Non-perturbative improvement and renormalization of Wilson fermions using position space correlators

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work done in collaboration with Gunnar Bali and Jakob Simeth
for the RQCD collaboration

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Introduction

Improvement of LQCD with Wilson fermions

Reliable extraction of physical estimates of hadronic matrix elements of electro-weak currents from simulations with Wilson discretization of fermions requires appropriate renormalization constants and improvement coefficients.

Lattice artefacts can be accounted for using Symanzik's expansion

$$S_{\mathrm{QCD}}\big(a(\beta)\big) = S_{\mathrm{continuum}} + aS_1 + a^2 S_2 + \dots$$

For the operators we use

$$A_\mu^{jk,\mathrm{I}}(x) = \bar{\psi}_j(x)\gamma_\mu\gamma_5\psi_k(x) + a c_A \partial_\mu^{\mathrm{sym}} P^{jk}(x)$$

and

$$A_\mu^{jk,\mathrm{R}}(x) = Z_A\big(1 + a b_A m_{jk} + a 3\tilde{b}_A \overline{m}\big) A_\mu^{jk,\mathrm{I}}(x)$$

$\Rightarrow$ Z_A and b_A, $\tilde{b}_A$ can be extracted non-perturbatively from $\langle A_\mu^{jk,\mathrm{R}}(x)\langle A_\mu^{jk,\mathrm{R}}(0)\rangle$ correlators in position space.

Introduction

CLS ensembles

The CLS initiative is currently generating ensembles with $N_f = 2+1$ flavours of non-perturbatively improved Wilson Fermions and the tree-level Lüscher-Weisz gauge action at $\beta = 3.4, 3.46, 3.55$, 3.7, 3.85. This corresponds to lattice spacings of $a \in [0.039, 0.086]\,\text{fm}$.

Improvement coefficients

- $c_{\rm SW} \to$ Bulava, Schaefer, '13
- $c_A \to$ Bulava, Della Morte, Heitger, Wittemeier '15
- $b_\Gamma \to$ Bali, P.K. '17
- $\tilde{b}_\Gamma$, $Z_A \to$ this talk

The mass dependence of physical observables can be parameterized in terms of the average quark mass

$$m_{jk} = \frac{1}{2}(m_j + m_k)\,, \quad m_j = \frac{1}{2a}\left(\frac{1}{\kappa_j} - \frac{1}{\kappa_{\rm crit}}\right), \quad \overline{m} = \frac{1}{3}\left(m_s + 2m_\ell\right)\,.$$

Reminder

We define connected Euclidean current-current correlation functions in a continuum renormalization scheme R, e.g., $R = \overline{\mathrm{MS}}$, at a scale μ:

$$G^R_{J^{(jk)}}(x, m_\ell, m_s; \mu) = \left\langle \Omega \middle| T\, J^{(jk)}(x)\bar{J}^{(jk)}(0) \middle| \Omega \right\rangle^R .$$

Renormalization constants

We impose the following renormalization condition at $\mu = 1/x_0$

$$\lim_{a\to 0} G^X_{J^{(jk)}}(x, m_\ell = 0, m_s = 0)\Big|_{x^2=x_0^2} = G^{\mathrm{free}}_{J^{(jk)},\mathrm{latt}}(x_0, m_\ell = 0, m_s = 0),$$

The renormalized operator in the X-scheme is

$$J^{X,(jk)}(x, \mu = 1/x_0) = Z^X_J(\mu = 1/x_0) J^{X,(jk)}(x),$$

$$Z^X_J(\mu = 1/x_0) = \sqrt{\frac{G^{\mathrm{free}}_{J^{(jk)},\mathrm{latt}}(x_0, 0, 0)}{G_{J^{(jk)}}(x_0, 0, 0)}}.$$

Reminder

Improvement coefficients

The continuum Green function G^R can be related to the corresponding Green function G obtained in the lattice scheme at a lattice spacing $a = a(g^2)$ as follows:

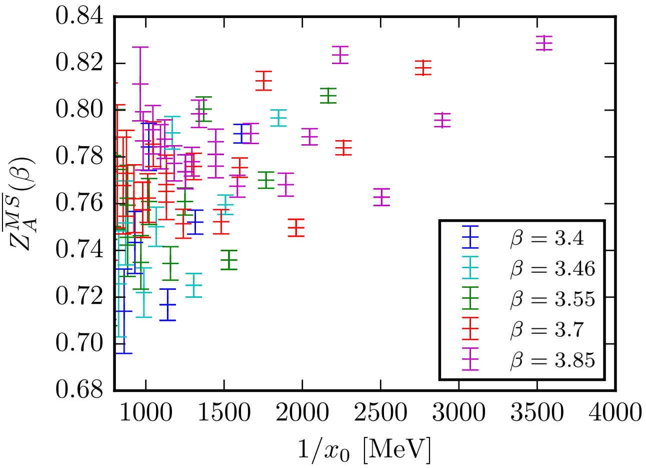
$$
\begin{aligned}
G^R_{J(jk)}(x, m_\ell, m_s; \mu) = & \\
& = G^R_{J(jk)}(x, 0, 0; \mu) \times \left[1 + \mathcal{O}\left(m^2 x^2, m\langle\overline{\psi}\psi\rangle x^4, \dots\right)\right] \\
& = \left(Z^R_J\right)^2(\tilde{g}^2, a\mu) \times \left(1 + 2b_J a m_{jk} + 6\bar{b}_J a\overline{m}\right) \times \\
& \qquad G_{J(jk),I}(n, am_{jk}, a\overline{m}; g^2)
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\frac{G_{J(jk)}\left(n, am^{(\rho)}_{jk}, a\overline{m}^{(\rho)}; g^2\right)}{G_{J(rs)}\left(n, am^{(\sigma)}_{rs}, a\overline{m}^{(\sigma)}; g^2\right)} = & \, 1 + 2b_J a\left(m^{(\sigma)}_{rs} - m^{(\rho)}_{jk}\right) \\
& + 6\tilde{b}_J a\left(\overline{m}^{(\sigma)} - \overline{m}^{(\rho)}\right) + \mathcal{O}\left(a^2, x^2\right)
\end{aligned}
$$

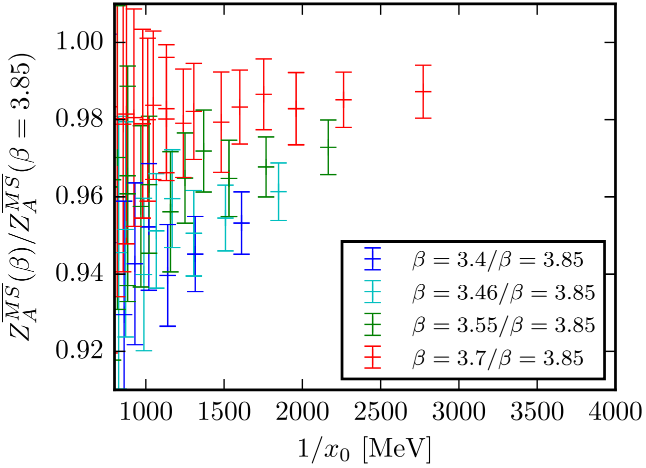
Window problem

Straightforward implementation of the renormalization condition



Improved strategy

Cancellation of lattice artefacts in the ratios



Improved strategy

Factorization

The renormalization constant $Z_J(\beta, a\mu = 1/n)$ can be factorized into:

- a renormalization constant evaluated at the fine lattice spacing $Z_J(\hat{\beta}, \hat{a}\hat{\mu} = 1/n_0)$
- a ratio describing the running of Z_J from $\hat{\beta}$ to β at fixed $1/n$

Hence,

$$Z_J(\beta, a\mu = 1/n) = \hat{Z}_J(\hat{\beta}, \mu') \frac{Z_J(\beta, 1/n)}{Z_J(\hat{\beta}, 1/n)},$$

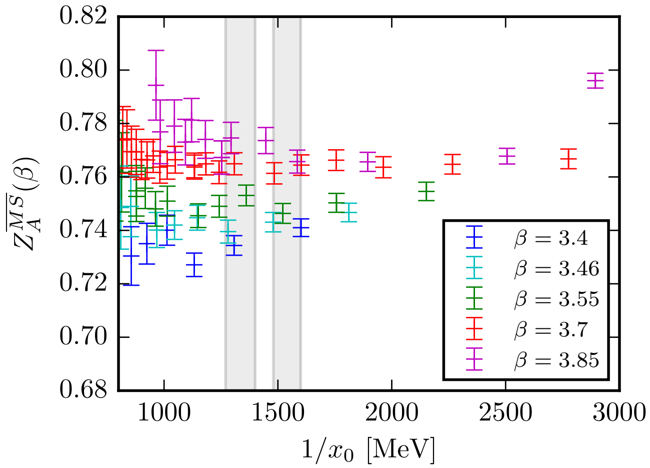
where

$$\hat{Z}_J(\hat{\beta}, \mu') = Z_J(\hat{\beta}, \hat{a}\hat{\mu} = 1/n_0) R(\hat{a}\mu' = 1/n, \hat{a}\hat{\mu} = 1/n_0),$$

and $R(\mu', \hat{\mu})$ is a perturbative factor describing the running of Z_J from the scale $\hat{\mu}$ to $\mu' = 1/(\hat{a}n)$.

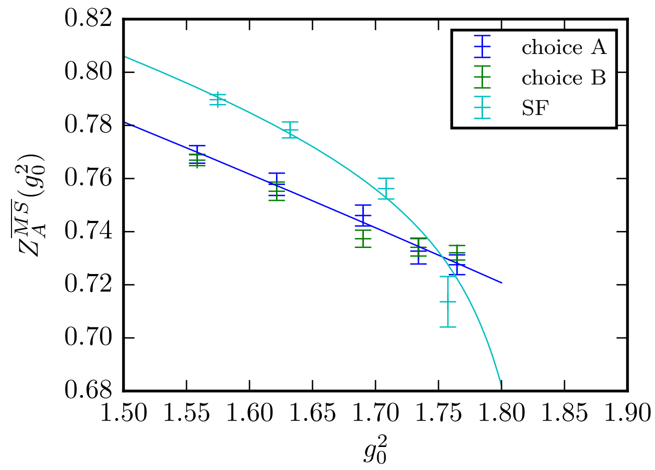
Improved strategy

Improved implementation of the renormalization condition



Improved strategy

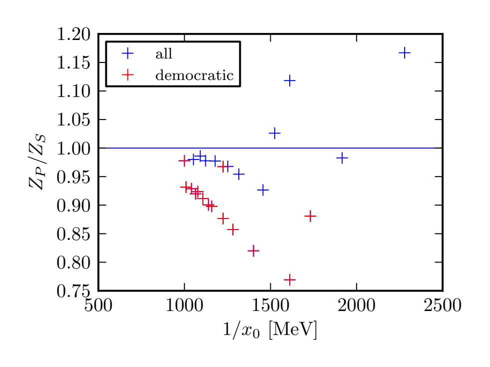
Comparison with Schrödinger Functional scheme



One-loop lattice artefacts

Numerical Stochastic Perturbation Theory

We implemented the tree-level Lüscher-Weisz gauge action and clover fermions in NSPT. We generated 24^4, 32^4, 48^4 and 64^4 ensembles of 20 configurations at $\epsilon = 0.005, 0.01, 0.015$. We measured the corresponding current-current correlation functions and can estimate 1-loop cutoff effects with $< 1\%$ precision.



Improved estimators

Truncated Solver method

We factorize the estimated propagator into two parts with $N_1 > N_2$

$$D^{-1}(x-y) \approx \frac{1}{N_1}\sum_{i=1}^{N_1} D_{n_t}^{-1}(x-y) + \frac{1}{N_2}\sum_{i=1}^{N_2}\left\{D_{\mathrm{exact}}^{-1}(x-y) - D_{n_t}^{-1}(x-y)\right\}$$

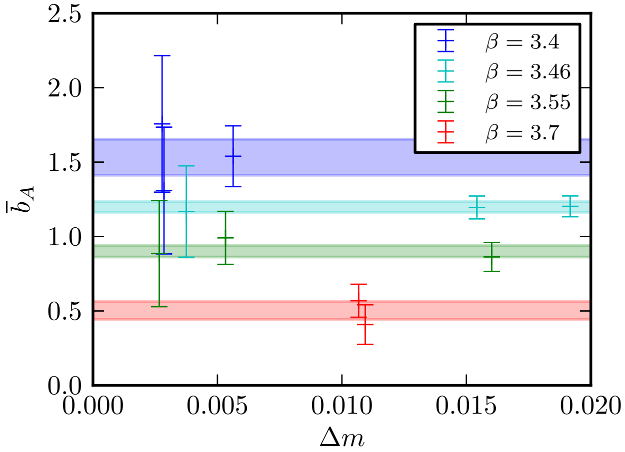
where $D_{n_t}^{-1}(x-y)$ is the propagator obtained after n_t solver iterations.

ensemble		truncated	exact	reduction
H101	$32^3 \times 96$	6.4 s	20.9 s	$\times$3.3
H200	$32^3 \times 96$	6.3 s	17.9 s	$\times$2.8
H400	$32^3 \times 96$	6.3 s	18.7 s	$\times$3.0
N300	$48^3 \times 128$	17.2 s	55.8 s	$\times$3.2
J500	$64^3 \times 192$	47.1 s	137.9 s	$\times$2.9

32 or 64 measurements per conf. $\Rightarrow$ 10 times smaller stat. uncertainties!

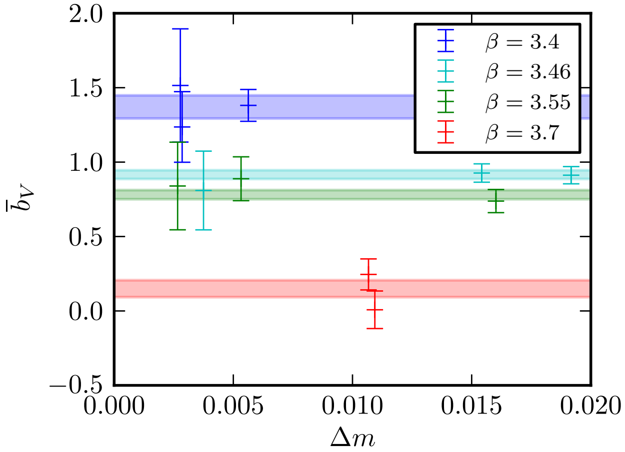
Improvement coefficients

$\bar{b}_A$ coefficients: statistical errors only!



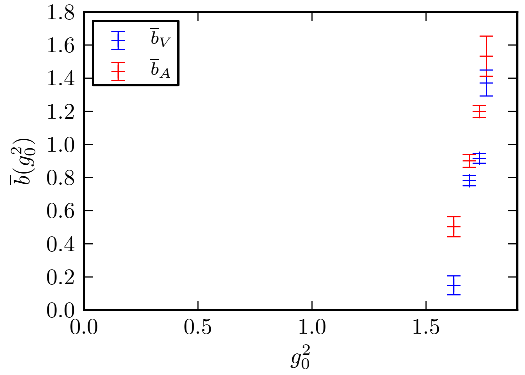
Improvement coefficients

$\bar{b}_V$ coefficients: statistical errors only!



Improvement coefficients

Running: PRELIMINARY!



- although $\overline{b}_J = \mathcal{O}(g_0^4)$ in perturbation theory
- very large at the coarse $\beta = 3.4$
- need to be taken into account

Conclusions

Renormalization constants

- position space correlators can be useful in estimating renormalization constants and improvement coefficients
- lattice artefacts in the renormalization constants can be controlled
- NSPT estimation of cut-off effects at order g_0^2
- democratic points may not be the best after all

Improvement coefficients

- enough statistical precision for $\bar{b}_J$ coefficients
- OPE expansion with Wilson coefficients at at least one loop
- NSPT estimation of cut-off effects at order g_0^2

Near future

- more complicated operators: bilinears with derivatives needed for the estimation of moments of distribution amplitudes
- extension to singlet operators

Acknowledgements

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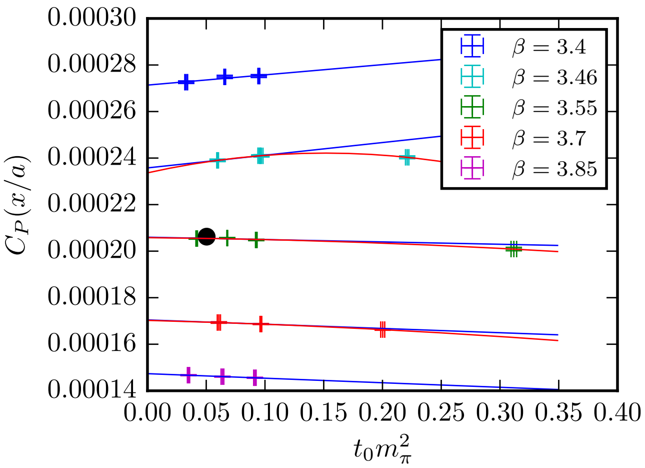
Backup slides: chiral extrapolation

Z_P



Backup slides: chiral extrapolation

Z_S