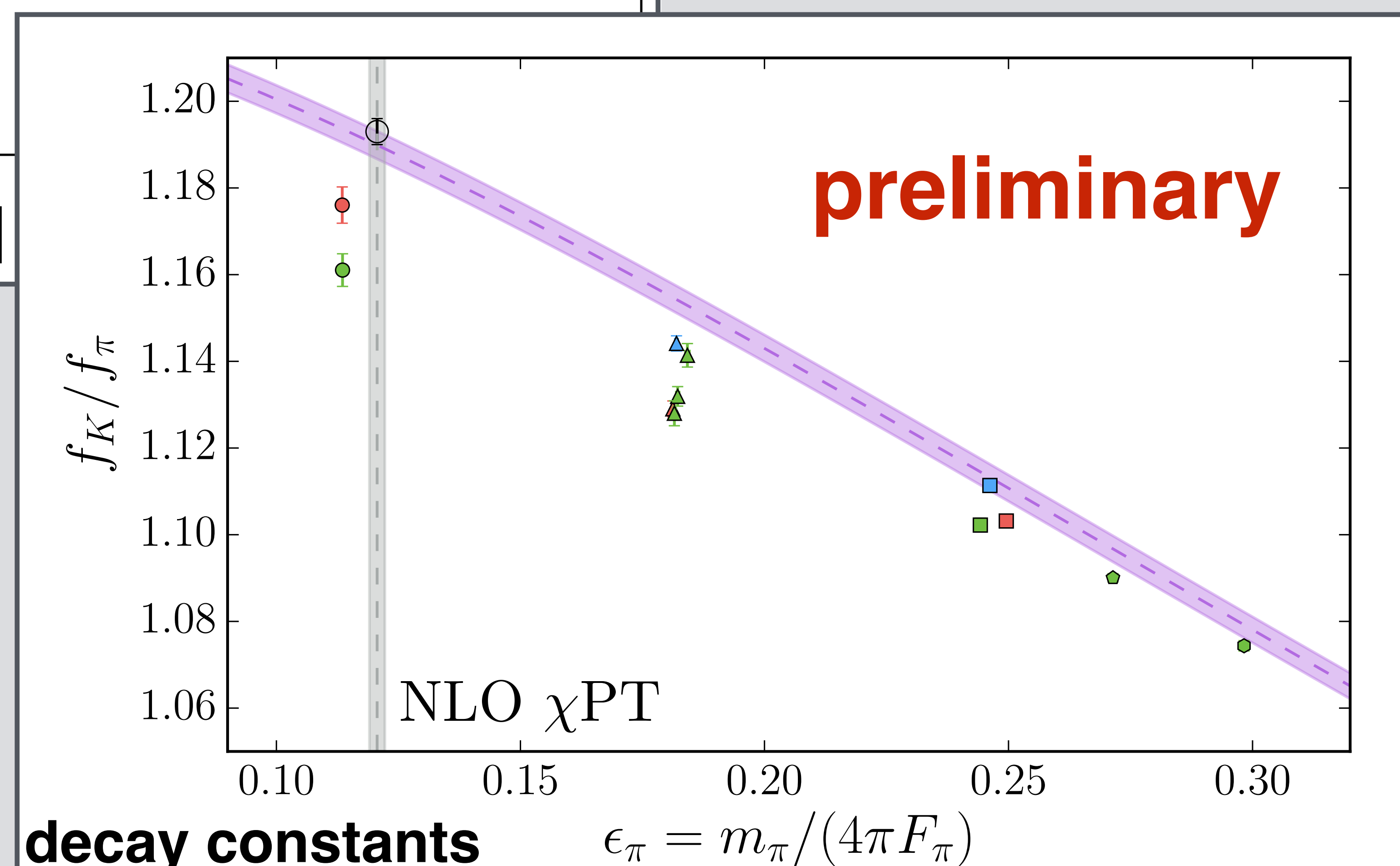


decay constants

arXiv: [1701.07559]

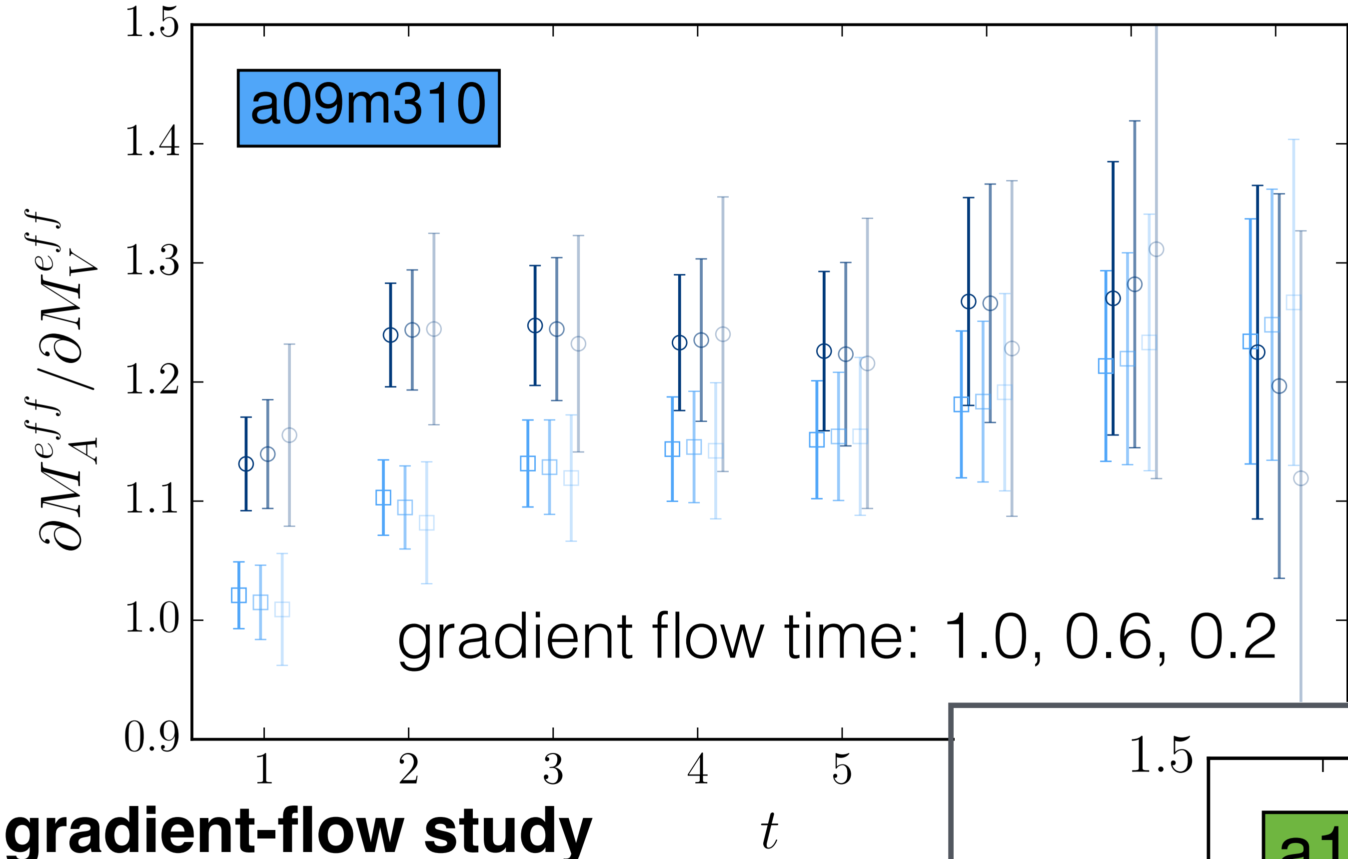
full statistics preliminary results





dashed	$800 + m_\pi$
black band	linear Taylor
purple	NNLO χPT

axial coupling correlator



gradient-flow study

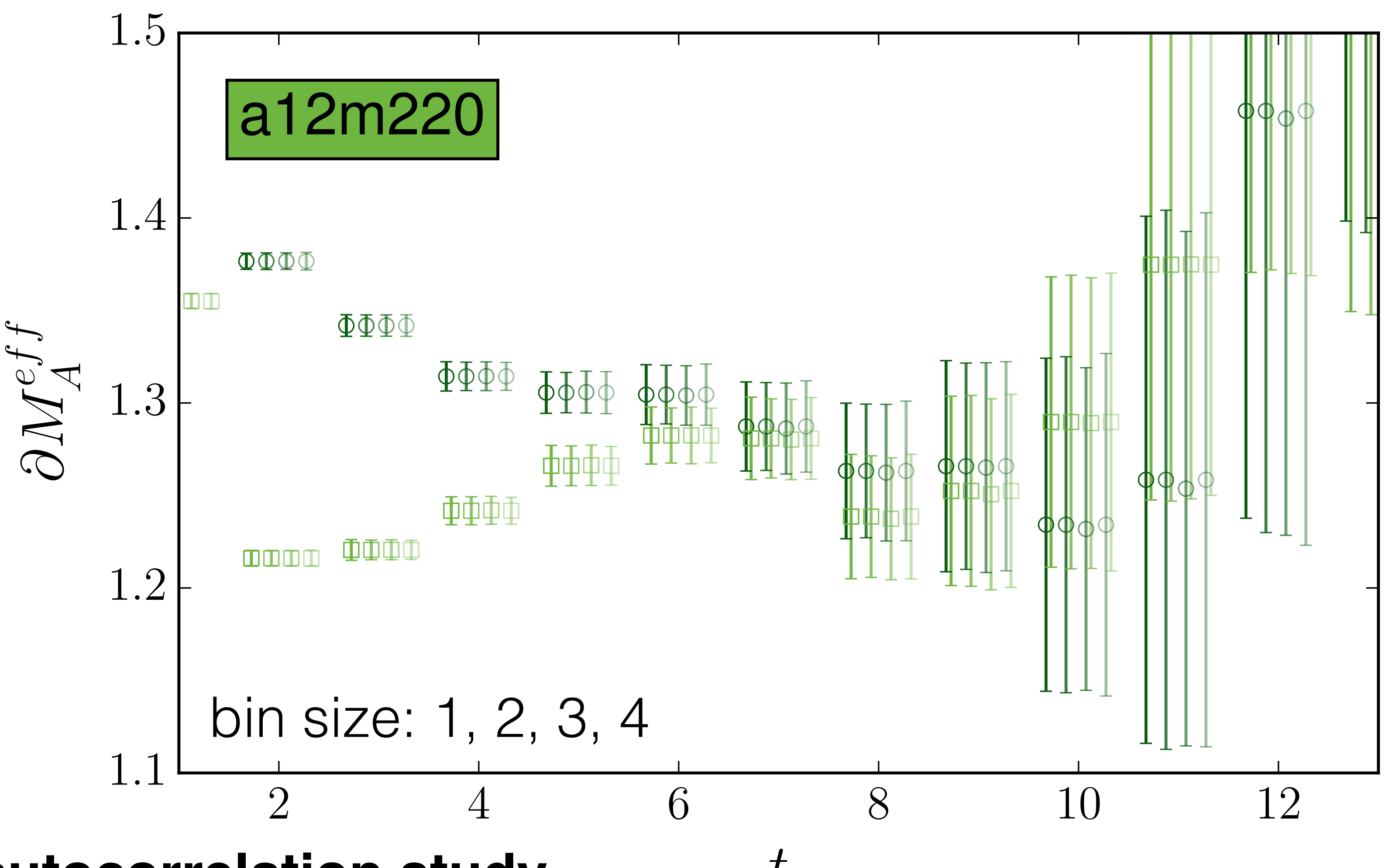
interpolating operators

$$\bar{\mathcal{N}}_{\gamma'} = \epsilon_{i'j'k'} \left(\bar{u}^{i'}_{\alpha'}(x) \Gamma^{\mathrm{src}}_{\alpha'\beta'} \bar{d}^{j'}_{\beta'}(x) \right) P^{\mathrm{src}}_{\gamma'\rho'} \bar{u}^{k'}_{\rho'}(x)$$

$$\mathcal{N}_{\gamma} = -\epsilon_{ijk} \left(u^{i}_{\alpha}(y) \Gamma^{\mathrm{snk}}_{\alpha\beta} d^{j}_{\beta}(y) \right) P^{\mathrm{snk}}_{\gamma\rho} u^{k}_{\rho}(y)$$

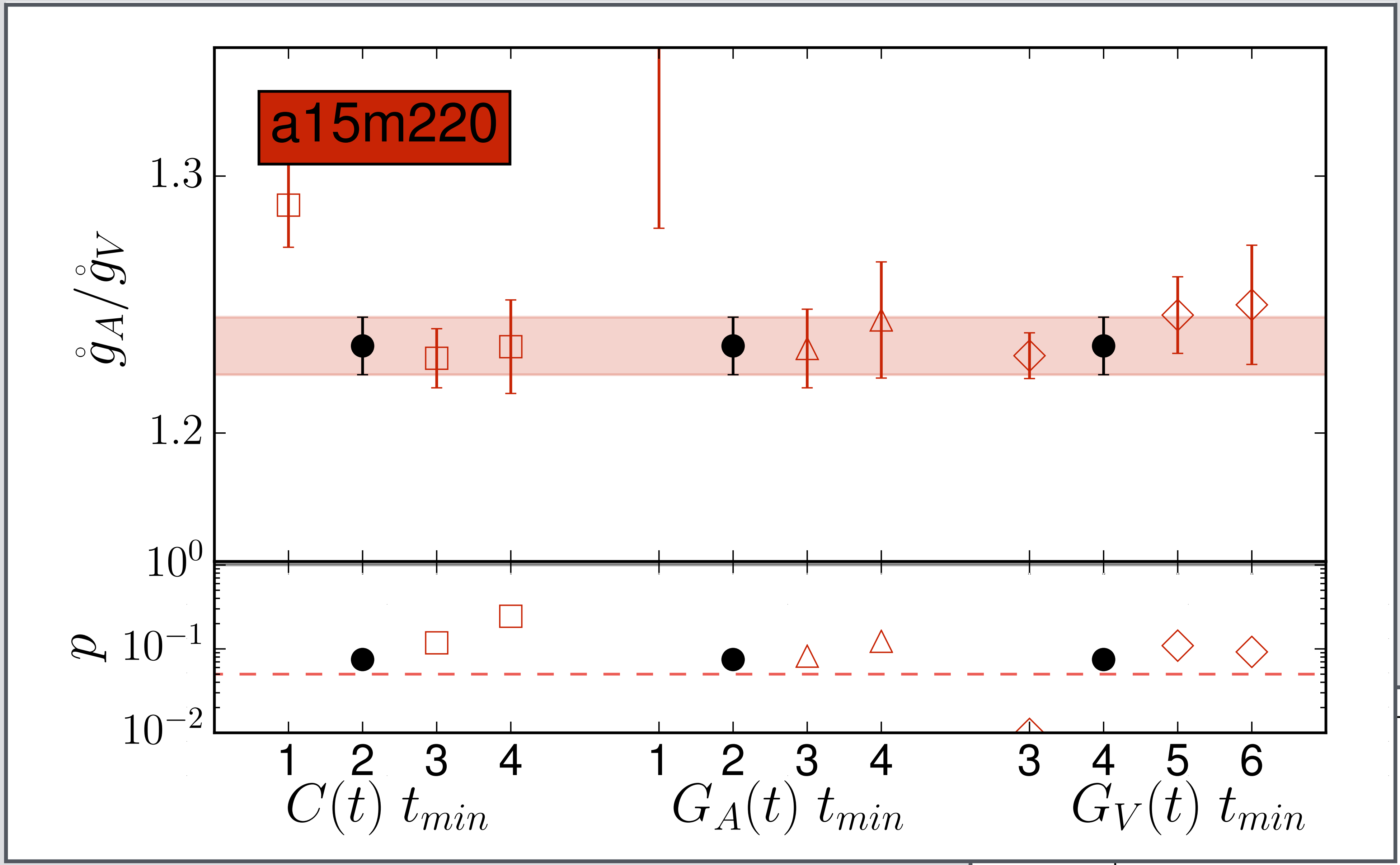
local current operator

$$\mathcal{J} = \bar{q}\gamma_\mu\gamma_5 q$$



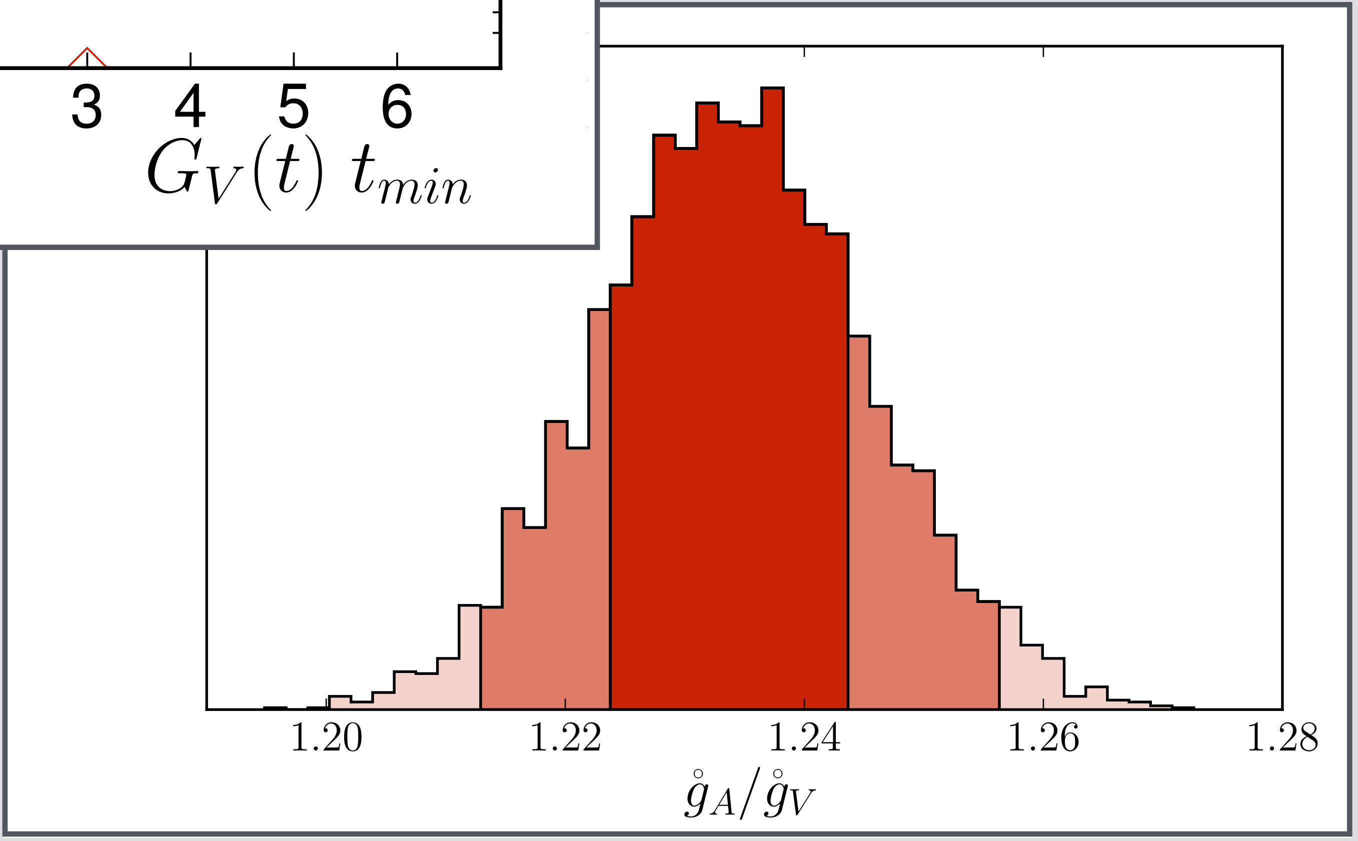
autocorrelation study

correlator analysis



requirements

p-value > 0.05
stable
Gaussian

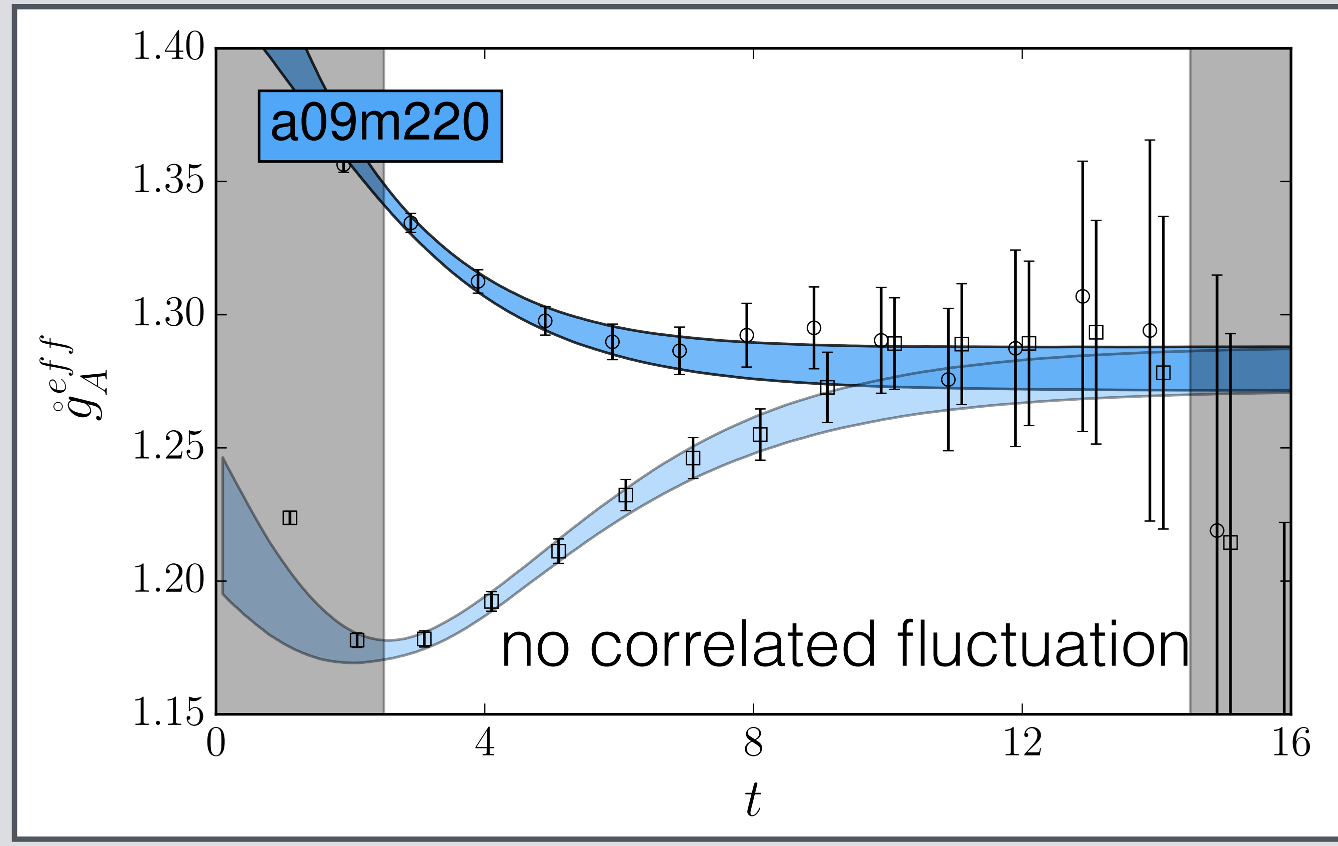


strategy

construct FH ratio
simultaneous fit to 6 correlators: (2,A,V)x(P,S)
Bayes fit → 2 state unconstrained fit
5000 bootstrap resamples

nucleon signal-to-noise

light (pion/kaon)	s/n $\propto e^{-[\mathrm{MeV}]t}/e^{-[\mathrm{MeV}]t}$
heavy-light (B/D)	s/n $\propto e^{-[\mathrm{GeV}]t}/e^{-[\mathrm{GeV}]t}$
nucleon	s/n $\propto e^{-[\mathrm{GeV}]t}/e^{-[\mathrm{MeV}]t}$

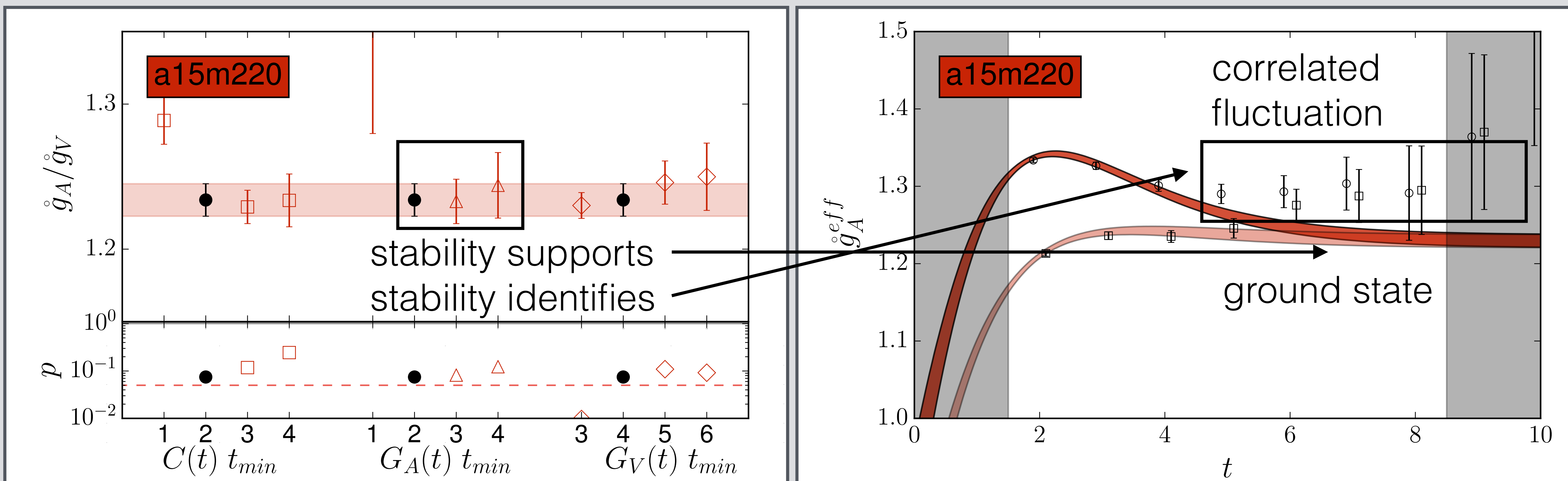


large correlated fluctuations in nucleon effective masses

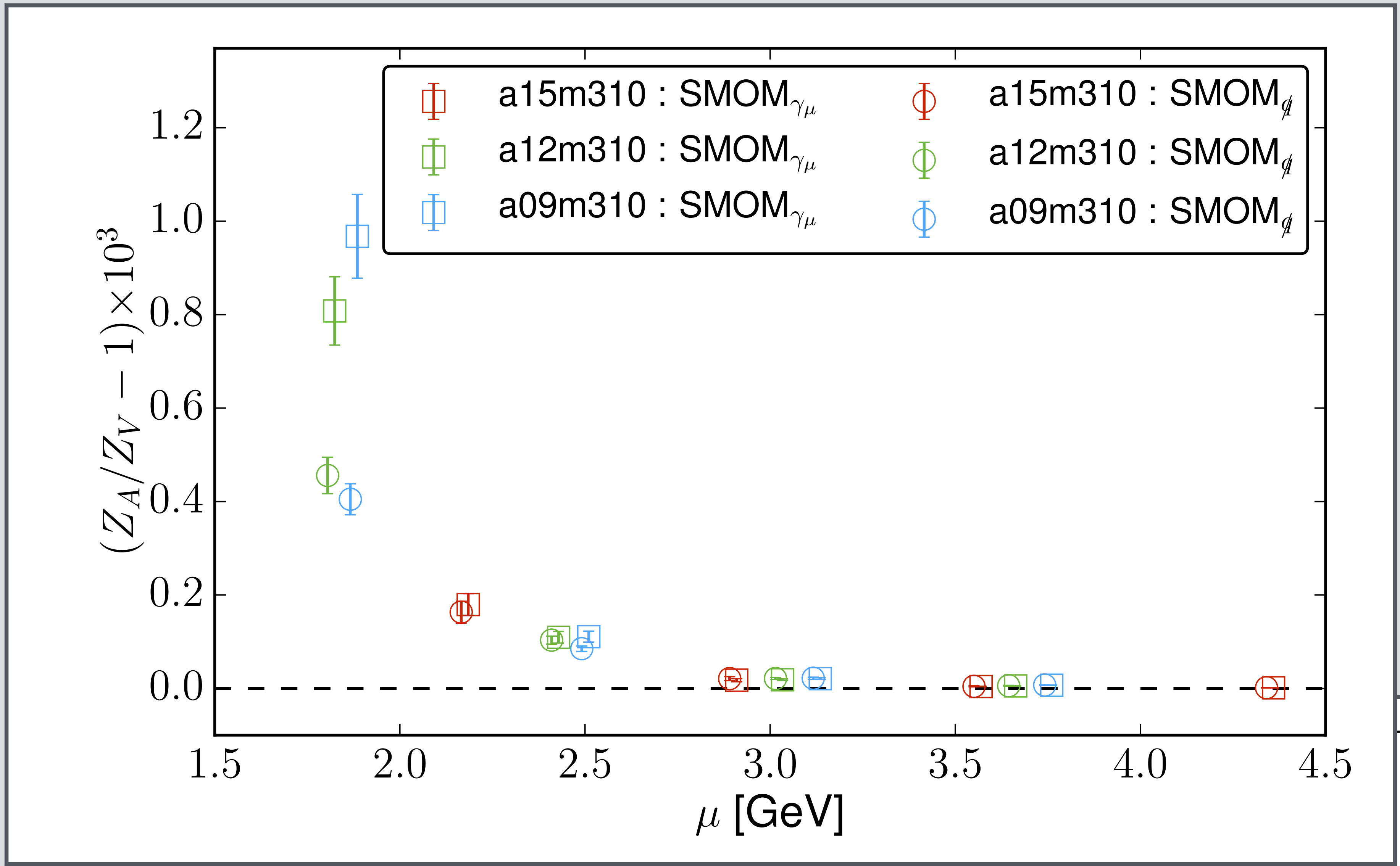
e.g. [0806.4549][0905.0466*][0912.4243][1301.1114]

*292,500 measurements

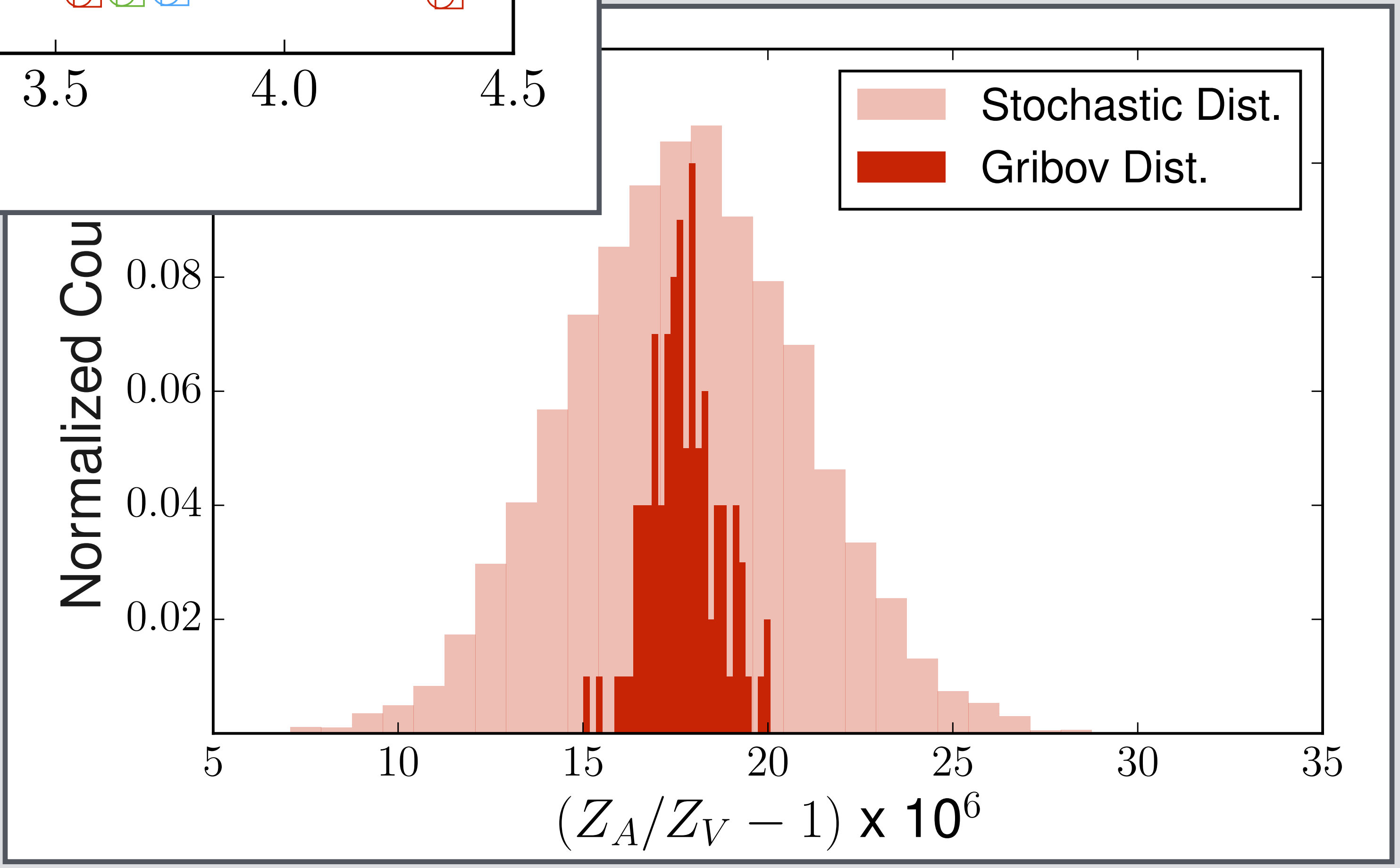
fit to exponentially cleaner signal



renormalization



$$\frac{Z_A}{Z_V}\frac{\mathring{g}_A}{\mathring{g}_V}$$



$Z_A / Z_V = 1$

@ one part in 10,000

chiral-continuum fit function

SU(2) NNLO baryon χPT

m_π^2 analytic	$g_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4$
non-analytic	$-\epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + g_0 c_3 \epsilon_\pi^3$
a^2 analytic	$a_2\epsilon_a^2 + b_4\epsilon_\pi^2\epsilon_a^2 + a_4\epsilon_a^4$
NLO FV	$(8/3)\epsilon_\pi^2[g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)]$

parameterize with

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi} \qquad \epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{\omega_0^2}$$

additional

$\mathrm{O}(m_{\mathrm{res}})$	$a_1 a/\omega_0$
gen. one-loop	$s_2\alpha_s\epsilon_a^2$

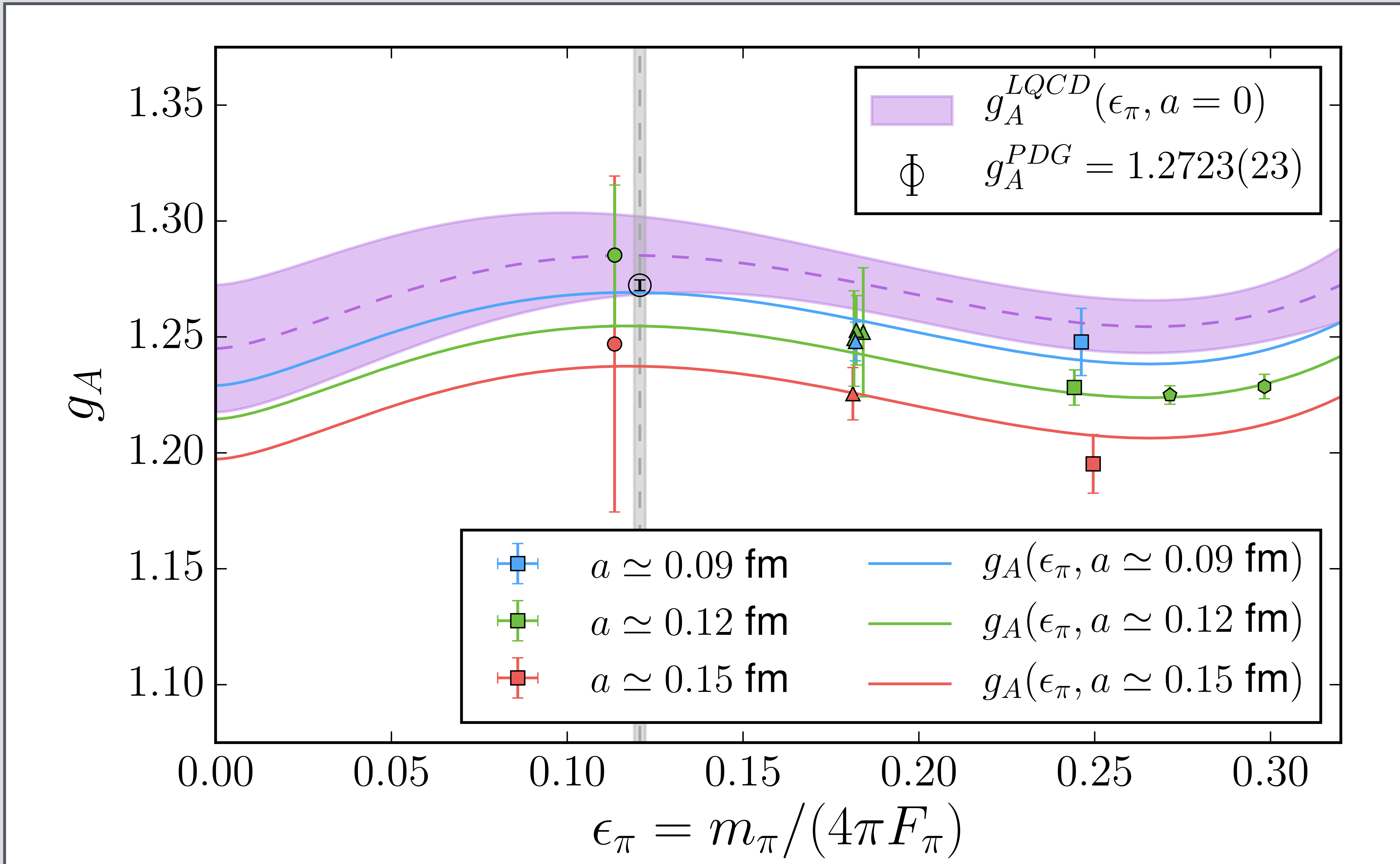
fit strategy

use Bayes Theorem as method of uncertainty quantification

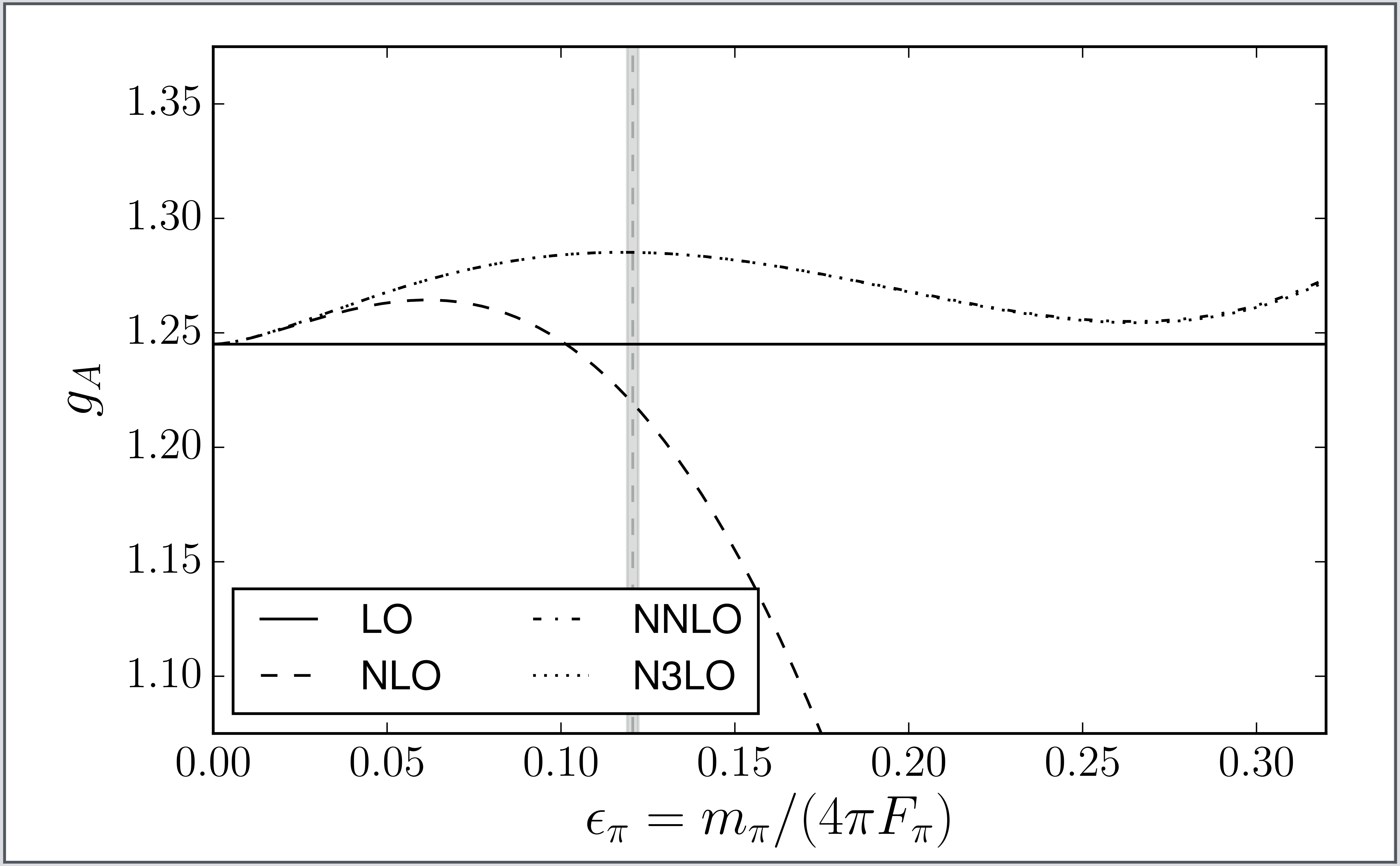
LEC priors motivated by χPT power counting

show stability of chiral-continuum extrapolation under varying models

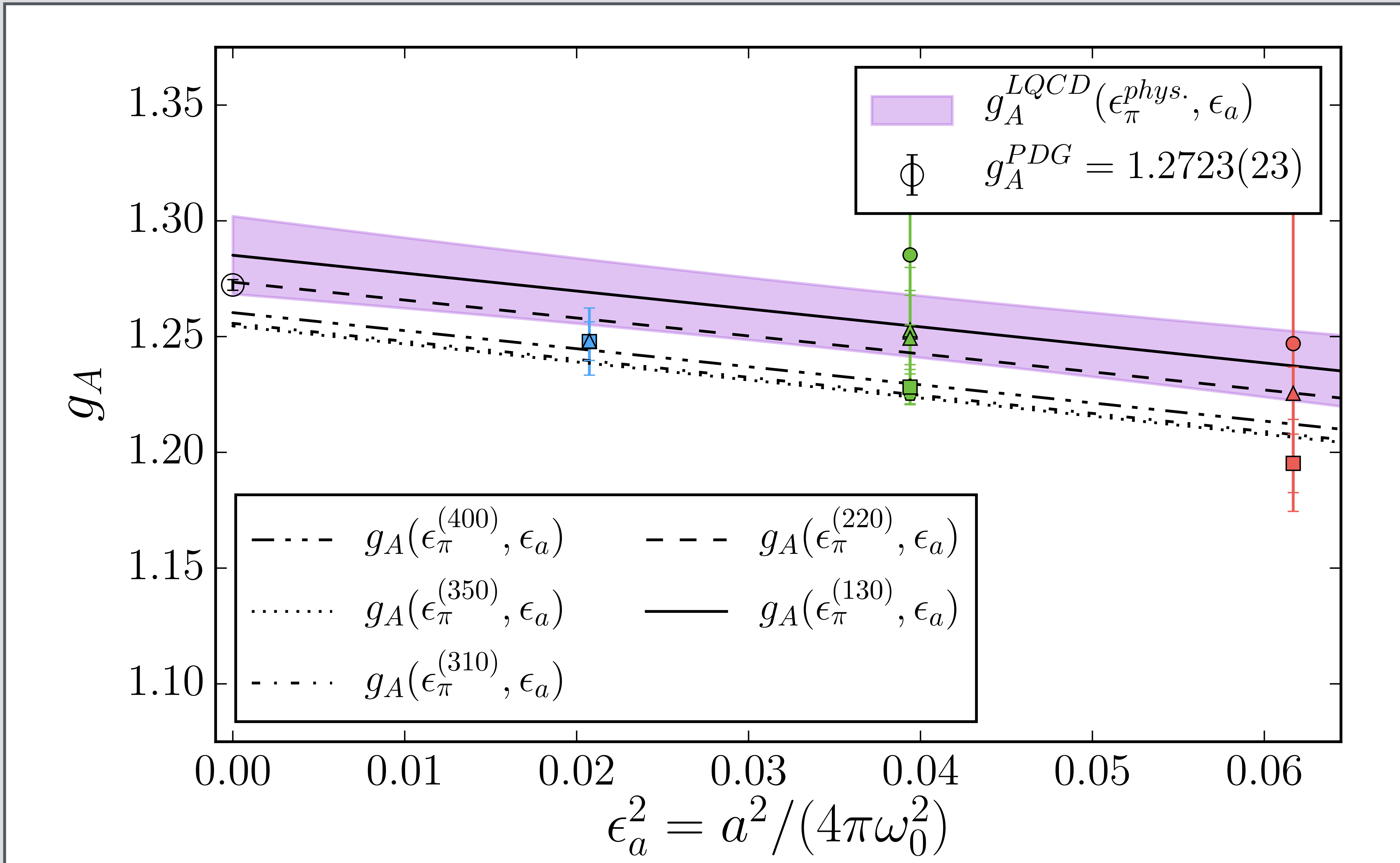
chiral-continuum extrapolation



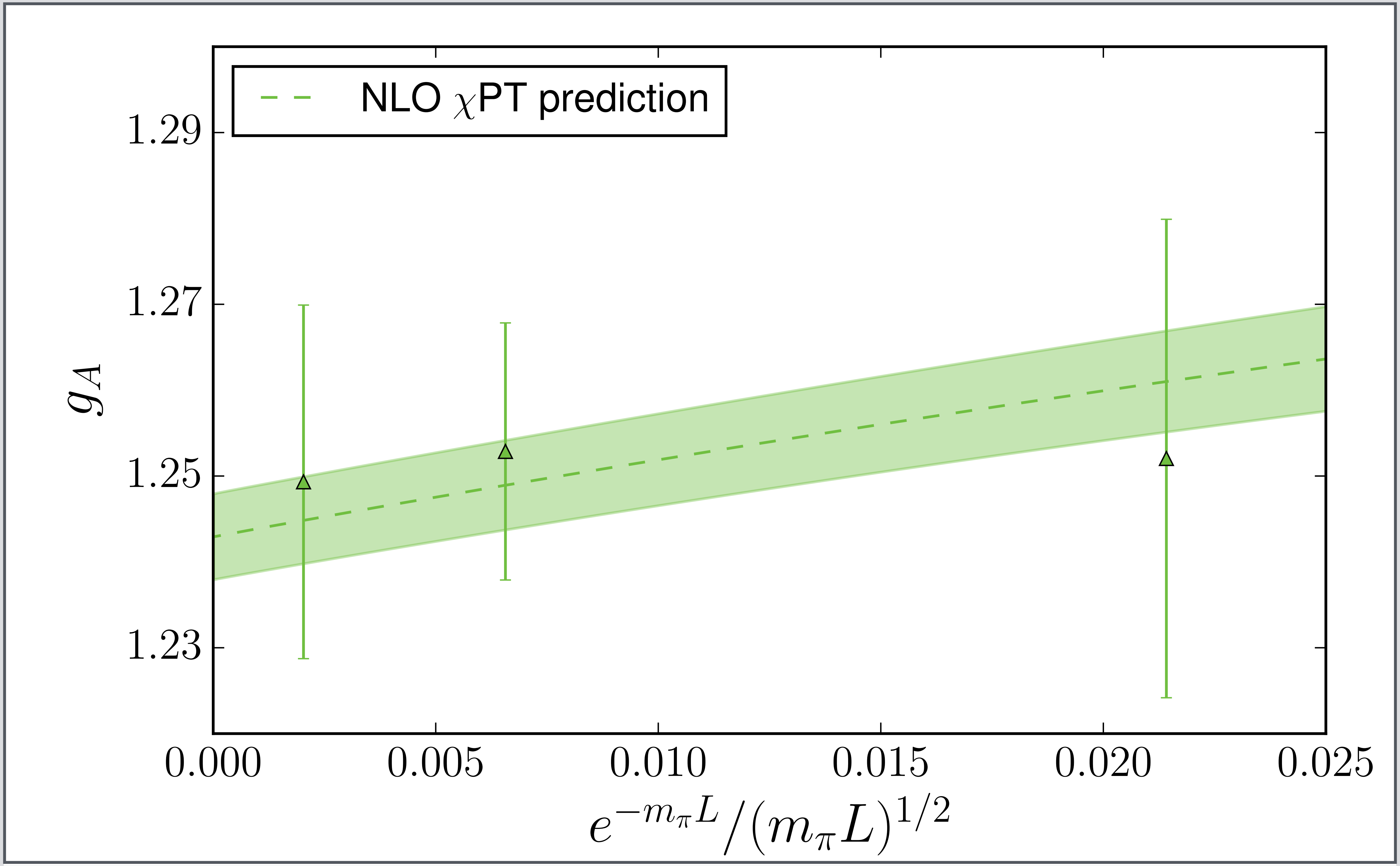
chiral fit convergence



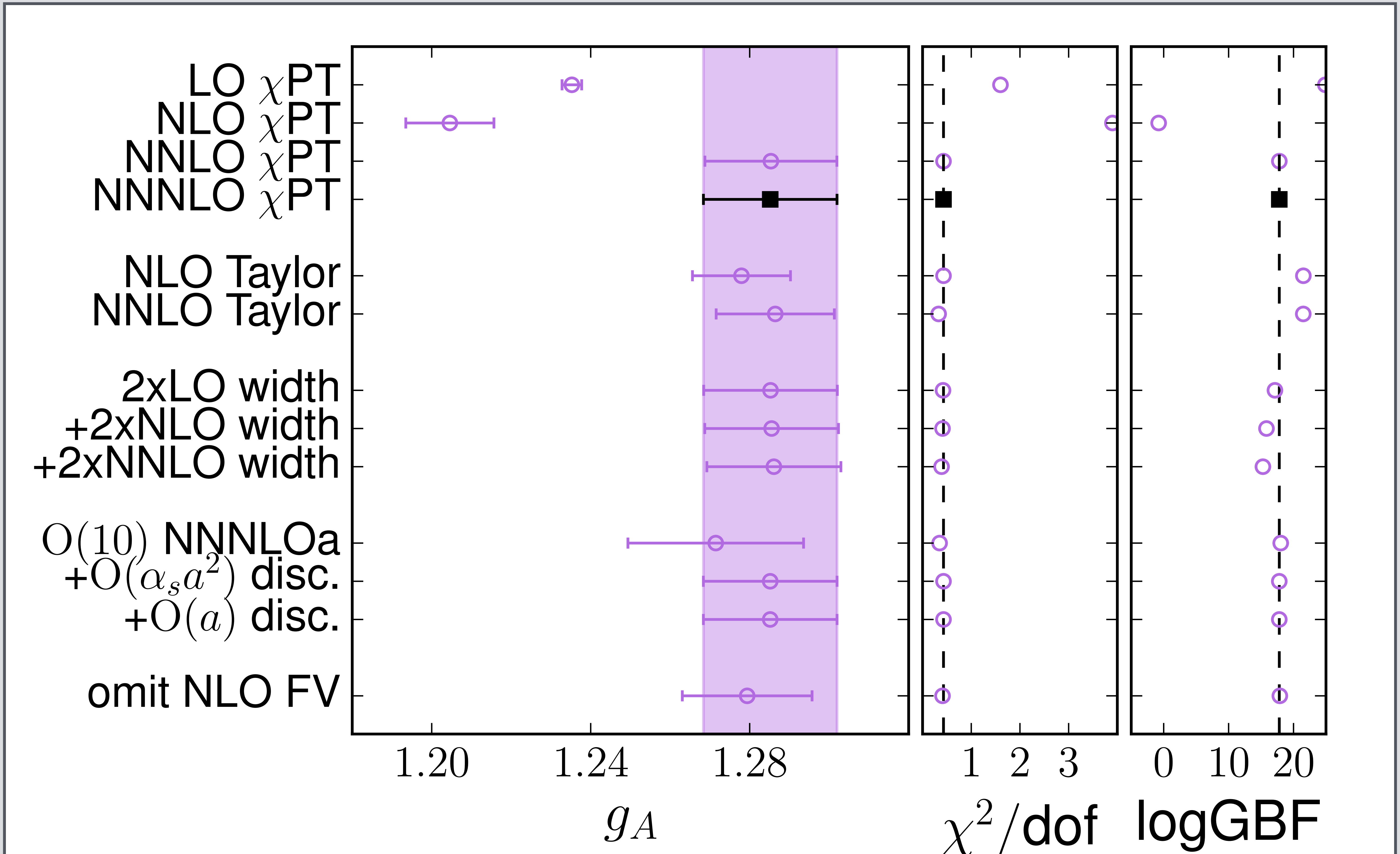
continuum extrapolation



NLO finite volume prediction



χPT stability plot



error budget

sources of uncertainty

statistical
g_A, g_V, m_π, PDG m_π & F_π

chiral extrapolation
c_4

continuum extrapolation
a_4, b_4, a/ω_0

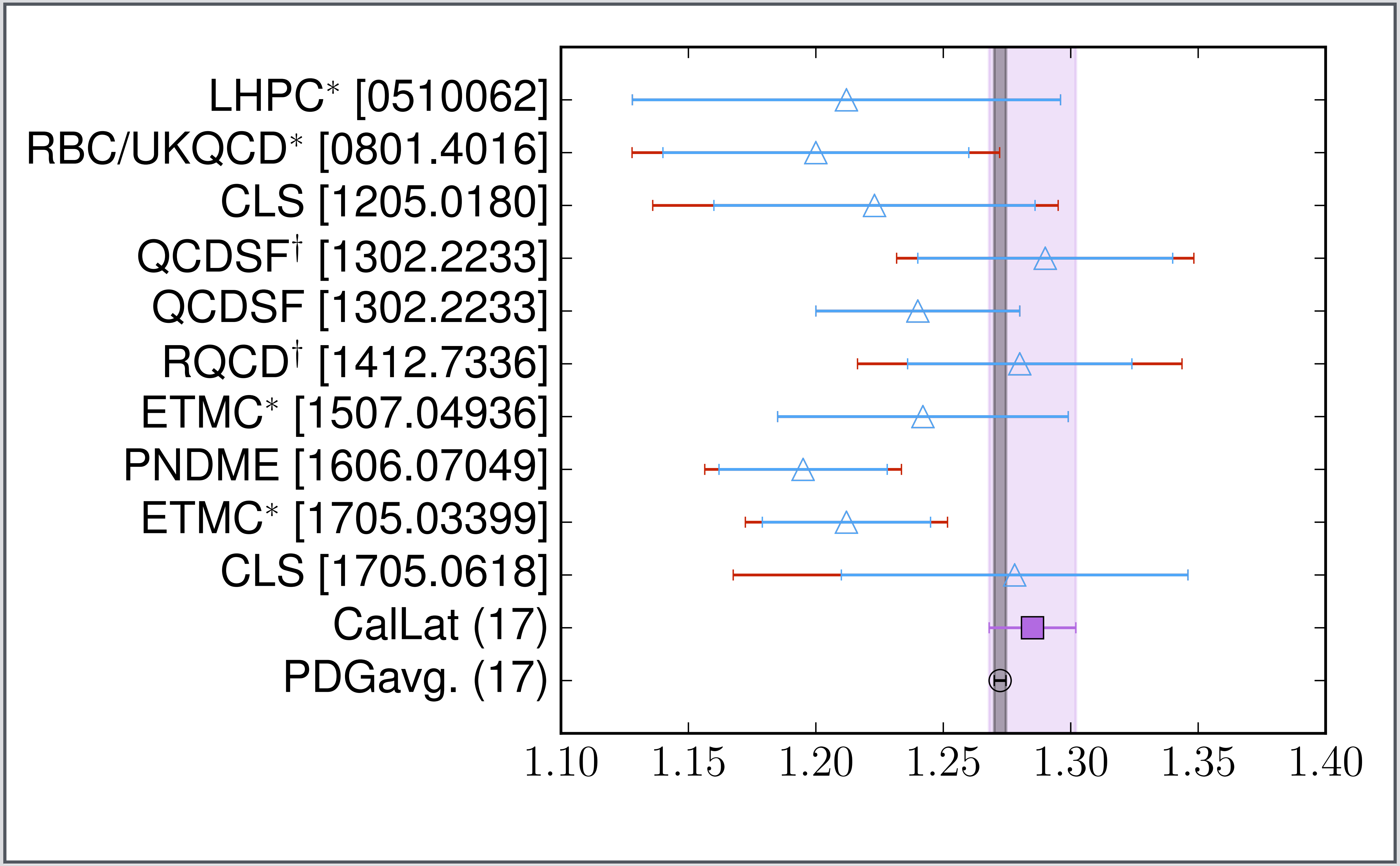
finite volume
half the difference between
NLO FV vs. no FV

isospin breaking
$(m_d - m_u)^2$ for isovector
$O(\alpha_{EM}/\pi)^2$ exp. result has radiative and recoil corrections

statistical	1.29%
chiral extrapolation	0.21%
continuum extrapolation	0.10%
infinite volume extrap.	0.23%
isospin breaking	0.04%
total	1.33%

final result
$g_A = 1.285(17)$

summary plot



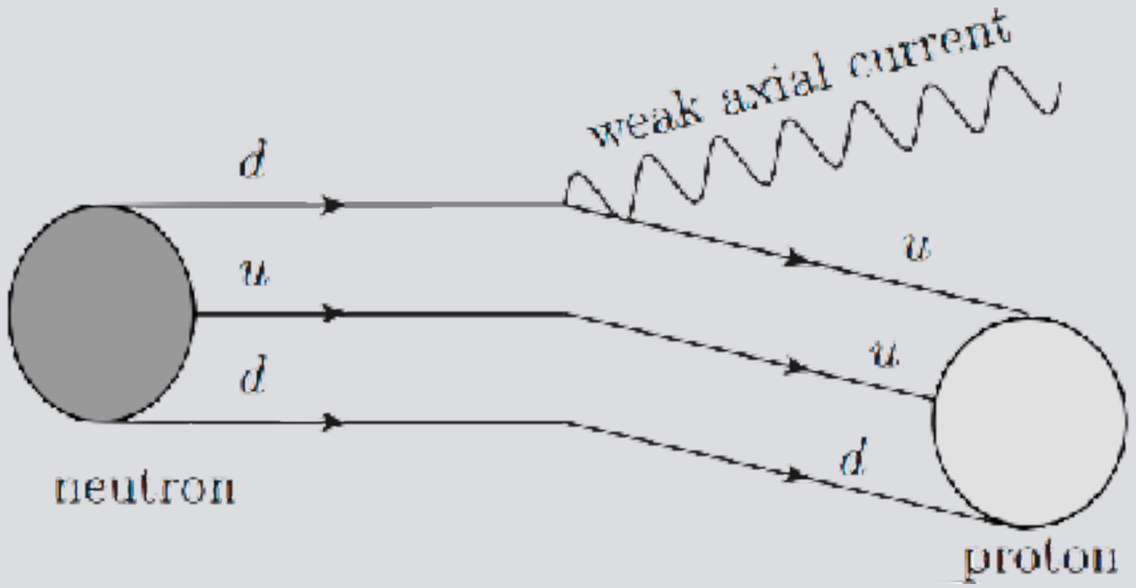
summary of recent arxiv'd papers of the nucleon axial coupling from lattice QCD.
apologies if I am missing any result. please let me know if I am missing a result.
results with a dagger denotes gA/Fpi extrapolation.
results with an asterisk are performed at one lattice spacing.

summary and outlook

current work

percent-level nucleon axial coupling
control over all systematic uncertainty

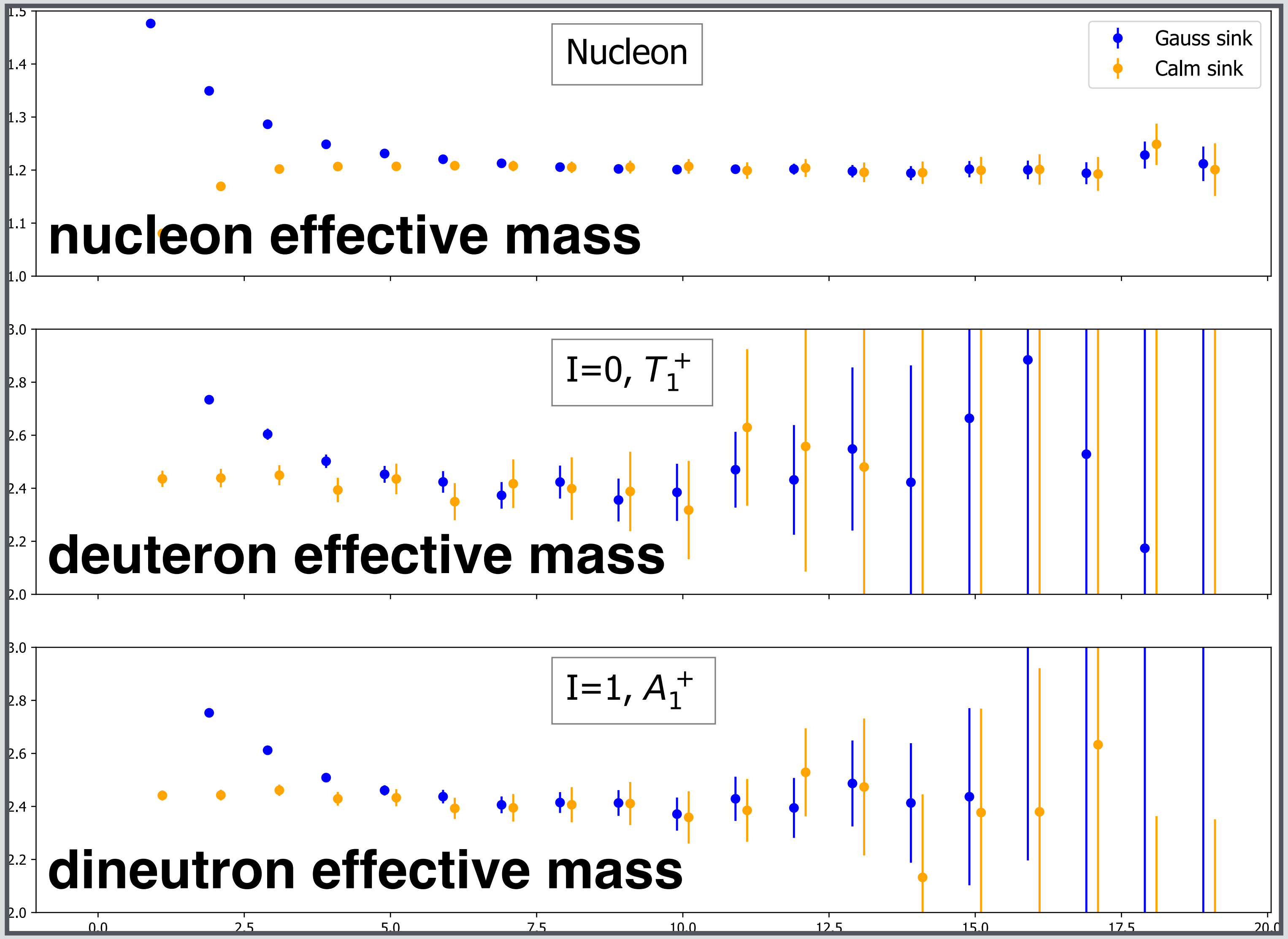
Feynman-Hellmann method
awesome ensembles (thanks MILC)
exp. suppressed mres (WF + Mobius)
exponentially precise corr. fit
(very) precise renormalization coeff.
phys. point extrap. is model indep.



future work

straightforward path towards:
- sub-percent gA
- semileptonic decays
- multi-current insertions
- other 1(+) baryon matrix elements

calm multi-baryon operators
Evan Berkowitz
Thur 17:50 Andalucía II



collaborators

Lattice2017

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Lawrence Livermore National Lab
Pavlos Vranas

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INCITE
Leadership Computing

Lawrence Livermore National Laboratory

supplemental

topological charge distribution