Lattice2017
35TH INTERNATIONAL
SYMPOSIUM ON
LATTICE FIELD
THEORY

Lattice study of area law for double-winding Wilson loops

Akihiro SHIBATA (KEK)

In collaboration with:
Kei-Ichi KONDO (Chiba University)
Seikou KATO (Oyama College)
Ryutaro MATSUDO (Chiba University)

Introduction

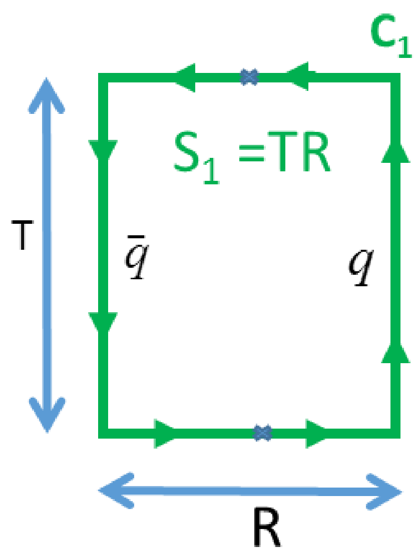
□Wilson loop is a gauge invariant and important operator for the lattice study.

□A single winding (usual) Wilson loop operator

- Quark and antiquark source of the fundamental representation
- The Wilson loop average gives the static potential between quark and antiquark

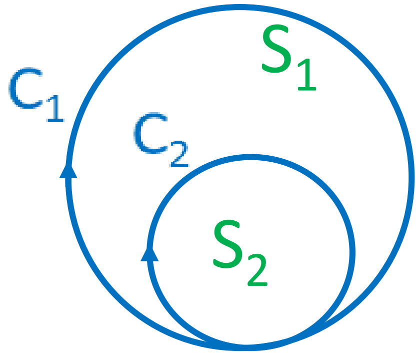
➔The area low or the linear potential

$$\langle W(C) \rangle \simeq \exp(-\sigma_1 S_1)$$



- For the study of quark confinement
  - Non-Abelian stokes theorem for Wilson loop operator
  - Dual super conductivity picture
    - ✓The restricted field dominance for the string tentsion
    - ✓The magnetic monopole dominance for the string tension
    - ✓Dual Meissner effect (Chromo-electric flux tube)

Introduction (II)

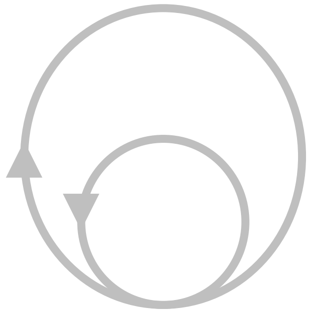


□ Double winding Wilson loop

Wilson loop operator with path C=(C1,C2)

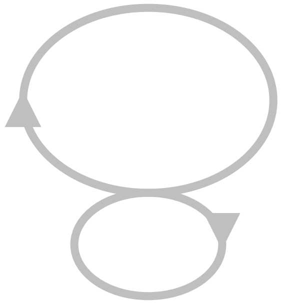
➢ How about string tension for the minimum surface of the Wilson loop ?

- Sum of areas behavior $\langle W(C_1 \times C_2)\rangle \simeq \exp[-\sigma(S_1 + S_2) - \mu P]$
- Differenece of area behavior $\langle W(C_1 \times C_2)\rangle \simeq \alpha \exp[-\sigma|S_1 - S_2|]$
- Others



➢SU(2) case [Greensite et.al PRD91 054509 (2015)]

- Difference of erea behavior
- cf U(1) , Abelian sum of areas behavior



How abuot SU(N) (N>2) case ?

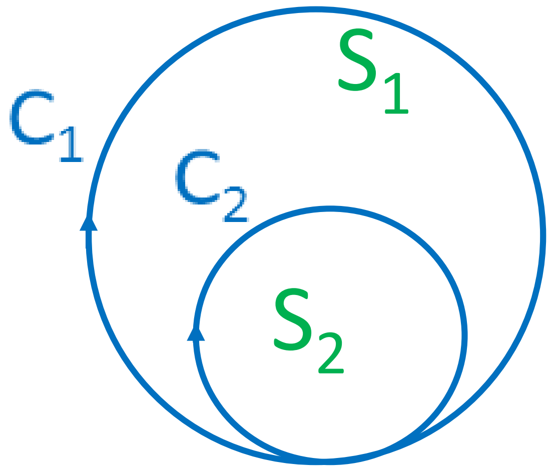
Double Winding Wilson loop

Contours which wind once around a loop C1 and once around a loop C2,

- ✓ the two co-planar loops share one point in common,
- ✓ C2 lies entirely in the minimal area of C1.
- ✓ C1 and C2 are in the same direction.
- ✓ S1 is the minimum area framed by C1
- ✓ S2 is the minimum area framed by C2

The purpose of this work is to investigate the double-winding Wilson loop of SU(N) Yang-Mills theory and to know the correct behavior of its expectation values such as:

- ◆Gauge group dependence
- ◆Relation to the N-arity
- ◆Relation to the non-Abelian stokes theorem and the dual superconductivity



CONTENTS

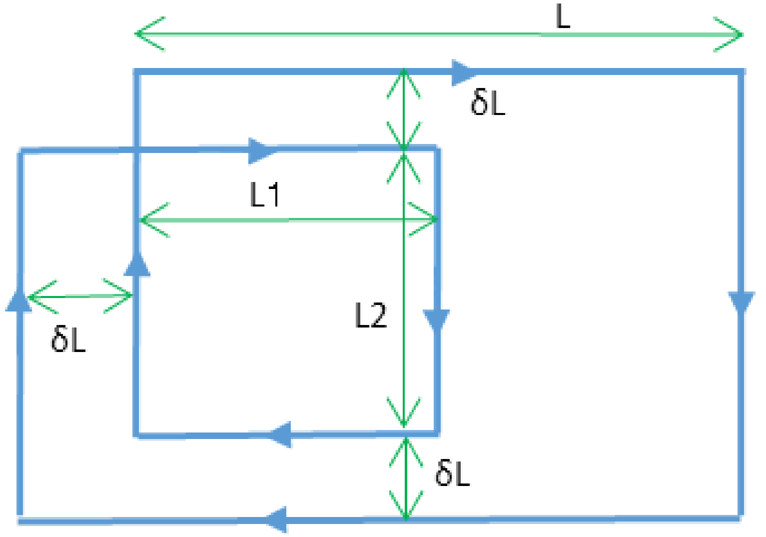
Study of Double-winding Wilson loop

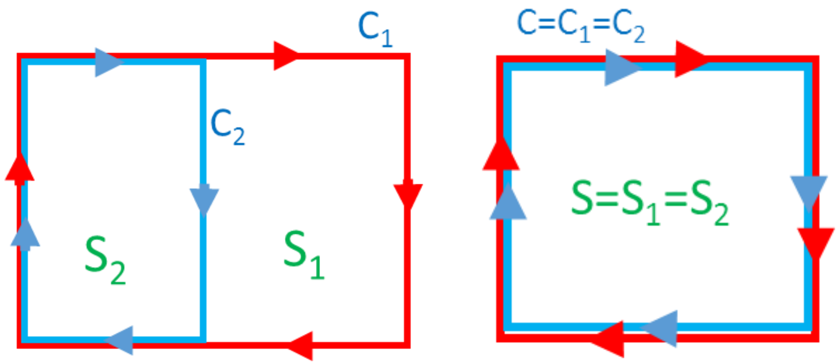
In this study we consider the double-winding Wilson loop as the right figures

- SU(N) Yang-Mills theory : Wilson action

$$S_g = \beta \sum_{\square} \mathrm{Re}\,\mathrm{tr}(1 - U_{\square}) \quad U_{\square} = U_{x,\mu} U_{x+\mu,\nu} U^{\dagger}_{x+\nu,\mu} U^{\dagger}_{x+\nu,\mu}$$

- discuss the expectation value of such double-winding loops.
  - ➢By using the strong coupling expansion
  - ➢By using numerical simulation





Strong coupling expansion

Strong coupling expansion

SU(N) group integral

$$\int dU 1 = 1$$

$$\int dU U_{ab} = 0$$

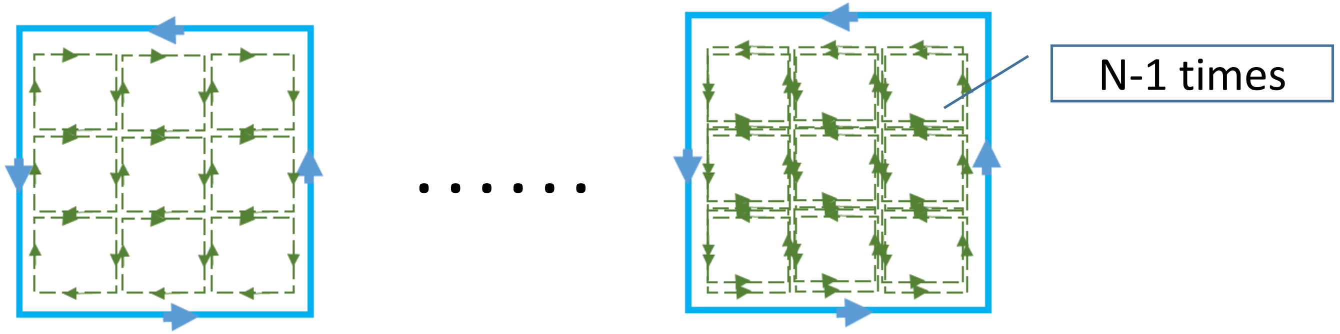
$$\int dU U_{ab} U^{\dagger}_{kl} = \frac{1}{N}\delta_{al}\delta_{bk}$$

$$\int dU U_{a_1b_1} U_{a_2b_2} \cdots U_{a_Nb_N} = \frac{1}{N!}\epsilon_{a_1a_2\cdots a_N}\epsilon_{b_1b_2\cdots b_N}$$

$$\int dU U_{a_1b_1} U_{a_2b_2} \cdots U_{a_Mb_M} = 0, \quad (M \neq 0 \mod N)$$

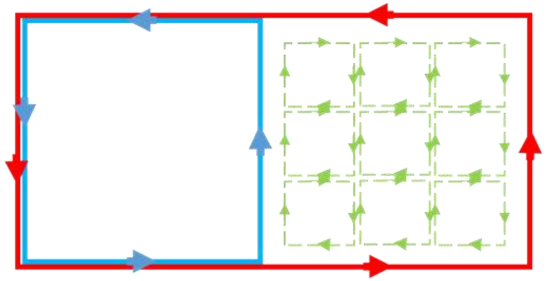
$$\int dU U_{ab} U_{cd} U^{\dagger}_{ij} U^{\dagger}_{kl} = \frac{1}{(N^2-1)}\left[\delta_{aj}\delta_{bi}\delta_{cl}\delta_{dk} + \delta_{al}\delta_{bk}\delta_{cj}\delta_{di} - \frac{1}{N}(\delta_{aj}\delta_{bk}\delta_{cl}\delta_{di} + \delta_{al}\delta_{bi}\delta_{cj}\delta_{dk})\right]$$

- Example which contribute to expectation of operators: leading and corrections.

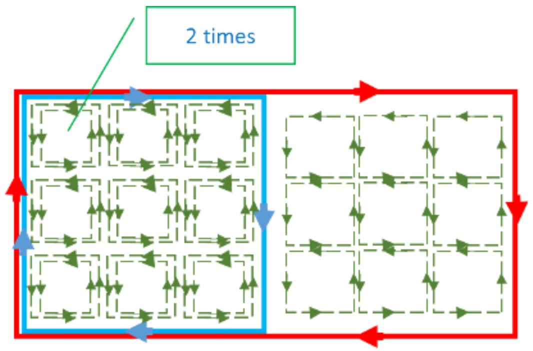


SU(2) case

- Leading term



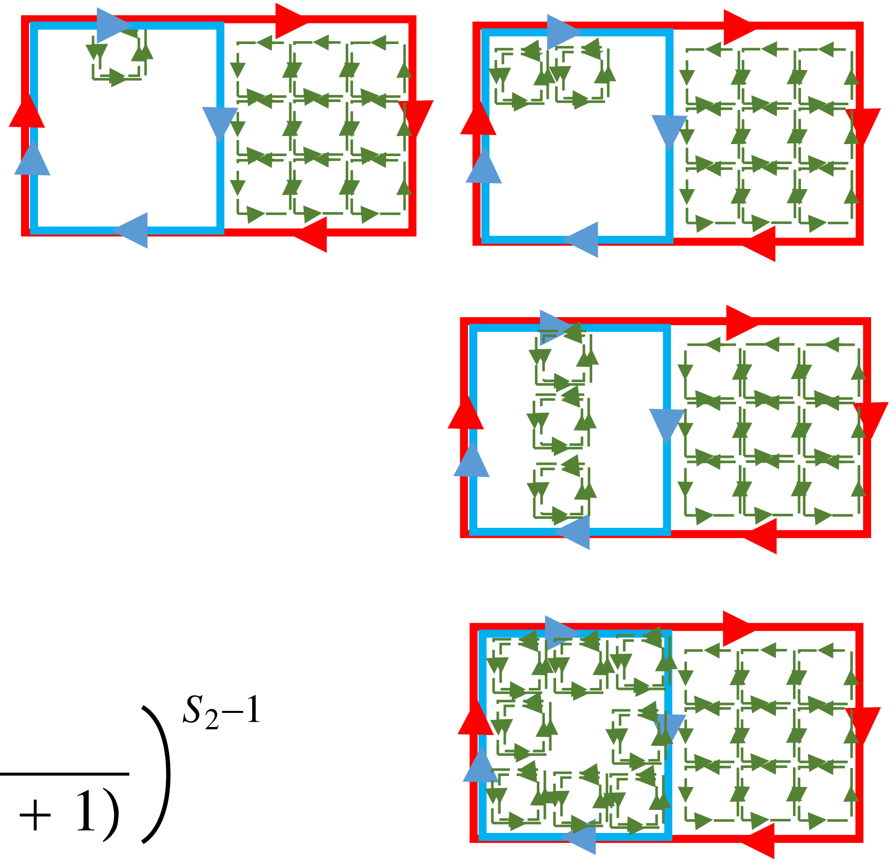
$$\langle W(C)\rangle|_{x2} = -2\left(\frac{\beta}{N_c}\right)^{S_1-S_2}$$



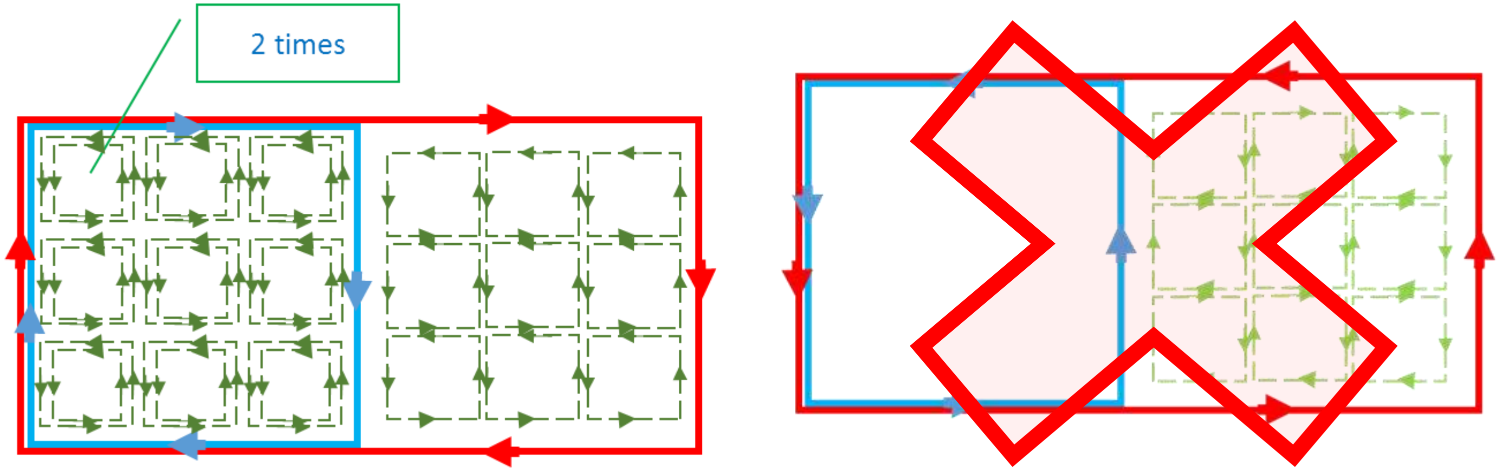
$$\langle W(C)\rangle|_{y_2} = -\frac{1}{2}q\beta^{2S_2}\left(\frac{\beta}{N_c}\right)^{S_1-S_2}$$

$$q = \left(\frac{1}{N_c(N_c-1)}\right)^{S_2-1} - \left(\frac{1}{N_c(N_c+1)}\right)^{S_2-1}$$

- Correction terms



Abelian, U(1) case



- Sum of areas behavior

$$\langle W(C_1 \times C_2)\rangle = \exp(-\sigma(S_1 + S_2))$$

Group integration

$$\int dg = 1$$

$$\int dgg = 0$$

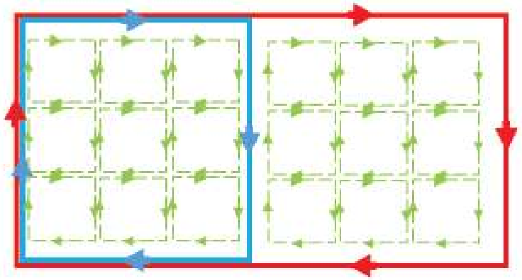
$$\int dggg^\dagger = 1$$

$$\int dggg = 0$$

$$\int dgggg^\dagger g^\dagger = 1$$

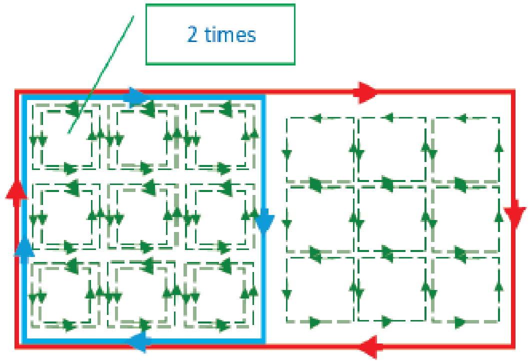
SU(3) case

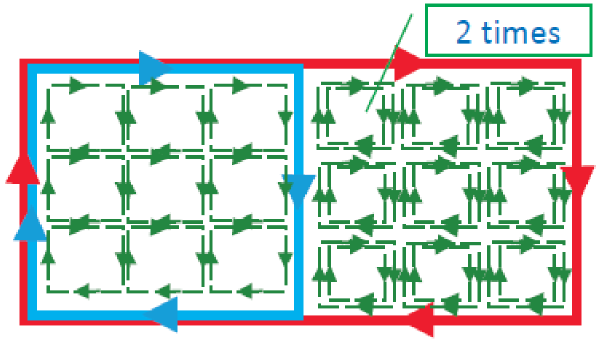
Leading term



$$\langle W(C)\rangle\big|_{x_3} = -\left(\frac{\beta}{N_c}\right)^{S_1}$$

Corrections



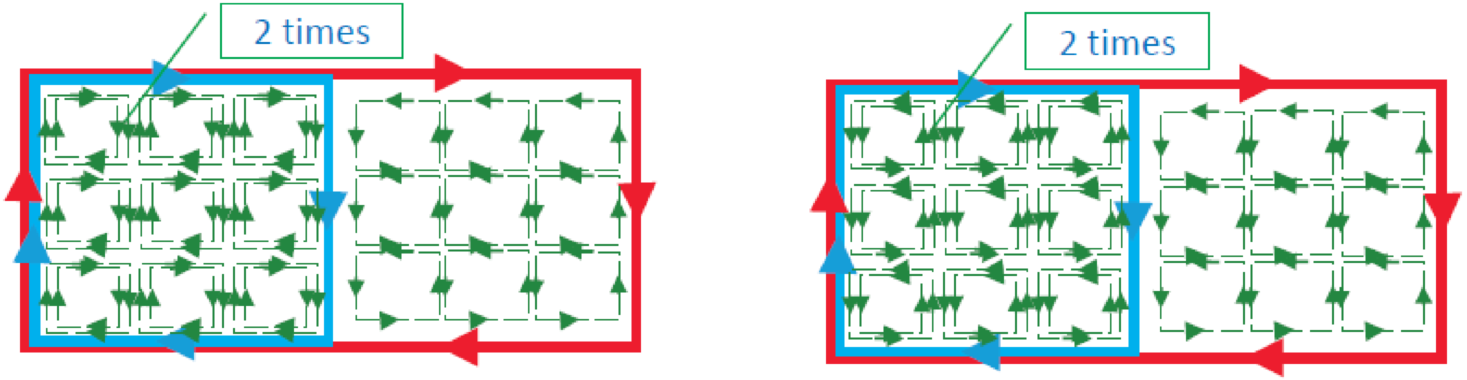


$$\langle W(C)\rangle\big|_{y_3} = -\frac{1}{2}q(S_2)\beta^{2S_2}\left(\frac{\beta}{N_c}\right)^{S_1-S_2}$$

$$\langle W(C)\rangle\big|_{z_3} = -\frac{1}{2}q(S_1 - S_2)\left(\frac{\beta}{N_c}\right)^{S_2}\beta^{2(S_1-S_2)}$$

$$q(S) = \left(\frac{1}{N_c(N_c-1)}\right)^{S-1} - \left(\frac{1}{N_c(N_c+1)}\right)^{S-1}$$

SU(4)



Both graphs give the equal contribution:

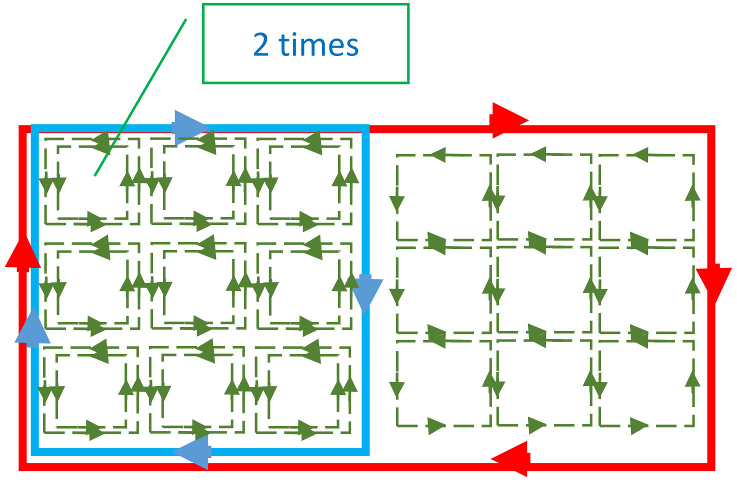
$$\langle W(C)\rangle\big|_{x_4} = -\frac{1}{2}q(S_2)\beta^{2S_2}\left(\frac{\beta}{N_c}\right)^{S_1-S_2}$$

$$\langle W(C)\rangle\big|_{y_4} = -\frac{1}{2}q(S_2)\beta^{2S_2}\left(\frac{\beta}{N_c}\right)^{S_1-S_2}$$

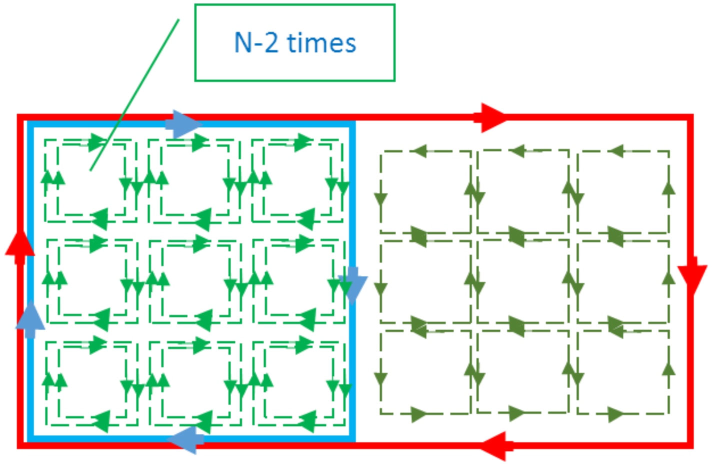
$$q(S) = \left(\frac{1}{N_c(N_c-1)}\right)^{S-1} - \left(\frac{1}{N_c(N_c+1)}\right)^{S-1}$$

SU(Nc) Nc > 4 case

- Leading term



- Correction term



In case of Nc > 4, the leading graph is replaced.

$$\langle W(C)\rangle|_{y_{N_c}} = -\frac{1}{2}q(S_2)\beta^{2S_2}\left(\frac{\beta}{N_c}\right)^{S_1-S_2}$$

$$q(S) = \left(\frac{1}{N_c(N_c-1)}\right)^{S-1} - \left(\frac{1}{N_c(N_c+1)}\right)^{S-1}$$

$$q \to \left(\frac{1}{N_c}\right)^{2S_2-1} \quad \text{for large } N_c$$

$$\langle W(C)\rangle|_{y_{N_c}} \to \left(\frac{\beta}{N_c}\right)^{S_1+S_2}$$

Summary

- SU(2) case : difference of area low

$$\langle W(C)\rangle \simeq \exp(-\sigma_2(S_1 - S_2))$$

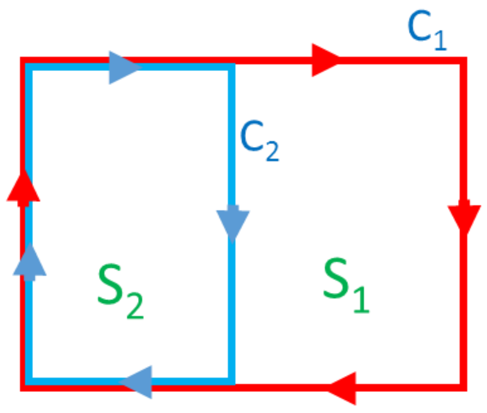
- SU(3) case: neither of difference of areas lows sum of areas lows

$$\langle W(C)\rangle \simeq \exp(-\sigma_3 S_1)$$

- SU(N) (N>3) case : sum of area low

$$\langle W(C)\rangle = -\frac{1}{2}\exp\{-\sigma'_{N_c}(2S_2) - \sigma_{N_c}(S_1 - S_2)\}$$

$$\sigma_{N_c} \simeq \sigma'_{N_c} \quad \text{for large } N_c$$



Numerical Simulation

Numerical simulation

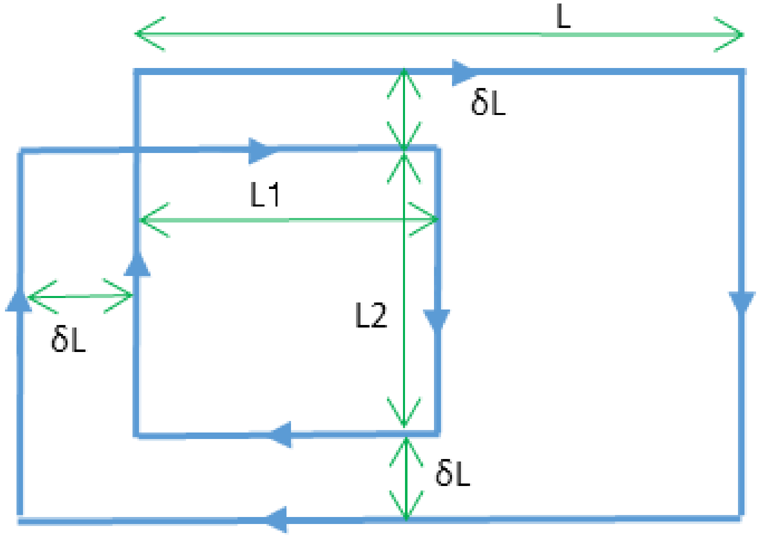
SU(2) case

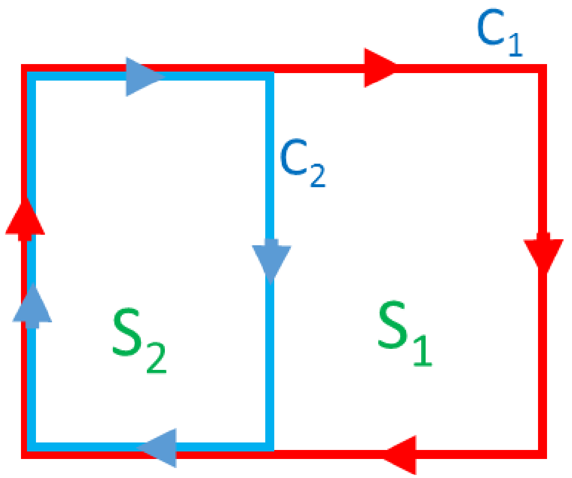
- Standard Wilson action
- 32^4 lattice β=2.6 pseudo heat bath
- Gauge links are smeared by using APE smearing method

SU(3) case

- Standard Wilson action
- 24^4 lattice β=6.2 pseudo heat bath with over relaxation
- Gauge links are smeared by using APE smearing method

SU(2) case (δL=0)

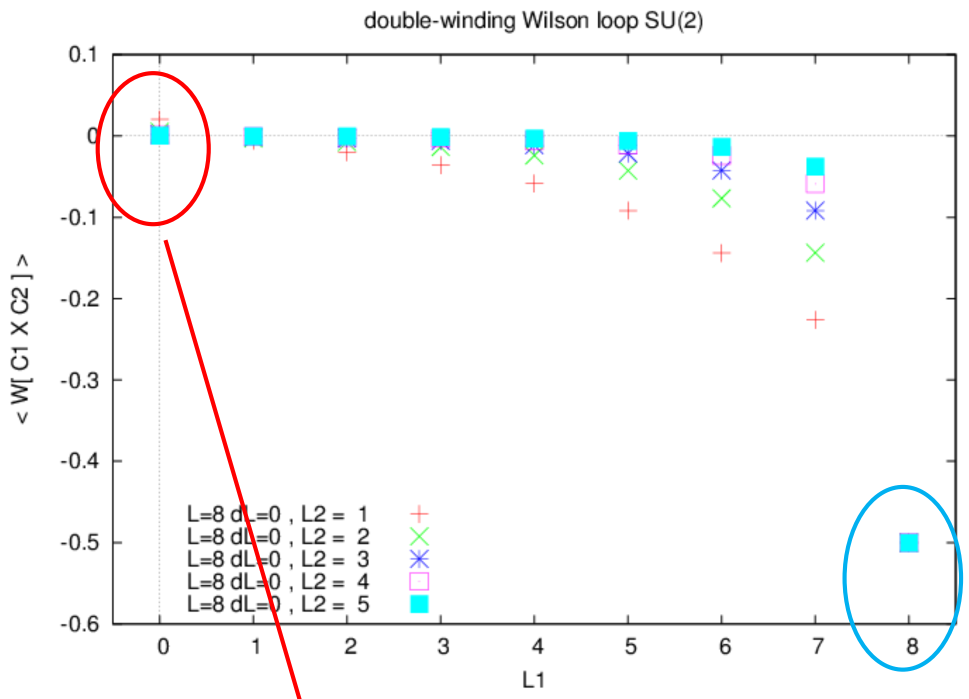




Double Wilson loop : SU(2) case

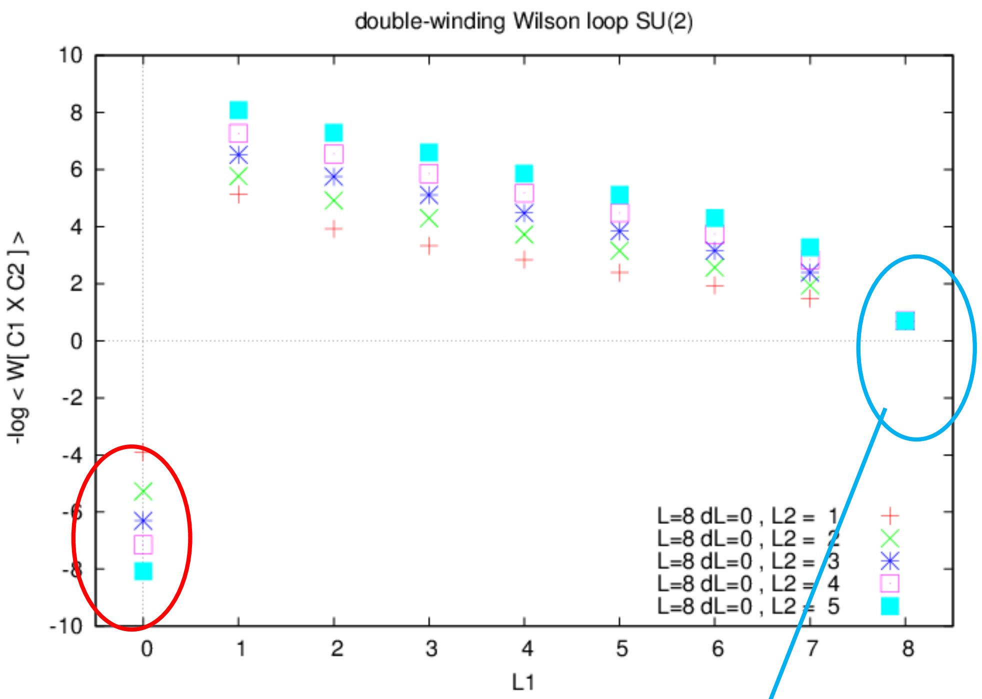
The result by Greensite et.al. PRD91 054509 (2015)] is reproduced.

Normal scale L=8 dL=0



L1=0 , single-winding

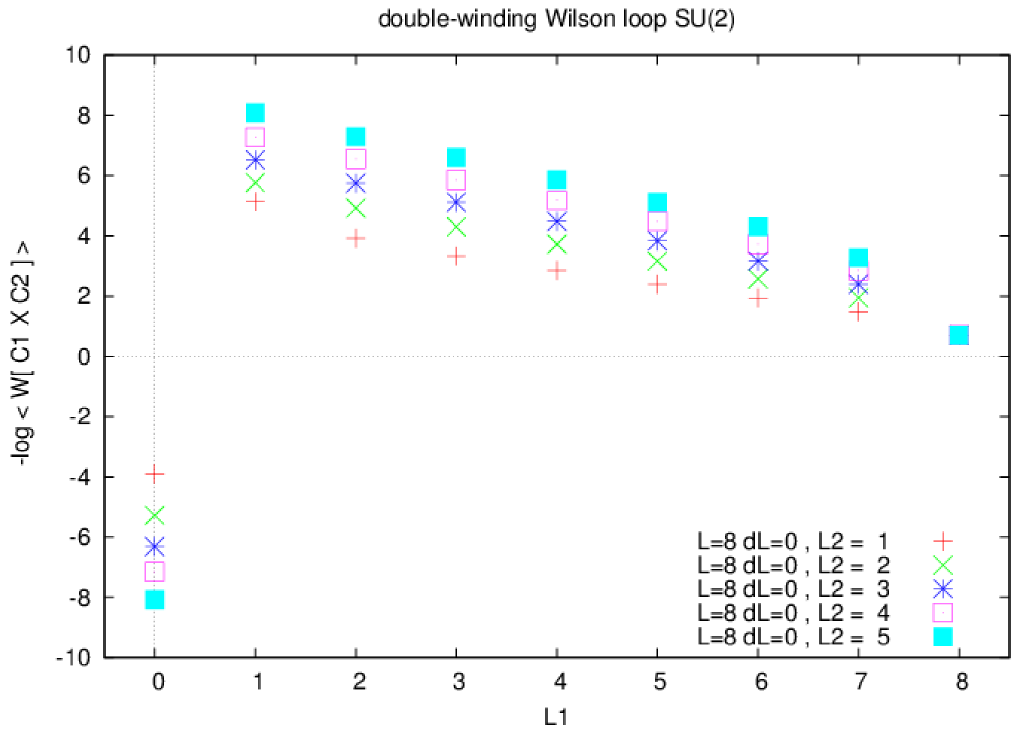
Log scale L=8, dL=0



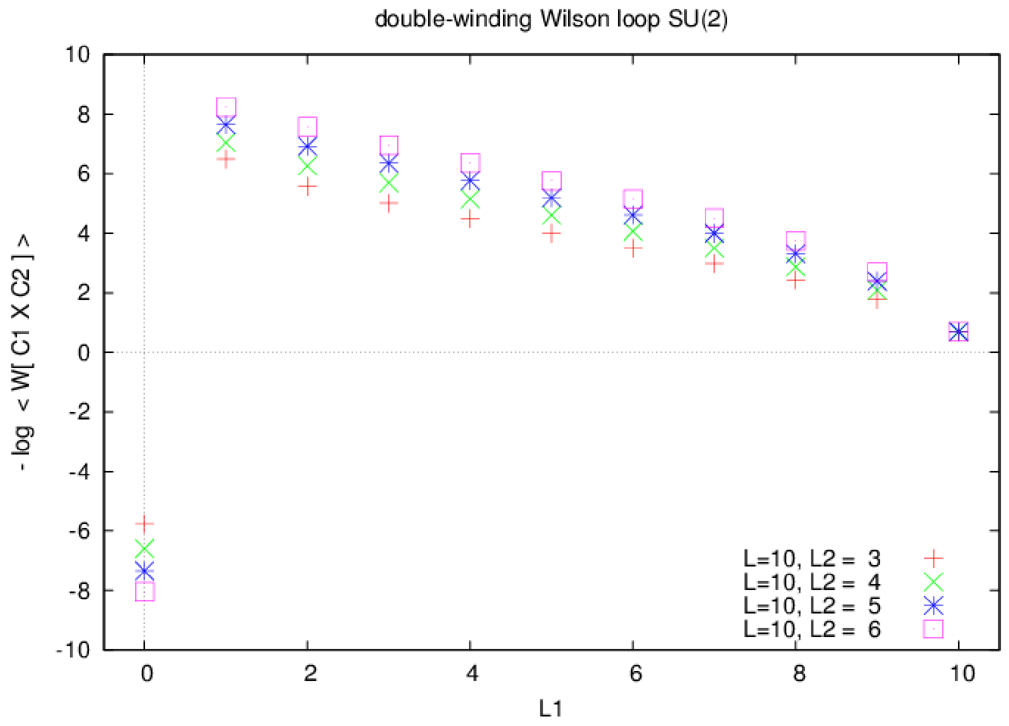
L1=8 double-winding with identical loops

Double Wilson loop : SU(2) case

Log-scale L=8 dL=0



Log-scale L=10, dL=0



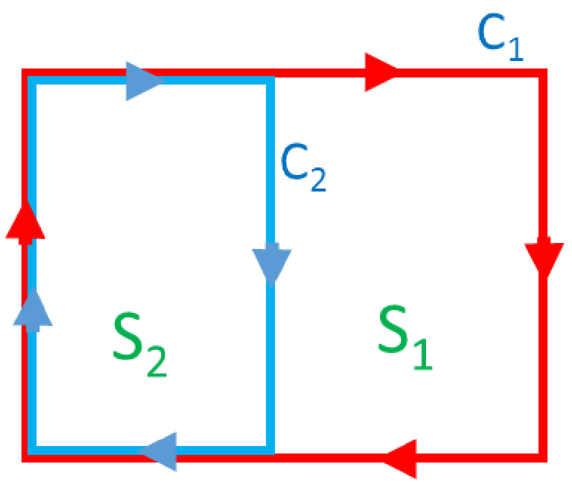
SU(3) case

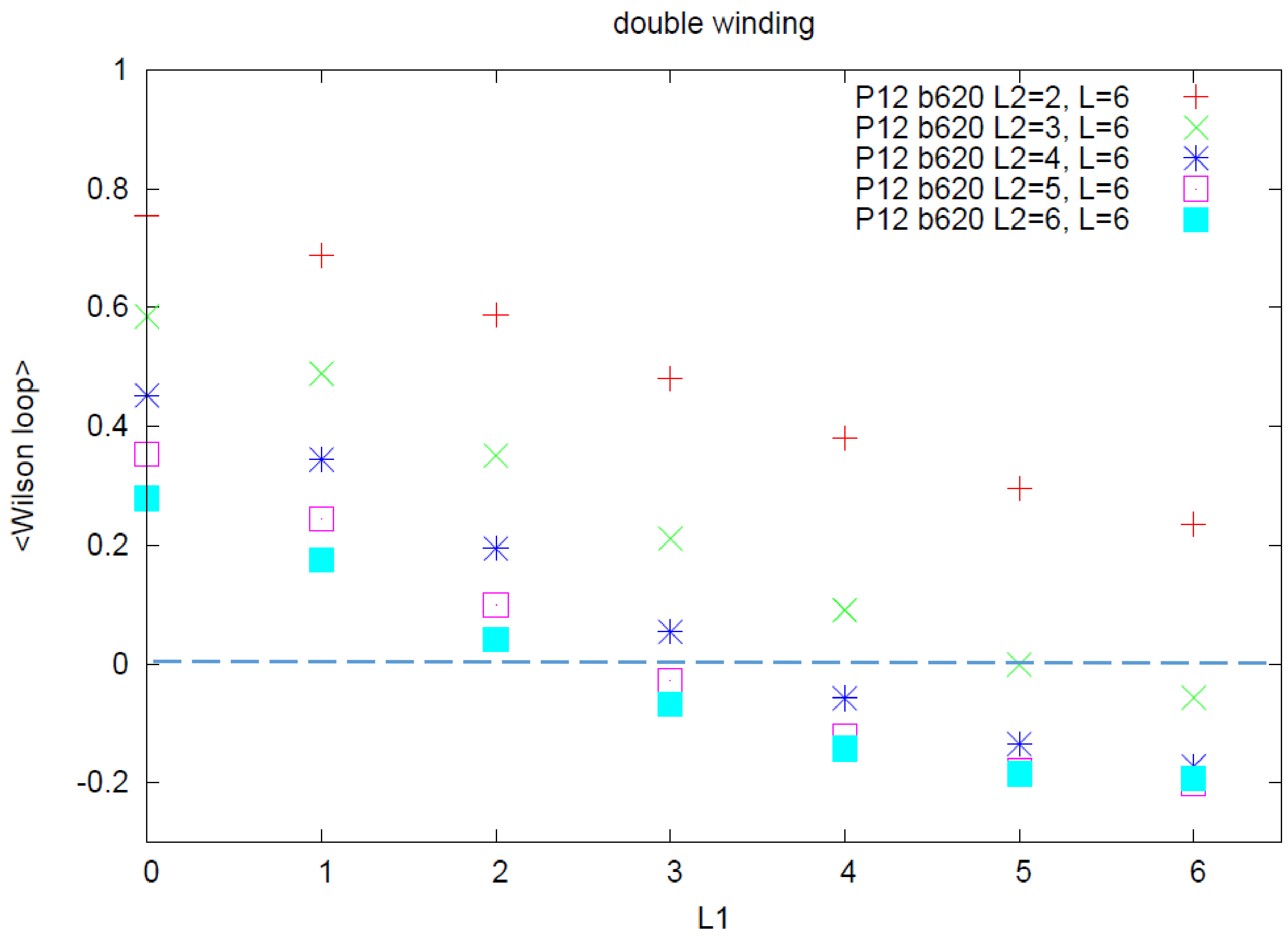
For L=6 the APE smearing with weight = 0.2 are applied 8 times.

For L=8 the APE smearing with weight = 0.1 are applied 8 times.

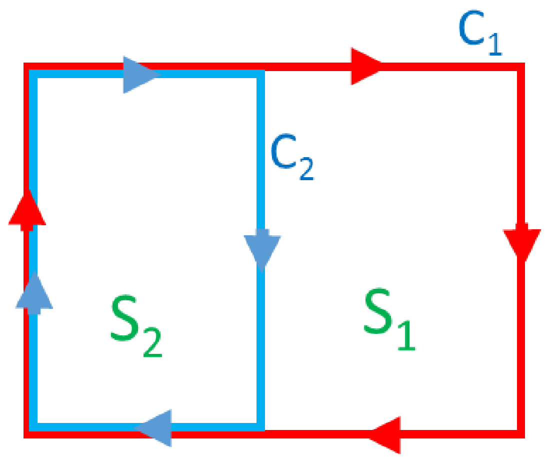
The numerical results are preliminary

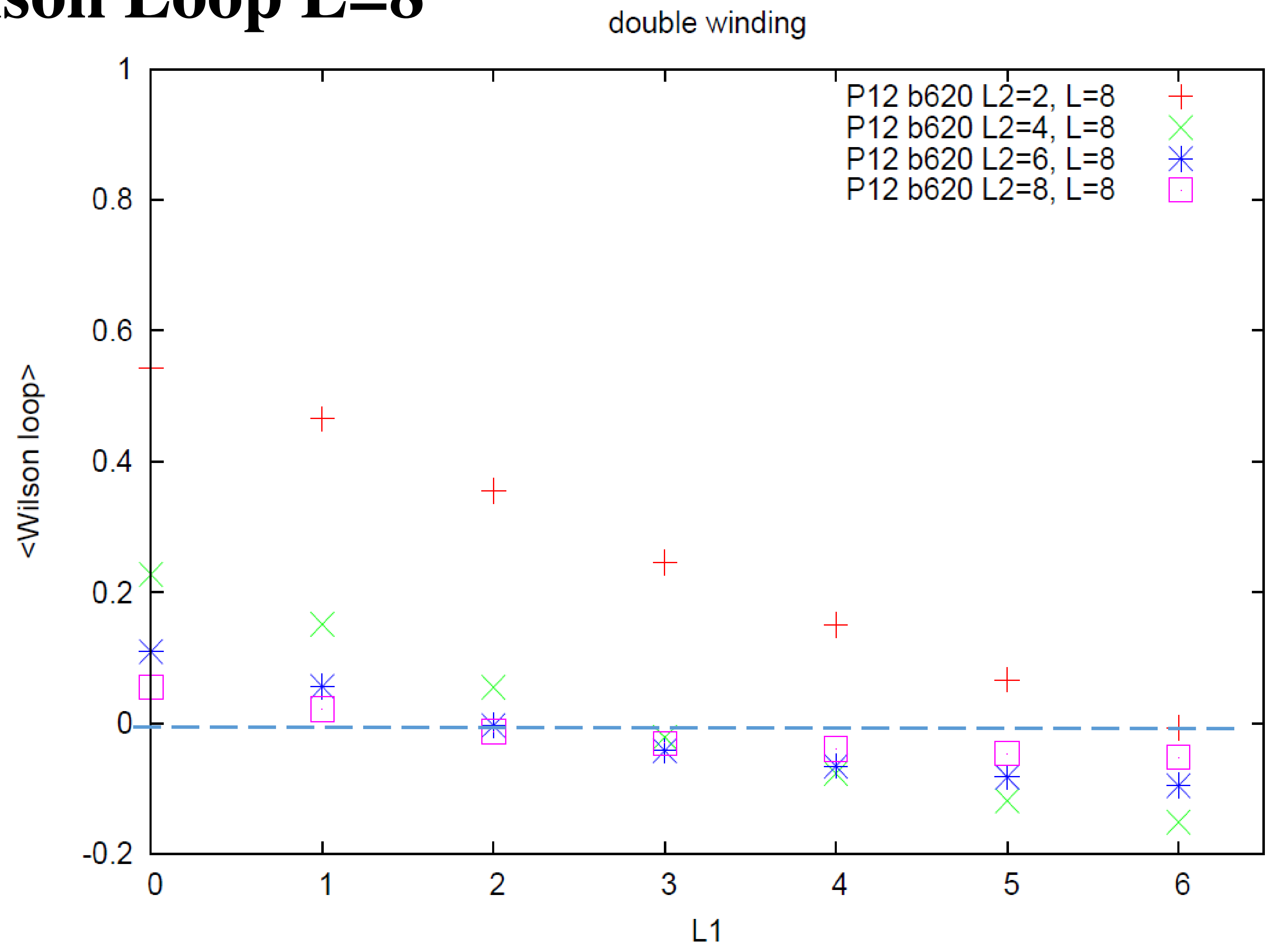
Double-winding Wilson loop L=6



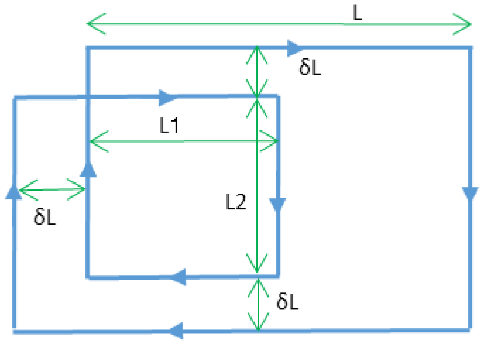


Double-winding Wilson Loop L=8

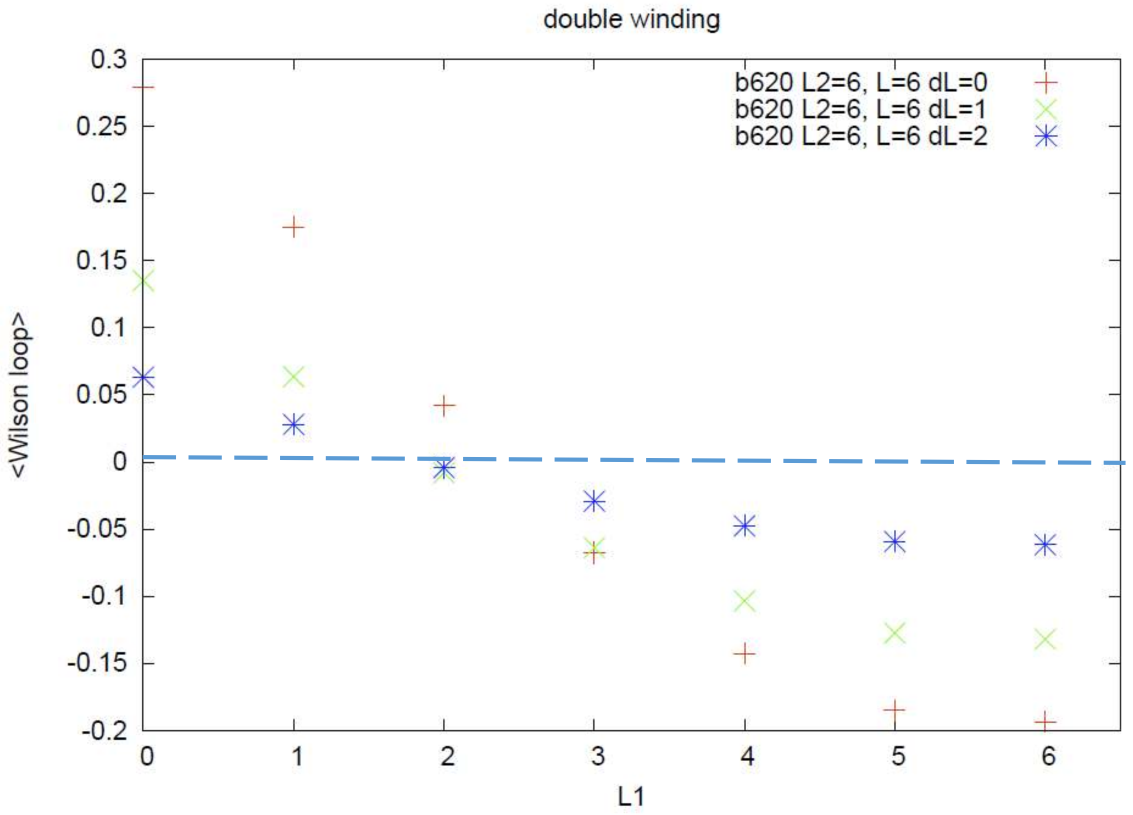




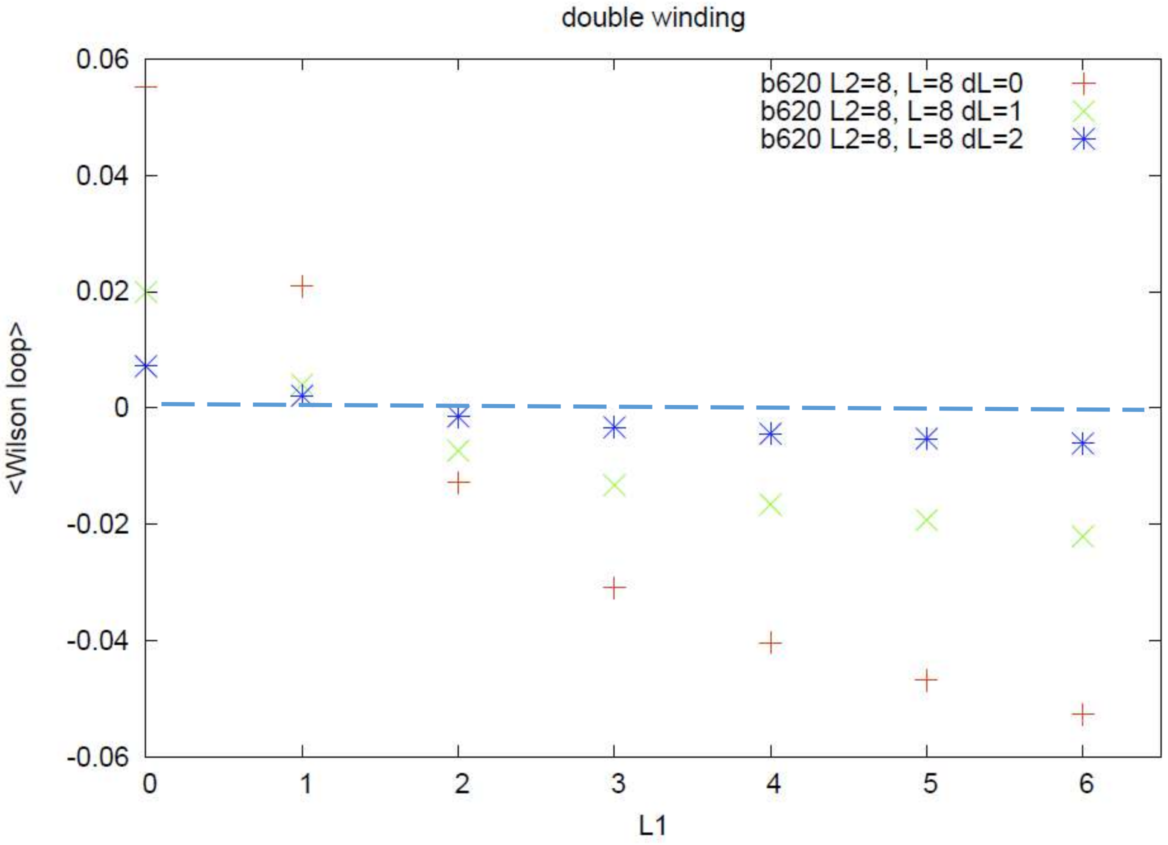
Double Winding Wilson loop (δL dependence)



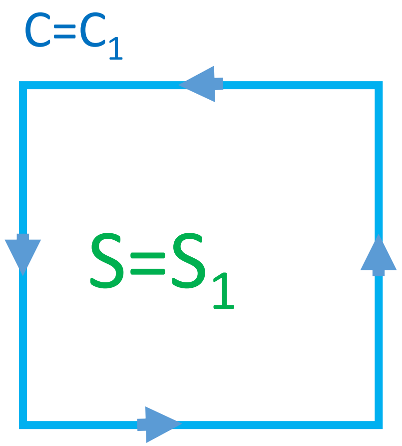
L=6

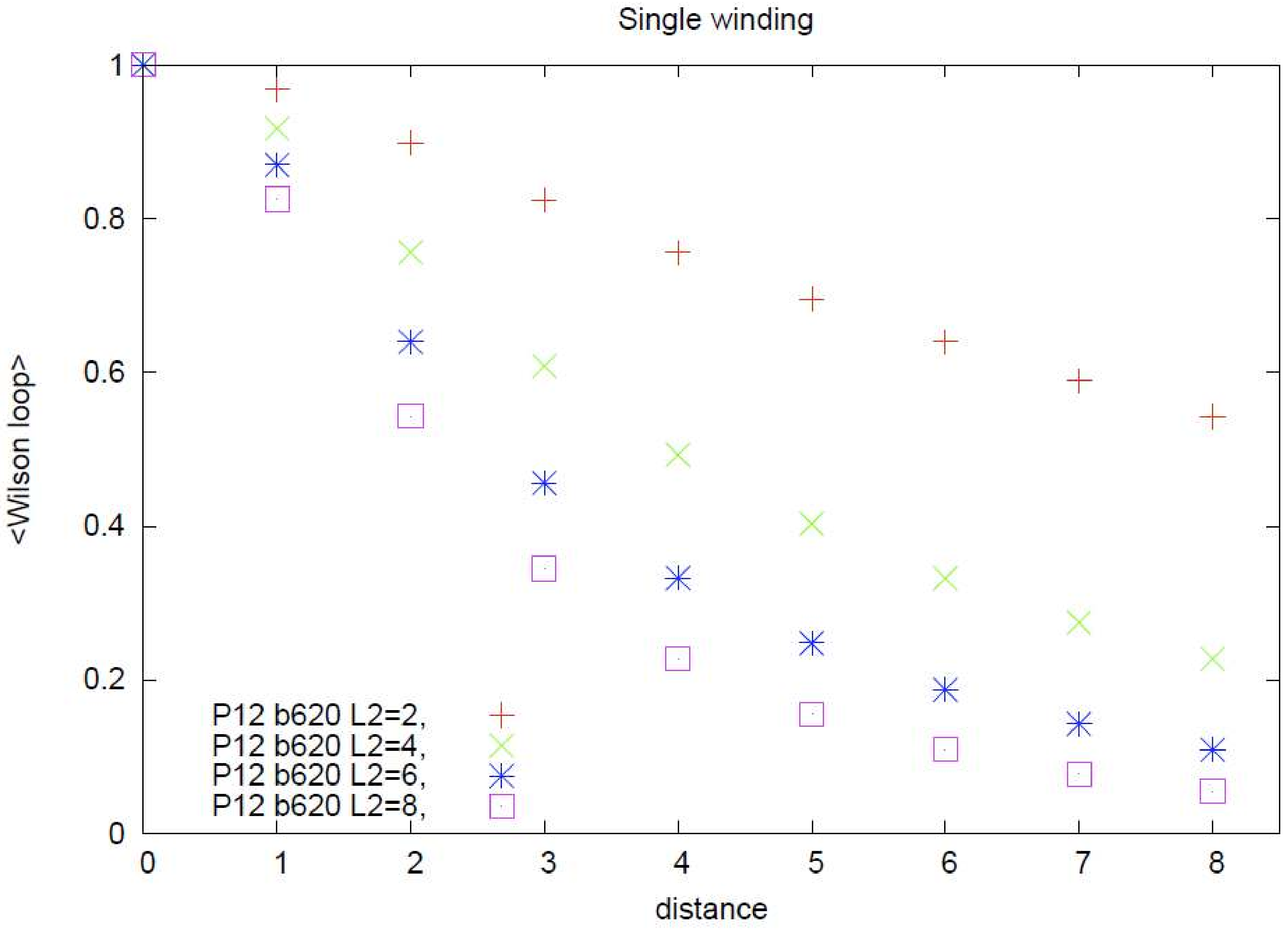


L=8

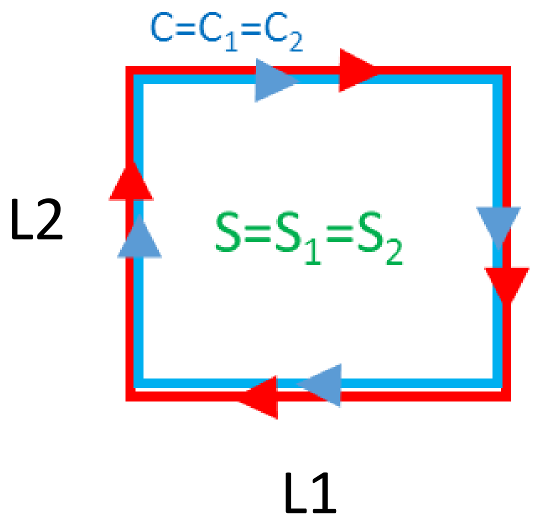


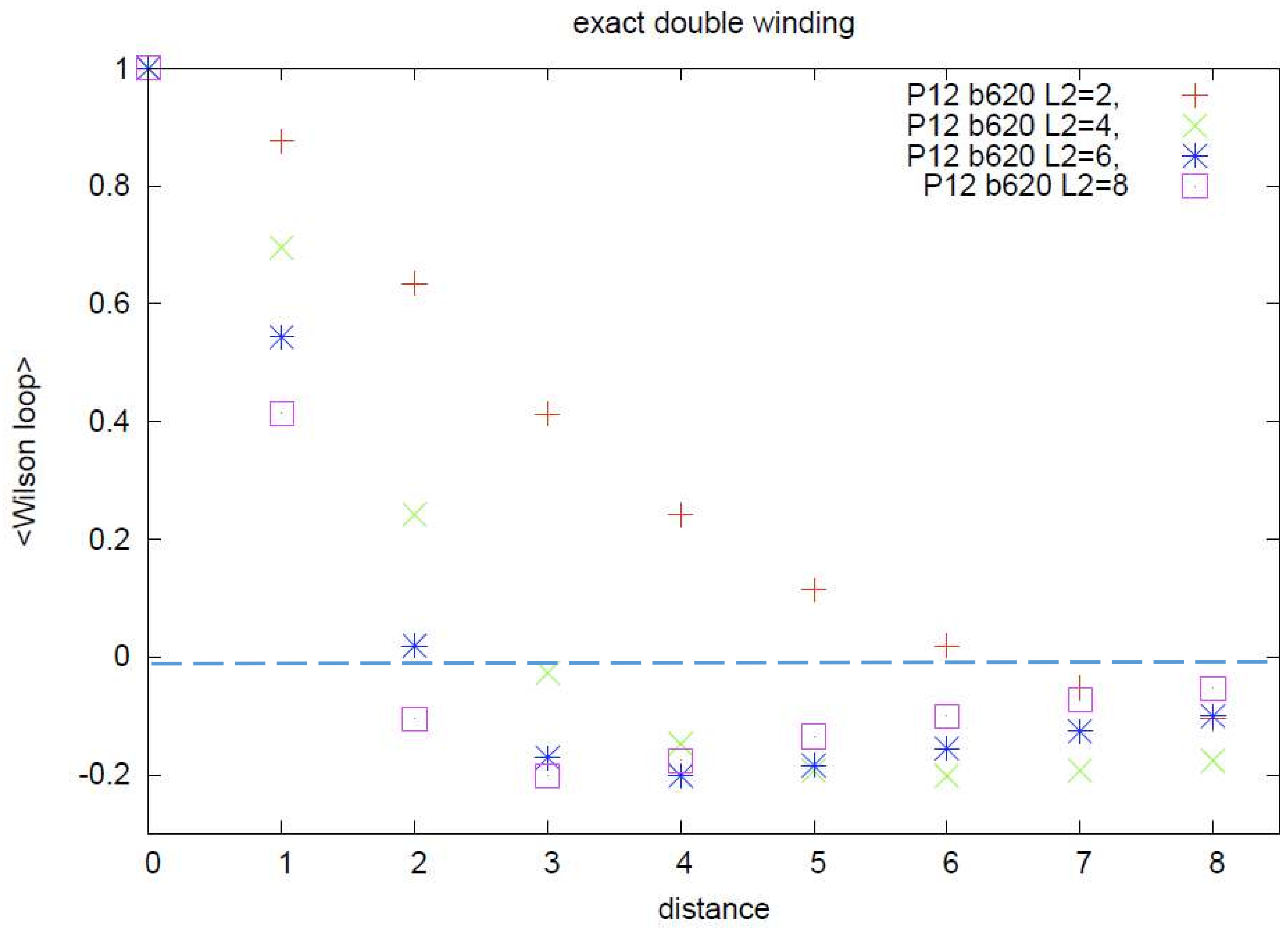
Single-Winding Wilson loop : SU(3)





Double-winding Wilson loop with the identical loop (C1=C2)

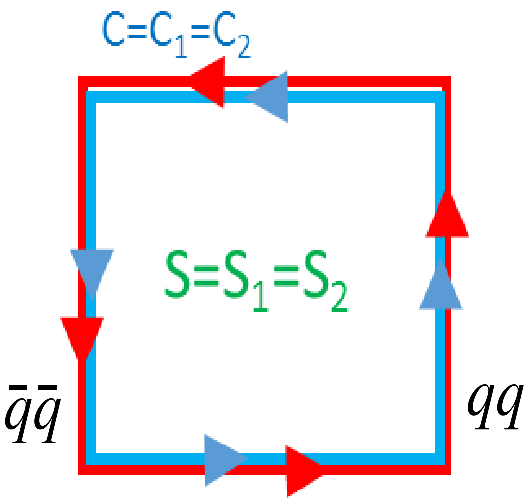




Summary and discussion

- We have investigated the double-winding Wilson loop average for SU(N) Yang-Mills theory by using the strong coupling expansion and lattice simulation.
- By using the strong coupling expansion we obtain:
  - ➢For SU(2) case, the difference of area behavior
  - ➢For SU(N) (N>2) , neither difference of area behavior nor sum of the area behavior. In the large Nc limit, sum of the area behavior is expected.
- By numerical simulation, we confirm the result of the string coupling expansion for large area of S1.
- These results are consistent with another analysis: (talk by Matsudo)

Double Winding Wilson loops and high dimensional representation



$$SU(2)\text{ case}:\ 2\otimes 2 = 2\otimes 2^* = 1\oplus 3$$

$$\langle W(C\times C)\rangle = -\frac{1}{2}+\frac{3}{2}\langle W_{\mathrm{adj}}(C)\rangle$$

$$SU(3)\text{ case}:\ 3\otimes 3 = 3^*\oplus 6$$

$$\langle W(C\times C)\rangle = -\langle W(C)_{[0,1]}\rangle + 2\langle W(C)_{[2,0]}\rangle$$

$$SU(N)\text{case}:\quad N\otimes N = \left(\frac{N(N-1)}{2}\right)_A \oplus \left(\frac{N(N+1)}{2}\right)_S$$

$$\langle W(C\times C)\rangle = -\frac{N-1}{2}\langle W(C)_{[0,1,\dots\dots,0]}\rangle + \frac{N+1}{2}\langle W(C)_{[2,0,\dots.0]}\rangle$$

See Matsudo's talk

Outlook

- Relation to N-ality
- Relation to the non-Abelian stokes theorem for quarks in the higher dimensional representation.
- Understanding in view of the dual-superconductivity