Semileptonic B-meson decays to light pseudoscalar mesons on the HISQ ensembles

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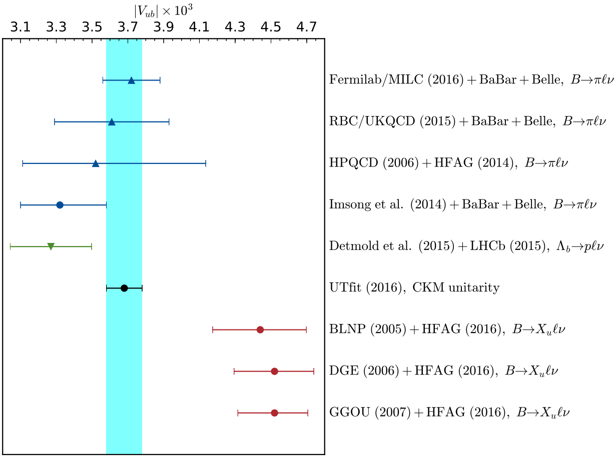
Overview

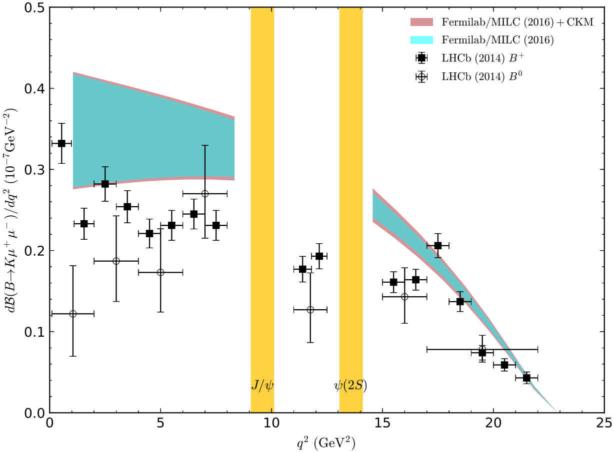
1. Introduction
   - a. Motivations
   - b. Standard Model decay rates: $B \to \pi l \nu$, $B_s \to K l \nu$, $B \to K l^+ l^-$
   - c. Form factor definitions
2. Analysis
   - a. Actions and parameters
   - b. Correlator functions
   - c. Form factor fits
3. Outlook
   - a. Form factors in the chiral-continuum limit

Status of $|V_{ub}|$



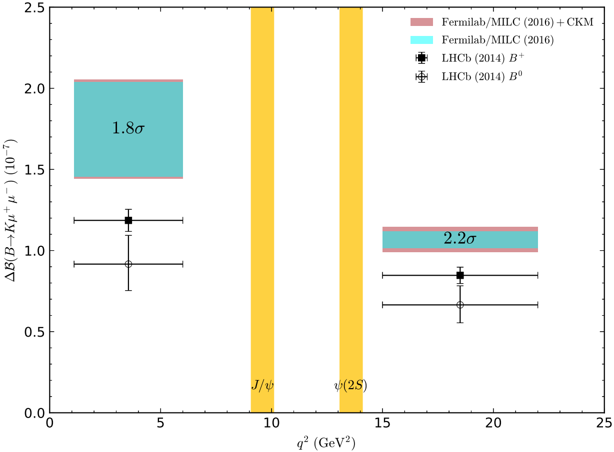
Update of plot in arXiv:1503.07839

Tension in $B \to K\mu^+\mu^-$



Older, less precise experiments omitted; cf. arXiv:1510.02349

Tension in $B \to K\mu^+\mu^-$



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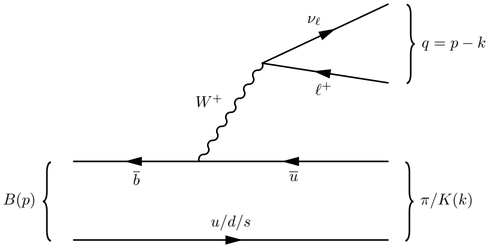
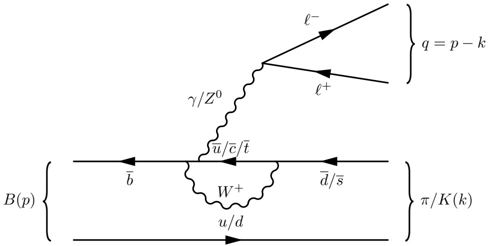
Charged-current semileptonic decay



$$\frac{d\Gamma}{dq^2} = \frac{C_P G_F^2 |V_{ub}|^2}{3\,(2\pi)^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 |\boldsymbol{k}| \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) |\boldsymbol{k}|^2 \left| f_+(q^2) \right|^2 + \frac{3m_\ell^2 \left(M_B^2 - M_P^2\right)^2}{8q^2 M_B^2} \left| f_0(q^2) \right|^2 \right],$$

where $B = B,\ B_s$ and $P = \pi,\ K$.

Flavor-changing neutral-current transition



$$\frac{d\Gamma}{dq^2} = \frac{C_P G_F^2 \alpha^2 |V_{tb} V_{tq}^*|^2}{4\,(2\pi)^5} \beta_\ell |\boldsymbol{k}| \left[\frac{2}{3} \beta_\ell^2 |\boldsymbol{k}|^2 \left| C_{10}^{\text{eff}} f_+(q^2) \right|^2 + \frac{m_\ell^2 \left(M_B^2 - M_P^2\right)^2}{q^2 M_B^2} \left| C_{10}^{\text{eff}} f_0(q^2) \right|^2 \right.$$

$$\left. + \left(1 - \frac{1}{3}\beta_\ell^2\right) |\boldsymbol{k}|^2 \left| C_9^{\text{eff}} f_+(q^2) + 2 C_7^{\text{eff}} \frac{m_b + m_q}{M_B + M_P} f_T(q^2) \right|^2 \right],$$

where $q = d,\ s$; C_i^{eff} are Wilson coefficients; and $\beta_\ell^2 = 1 - 4m_\ell^2/q^2$.

Form factors I

These transitions can be mediated by vector, scalar, and tensor currents.

Taking Lorentz and discrete symmetries into account:

$$\langle P(k)|\mathcal{V}^\mu|B(p)\rangle = f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2 - M_P^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_P^2}{q^2}q^\mu$$

$$= \sqrt{2M_B}\left[p_\perp^\mu f_\perp(E_P) + v^\mu f_\parallel(E_P)\right], \quad v = p/M_B$$

$$\langle P(k)|\mathcal{S}|B(p)\rangle = f_0(q^2)\frac{M_B^2 - M_P^2}{m_b - m_q}$$

$$\langle P(k)|\mathcal{T}^{\mu\nu}|B(p)\rangle = f_T(q^2)\frac{2}{M_B + M_P}\left(p^\mu k^\nu - p^\nu k^\mu\right)$$

PCVC ensures that the vector and scalar currents lead to the same f_0.

Form factors II

It is straightforward to extract the matrix elements

$$f_\perp(E_P) = \frac{\langle P|\mathcal{V}^i|B\rangle}{\sqrt{2M_B}} \frac{1}{k^i}$$

$$f_\parallel(E_P) = \frac{\langle P|\mathcal{V}^0|B\rangle}{\sqrt{2M_B}}$$

$$f_T(E_P) = \frac{M_B + M_P}{\sqrt{2M_B}} \frac{\langle P|\mathcal{T}^{0i}|B\rangle}{\sqrt{2M_B}} \frac{1}{k^i}$$

from three-point correlation functions.

Then f_+ and f_0 are linear combinations of $f_\perp$ and $f_\parallel$.

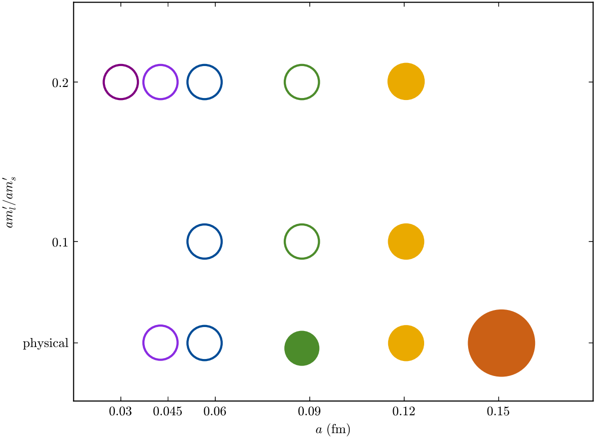
Actions and parameters

- MILC $N_f = 2+1+1$ ensembles
- Lüscher-Weisz gauge action $\rightarrow O(\alpha_s^2 a^2)$
- HISQ action for $q_l, s, c \rightarrow O(\alpha_s a^2)$
- Clover action with Fermilab interpretation for $b \rightarrow O(\alpha_s a, a^2)\, f\left((m_b a)^2\right)$
- $M_\pi L \gtrsim 3.6$

a (fm)	0.1509(14)	0.1206(14)	0.1206(11)	0.1206(11)	0.08750(80)
$N_{\text{cfg}} \times N_{\text{src}}$	3630×8	1053×8	1000×8	986×8	925×8
$N_s^3 \times N_4$	$32^3 \times 48$	$24^3 \times 64$	$32^3 \times 64$	$48^3 \times 64$	$64^3 \times 96$
am_l'	0.00235	0.0102	0.00507	0.00184	0.0012
am_s'	0.0647	0.0509	0.0507	0.0507	0.0363
am_c'	0.831	0.635	0.628	0.628	0.432
κ_b'	0.07732	0.08574	0.08574	0.08574	0.09569
w_0/a	1.1468(3)	1.3835(7)	1.4047(6)	1.4168(10)	1.9473(11)
$\alpha_V(2/a)$	0.45275	0.38138	0.38138	0.38138	0.31391

Ensembles

Sample sizes at a glance (area $\propto$ number of samples):



Filled circles: this work; open circles: other HISQ ensembles (future work).

Correlation functions and effective masses

For any pseudoscalar meson P

$$C_2(t;\boldsymbol{k}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left\langle \mathcal{O}_P(0,\mathbf{0})\, \mathcal{O}_P^\dagger(t,\boldsymbol{x}) \right\rangle$$

$$= \sum_m (-1)^{m(t+1)} \frac{\left|\langle 0|\mathcal{O}_P|P^{(m)}\rangle\right|^2}{2E_P^{(m)}} e^{-E_P^{(m)}t}$$

$$aM_{\text{eff}}(t;\boldsymbol{k}) = \cosh^{-1}\left(\frac{C_2(t+1;\boldsymbol{k}) + C_2(t-1;\boldsymbol{k})}{2C_2(t;\boldsymbol{k})}\right)$$

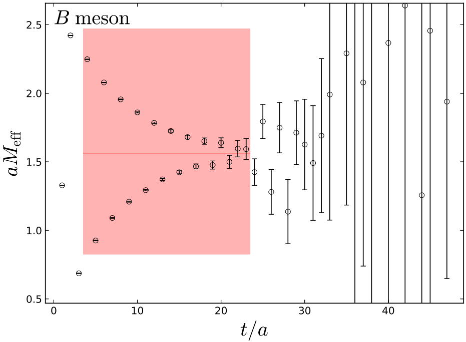
Suppress oscillating states characteristic of staggered fermions:

$$\overline{C}_2(t;\boldsymbol{k}) \equiv \frac{e^{-M_P^{(0)}t}}{4}\left[\frac{C_2(t;\boldsymbol{k})}{e^{-M_P^{(0)}t}} + \frac{2C_2(t+1;\boldsymbol{k})}{e^{-M_P^{(0)}(t+1)}} + \frac{C_2(t+2;\boldsymbol{k})}{e^{-M_P^{(0)}(t+2)}}\right]$$

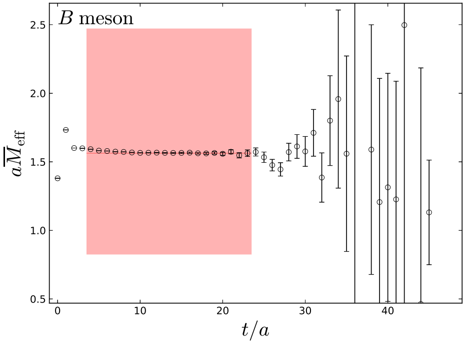
For corresponding effective mass with $C_2 \to \overline{C}_2$, write $a\overline{M}_{\text{eff}}$.

Correlation functions and effective masses

$B \to \pi$, $a \approx 0.15$ fm, $m_l/m_s =$ phys



oscillating states evident

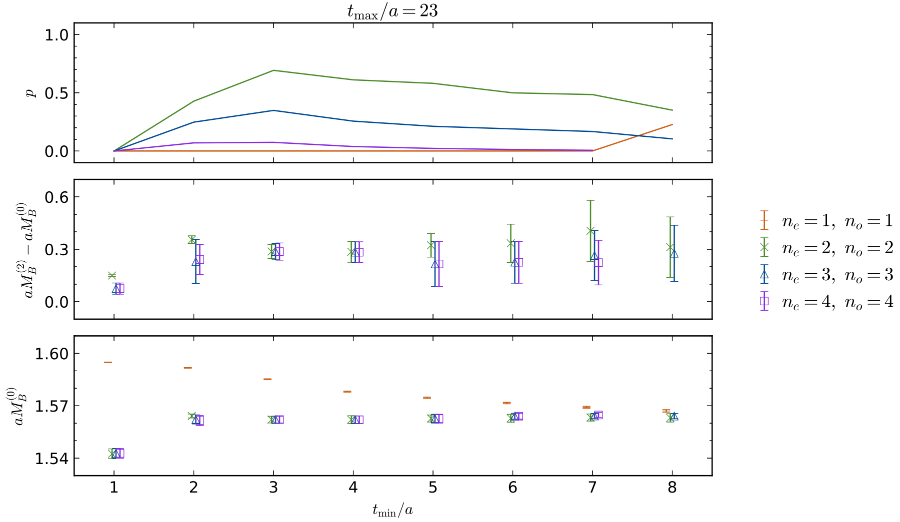


oscillating states suppressed

- Averaging procedure is effective at suppressing oscillations
- $\overline{C}_2$ and $\overline{M}_{\mathrm{eff}}(t)$ provide natural prior for the ground-state energy $E^{(0)}$; we take $\tilde{E}^{(0)} = \overline{M}_{\mathrm{eff}} \pm \frac{1}{2}\overline{M}_{\mathrm{eff}}$.

Stability of masses under variations in t_{fit} and N_{states}

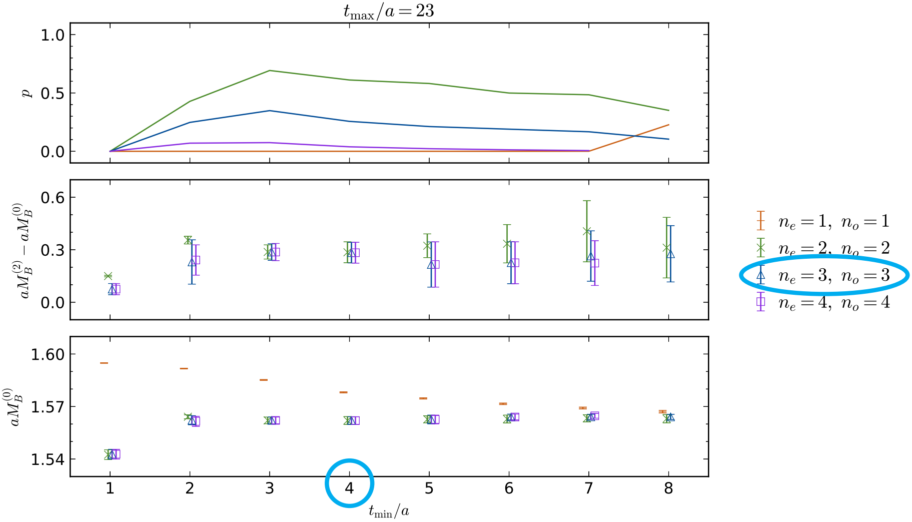
$B \to \pi$, $a \approx 0.15$ fm, $m_l/m_s =$ phys



p as defined in arXiv:1602.03560

Stability of masses under variations in $t_{\rm fit}$ and $N_{\rm states}$

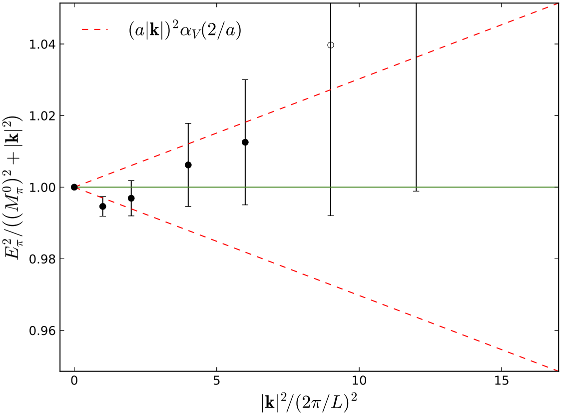
$B \to \pi$, $a \approx 0.15$ fm, m_l/m_s = phys



p as defined in arXiv:1602.03560

Dispersion relation tests

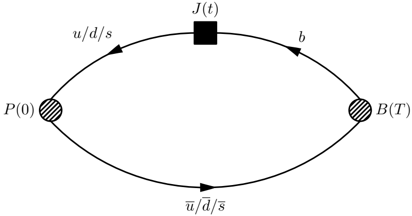
$B \to \pi$, $a \approx 0.15$ fm, $m_l/m_s =$ phys



$\Rightarrow$ Replace $E_P^{(0)}$ with $\mathcal{E}_P^{(0)} = \sqrt{\left(M_P^{(0)}\right)^2 + |\boldsymbol{k}|^2}$ in form factor fits

Three-point correlators

$$C_3^{\mu(\nu)}(t,T;\boldsymbol{k}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} \left\langle \mathcal{O}_P(0,\boldsymbol{0})\, J^{\mu(\nu)}(t,\boldsymbol{y})\, \mathcal{O}_B^\dagger(T,\boldsymbol{x}) \right\rangle$$



$$\overline{C}_3^{\mu(\nu)}(t,T;\boldsymbol{k}) \equiv \frac{e^{-E_P^{(0)}t}e^{-M_B^{(0)}(T-t)}}{8}\left[\frac{C_3^{\mu(\nu)}(t,T;\boldsymbol{k})}{e^{-E_P^{(0)}t}e^{-M_B^{(0)}(T-t)}} + \frac{2C_3^{\mu(\nu)}(t+1,T;\boldsymbol{k})}{e^{-E_P^{(0)}(t+1)}e^{-M_B^{(0)}(T-t-1)}} + \frac{C_3^{\mu(\nu)}(t+2,T;\boldsymbol{k})}{e^{-E_P^{(0)}(t+2)}e^{-M_B^{(0)}(T-t-2)}} + \{T\rightarrow T+1\}\right]$$

Three-point function fits

Form ratio from $\overline{C}_3^{\mu(\nu)}$, $\overline{C}_2$, and results of two-point fits:

$$\overline{R}^{\mu(\nu)} \equiv \frac{\overline{C}_3^{\mu(\nu)}(t,T;\boldsymbol{k})}{\sqrt{\overline{C}_{2,P}(t;\boldsymbol{k})\overline{C}_{2,B}(T-t;\boldsymbol{0})}}\sqrt{\frac{2\mathcal{E}_P^{(0)}}{e^{-\mathcal{E}_P^{(0)}t}e^{-M_B^{(0)}(T-t)}}} + O(\Delta M_P^2, \Delta M_P \Delta M_B, \Delta M_B^2)$$

$$\overline{R}^{\mu(\nu)} \stackrel{\text{fit}}{=} F^{\mu(\nu)}\left[1 - F_P e^{-\Delta M_P t} - F_B e^{-\Delta M_B (T-t)} - F_{\delta m} e^{-\delta m\, t}\right],$$

where δm accommodates (very small) differences between $\mathcal{E}_P^{(0)}$ and $E_P^{(0)}$.

$$f_\perp(E_P) = Z_\perp \frac{F^i(\boldsymbol{k})}{k^i}$$

$$f_\parallel(E_P) = Z_\parallel F^4(\boldsymbol{k})$$

$$f_T(E_P) = Z_T \frac{M_B + M_P}{\sqrt{2M_B}} \frac{F^{4i}(\boldsymbol{k})}{k^i}$$

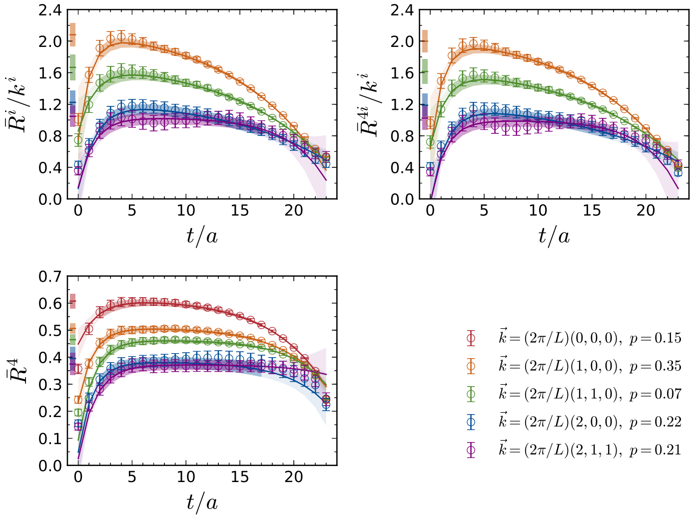
Mostly nonperturbative matching:

$$Z_J = (Z_{V_{bb}^4} Z_{V_{qq}^4})^{1/2} \rho_J;$$

apply blinding factor here.

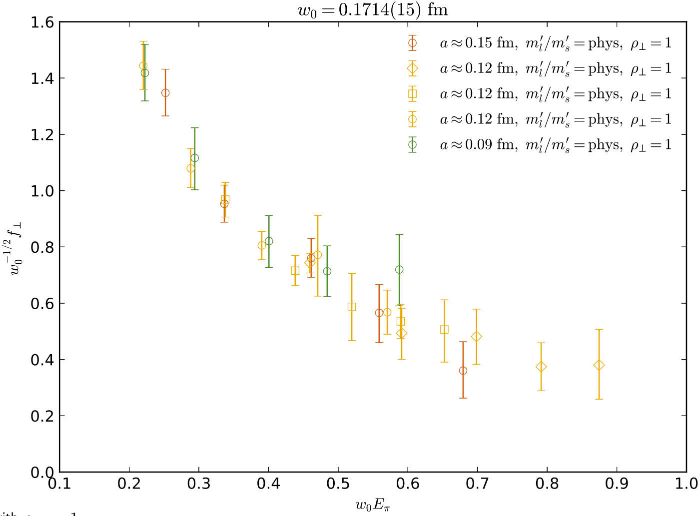
Form factor fits

$B \to \pi$, $a \approx 0.15$ fm, m_l/m_s = phys



Form factors (preliminary, with $\rho_J = 1$)

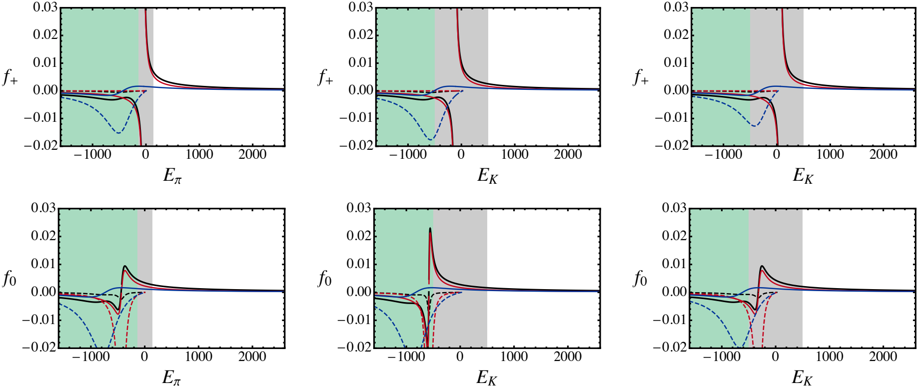
$B \to \pi$, $a \approx 0.15$ fm, m_l/m_s = phys



PRELIMINARY, with $\rho_J = 1$

Pololgy

Poles and cuts for νl scattering kinematics influence form factor everywhere:



When poles are in or near gray region, build them into fits of E_P dependence.

Chiral-continuum extrapolation

NLO $SU(2)$ hard-meson χPT description of form factors:

$$\frac{1}{\sqrt{w_0}} f_\perp = \frac{g_\pi}{w_0^2 f_\pi (E_P + \Delta_{B^*})} \left[C_0 (1 + \text{logs}) + C_l \chi_l + C_s \chi_s + C_c \chi_c + C_E \chi_E + C_{E^2} \chi_E^2 + C_{a^2} \chi_{a^2} \right],$$

with $\Delta_{B^*} = \left(M_{B^*}^2 - M_B^2 - M_P^2 \right) / (2M_B)$, where M_{B^*} is a 1^- or 0^+ mass.

Similar equations follow for $f_\parallel$ and f_T, except with no pole factor for $f_\parallel(B \to \pi)$.

$$\chi_l = \frac{2\mu m_l}{8\pi^2 f_\pi^2}, \qquad \chi_s = \frac{2\mu m_s}{8\pi^2 f_\pi^2}, \qquad \chi_c = \frac{2\mu m_c}{8\pi^2 f_\pi^2},$$

$$\chi_E = \frac{\sqrt{2} E_P}{4\pi f_\pi}, \qquad \chi_{a^2} = \frac{a^2 \overline{\Delta}}{8\pi^2 f_\pi^2}.$$

Outlook

1. Calculate ρ factors
2. Take chiral-continuum limit for form factors $(w_0 E \lesssim 1)$
3. Perform functional z expansion $\rightarrow$ full range of q^2
4. Construct complete error budget
5. Unblind current renormalization factors $\rho_\perp$, $\rho_\parallel$, ρ_T
6. Compare with experiment
   a. Determine $|V_{ub}|$ from charged-current decays
   b. Examine $\mathcal{B}$ observables from neutral-current decays
7. Analyze finer lattice spacings

Thank you!

Especially from Zech!