Multi-hadron spectroscopy in a large physical volume

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Can we do multi-hadron spectroscopy with sufficient precision in large physical volumes?

- want to perform Lüscher-type analysis to obtain scattering amplitudes
- need to resolve small energy shifts (near threshold, large volume)
  $$E_{2\pi} - 2m_\pi = \frac{4\pi a_0}{m_\pi L^3} + \mathcal{O}\left(L^{-4}\right)$$
- exponential finite-volume effects have to be negligible
- increasingly dense finite-volume spectrum
  $\Rightarrow$ many data points to constrain scattering amplitude
- volume-average vs. vanishing signal

TRY TO ANSWER ONE QUESTION

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Can we do multi-hadron spectroscopy with sufficient precision in large physical volumes?

The short answer is: yes (at least for mesons).

- all-to-all propagators required for multi-hadron interpolators with definite constituent momenta
- two key insights to treat all-to-all quark propagation:
  1. important physics is captured by a low-dimensional subspace
     $\rightarrow$ *distillation*
  2. achievable overall precision is limited by finite sampling of the path integral
     $\rightarrow$ stochastic estimators

- important contributions to the quark propagator are encoded in smaller subspace [Peardon et al ’08]

- spanned by $N_{\rm ev} \ll 12 \times N_s^3$ eigenvectors of covariant 3D Laplace operator

$$-\Delta v_n = \lambda_n v_n$$

- can be viewed as a low-pass filter – but for constant physical smearing

$$N_{\rm inv} \propto N_{\rm ev} \sim V$$

- use noisy estimation in the low-dimensional subspace [Morningstar et al '11]

- for random noise vectors $\eta_i^{(r)} \in Z_4, \quad i = 1, \ldots, N_{\rm ev}$

$$M'^{-1}_{ij} = \lim_{N_\eta \to \infty} \frac{1}{N_\eta} \sum_{r=1}^{N_\eta} X_i^{(r)} \eta_j^{(r)*}, \quad \text{where } M' X^{(r)} = \eta^{(r)}$$

- variance reduction using noise dilution [Foley et al '05]

$$\begin{pmatrix} 1 \\ \mathrm{i} \\ -1 \\ -1 \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \\ \vdots \end{pmatrix} + \begin{pmatrix} 0 \\ \mathrm{i} \\ 0 \\ -1 \\ \vdots \end{pmatrix}$$

$$\Rightarrow N_{\rm inv} \propto N_{\rm dil} \cdot N_\eta$$

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Breaking down our question

1. *What is the interplay between N_{dil}, N_η and V?*
2. *Can we get reasonably close to the distillation limit in a large volume?*
3. *Is that level of precision sufficient for a Lüscher-type analysis?*

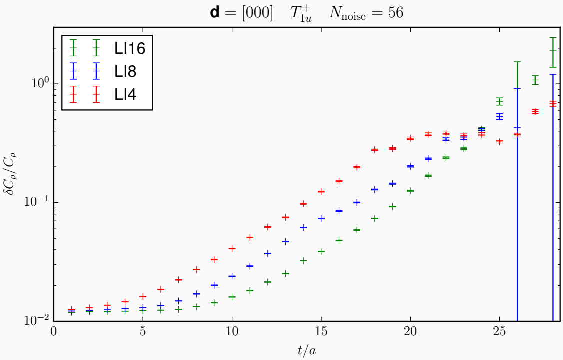
- **D101** (128×64^3) (CLS $N_\mathrm{f} = 2 + 1$) [Bruno et al '15, '17]

a [fm]	m_π [MeV]	$m_\pi L$	N_cfg
0.086	220	6.2	132

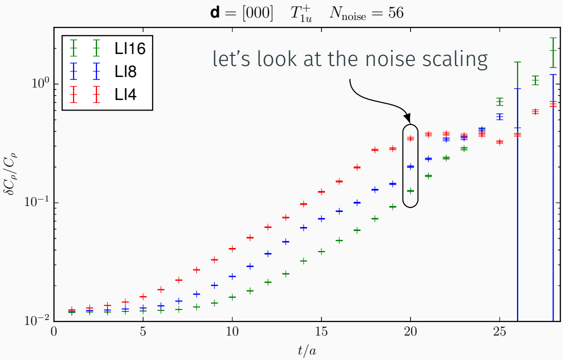
- stochastic LapH setup
  - $N_\mathrm{ev} = 928$
  - dilution schemes [Morningstar et al '11]
    fixed quark lines (TF, SF, LI16)
    relative quark lines (TI8, SF, LI16)
  - $N_{t_0} = 3$ $(t_0/a = 24, 44, 64)$
- check of volume scaling: **C101** (96×48^3)

$\Rightarrow N_\mathrm{inv} = 6 \cdot 3 \cdot 64 + 2 \cdot 8 \cdot 64 = 2\,176$ (exact distillation: $222\,720$)
per configuration

ρ-correlator: noise-to-Signal



- meson: $N_\eta \cdot (N_\eta - 1) = 56$ noise choices to evaluate correlator
- error bars from resampling in available noise combinations
  $\rightarrow$ not quite trustworthy at large times?!
- signal-to-noise problem of the ρ



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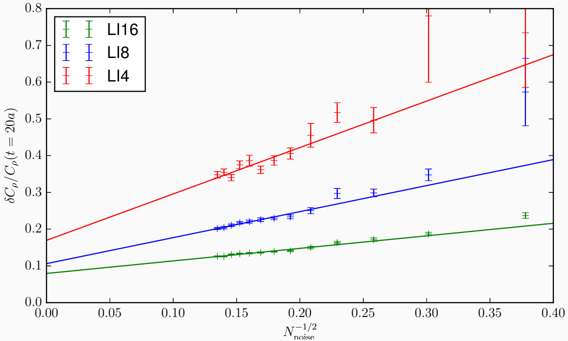
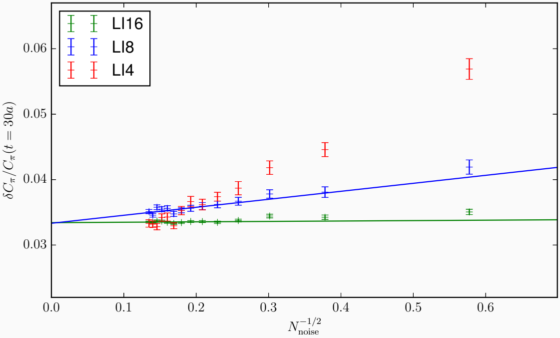
ρ-CORRELATOR: NOISE SCALING



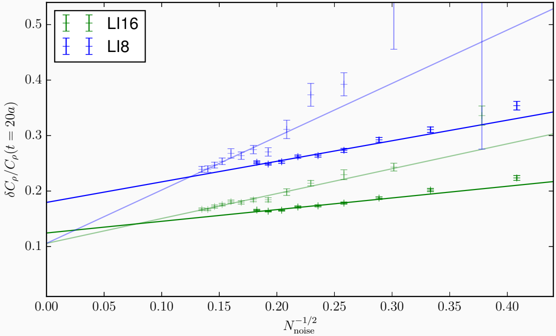
- we do not know the distillation result, but extrapolation in $1/\sqrt{N}$ more or less consistent

  (interestingly enough – measurements are not independent)
- smaller slope with more dilution $\Leftrightarrow$ dilution reduces variance
- level-interlace 16 gets reasonably close



- LI16 noise scaling almost flat beyond a handful of noise combinations
- possible metric to determine the ‘optimal’ N_{dil}?

ρ-CORRELATOR: VOLUME DEPENDENCE OF NOISE SCALING



- smaller volume (C101, thick lines) vs. larger volume (D101)
- no catastrophic breakdown of precision
- factor two improvement from LI8 $\rightarrow$ LI16 for fixed $N_{\rm noise}$, LI16 looks 'optimal'

- ✓ *What is the interplay between N_{dil}, N_η and V?*
- ✓ *Can we get reasonably close to the distillation limit in a large volume?*

3. *Is that level of precision sufficient for a Lüscher-type analysis?*

A Lüscher analysis

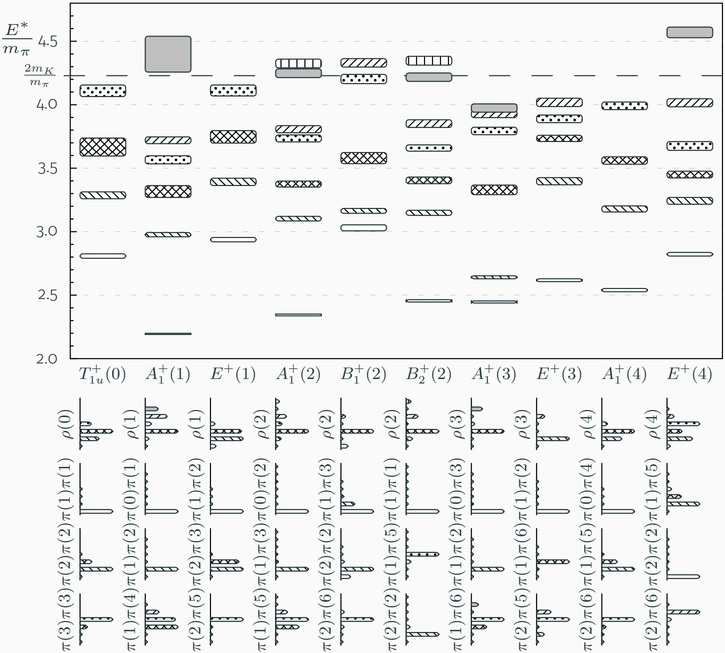
- ρ-resonance (isovector vector $\pi\pi$ scattering) as a testbed

  Toward pion-pion scattering amplitudes with controlled systematic errors

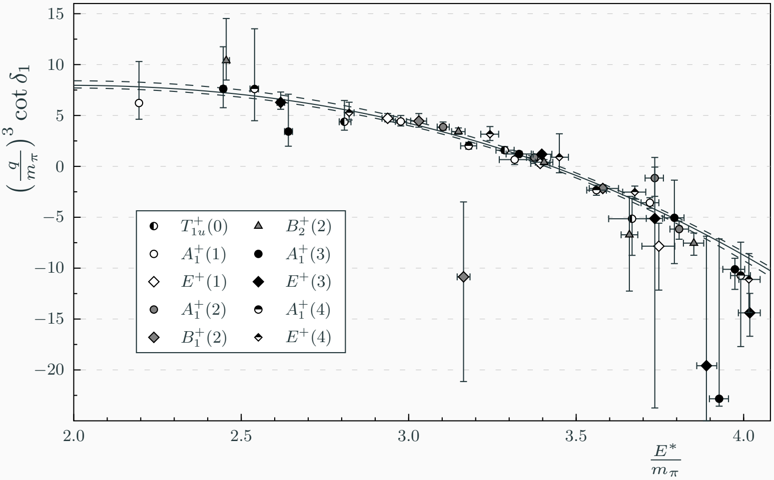
  [J. Bulava, Mon 17.20]

- analysis strategy in a nutshell [Bulava et al '16]
  - GEVP at fixed diagonalization times
  - various fit models: ratio fit, single-exponential fit
  - assessment of fitting-range systematics
  - neglect partial wave mixing in Lüscher analysis, but cf.

  Scattering from finite-volume energies including higher partial waves and multiple decay channels [C. Morningstar, Thu 17.30]

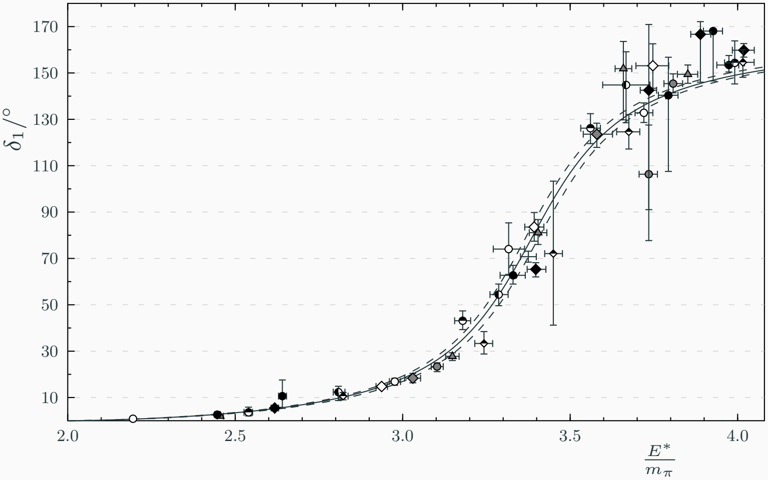


(not all interpolators shown)



Breit-Wigner: $m_r/m_\pi = 3.43(2), \quad g_{\rho\pi\pi} = 6.05(11), \quad \chi^2/\text{dof} = 0.90$

Isovector $\pi\pi$ scattering amplitude II



keep in mind: $N_{\text{cfg}} = 132$ $N_{\text{ev}} = 928$ $N_{\text{dil}} = 16$ $N_{\text{noise}} = 9$

Can we do multi-hadron spectroscopy with sufficient precision in large physical volumes?

- taken first step to better understand dilution
  - level-interlace 16 good choice in large volumes
  - noise average with noise recycling still beneficial

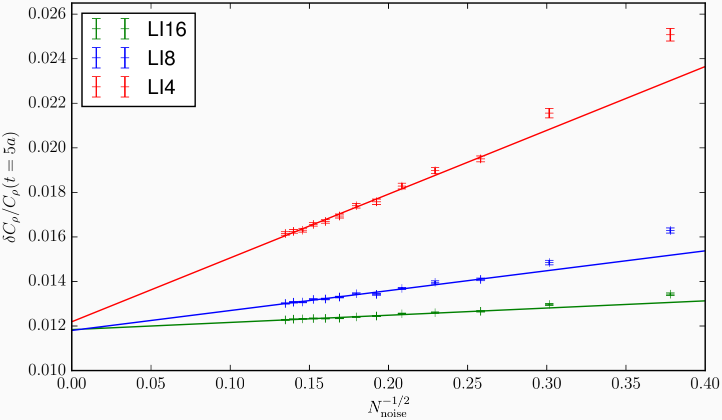
- next steps:
  - other observables, e.g. baryons
  - investigate dilution schemes other than interlacing

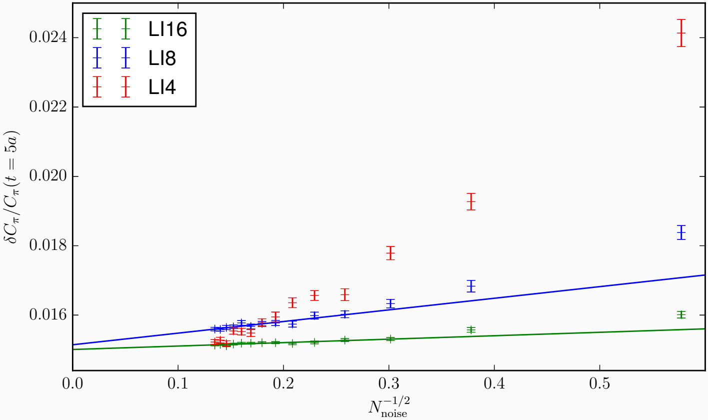
- practical point of view: can exploit volume average with existing methods
  - ongoing work on precise determination of ρ properties

[C. Andersen, J. Bulava, C. Morningstar, BH]

- encouraging for multi-hadron spectroscopy with close-to-physical pion mass
  - $m_\pi L = 4$ at physical pion mass $\Rightarrow N_{\mathrm{ev}} \approx 1000$ – we are right there!

ρ-CORRELATOR: NOISE SCALING AT EARLIER TIME




LI16
LI8
LI4
0.024
0.022
0.020
0.018
0.016
$\delta C_\pi / C_\pi(t = 5a)$
0.0
0.1
0.2
0.3
0.4
0.5
$N_{\mathrm{noise}}^{-1/2}$

Computational cost per config

stochastic LapH measurements:

- inversions: $120\,\mathrm{kch}$ [total: $16\,\mathrm{Mch}$] (BlueGene/Q)
- meson function construction: $8\,\mathrm{kch}$ [total: $1\,\mathrm{Mch}$] (Jureca)
- contractions negligible

compare to gauge generation:

- $170\,\mathrm{kch}$ per configuration
- we have measured on every other configuration of the chain
  $\Rightarrow$ effective cost of gauge generation $340\,\mathrm{kch}$ per configuration
- reasonable balancing of cost considering re-usability of quark propagators!