Two-baryon systems from HAL QCD method and the mirage in the temporal correlation of the direct method

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Refs HAL Coll.,
arXiv:1703.07210, **JHEP 1610(2016)101**[arXiv:1607.06371],
PoS(Lattice2016)107[arXiv:1610.09779]

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Hadron Interactions in Lattice QCD

1 **"Direct method"** — energy shift of two-particle in lattice

▶ Lüscher's formula ⇒ phase shift

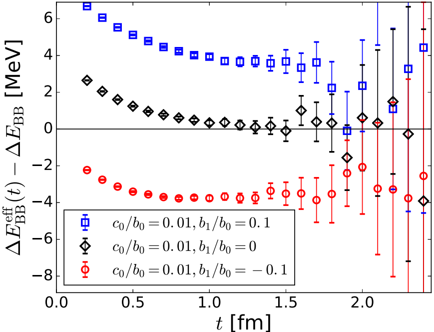
$$\Delta E_L = 2\sqrt{k^2+m^2} - 2m \implies k\cot\delta(k) = \frac{1}{\pi L}\sum_{\boldsymbol{n}\in\mathbb{Z}^3}\frac{1}{|\boldsymbol{n}|^2-(kL/2\pi)^2}$$

"plateau-like" structure of $\Delta E_L^{\rm eff}(t)$ would be "fake"

ground state saturation scale — $t \gg 2$ **fm!**

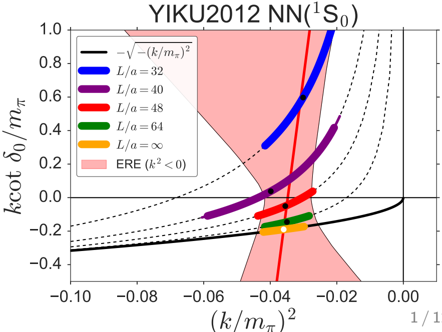
elastic states contamination

⇒ **"fake"**, i.e., mirage of plateau



unreasonable phase shift

eg. shallow but volume indep.



Hadron Interactions in Lattice QCD

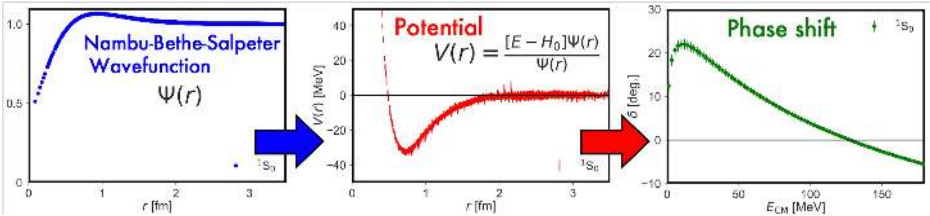
1. **"Direct method"** — energy shift of two-particle in lattice
   - ▶ Lüscher's formula ⇒ phase shift

$$\Delta E_L = 2\sqrt{k^2+m^2} - 2m \implies k\cot\delta(k) = \frac{1}{\pi L}\sum_{\boldsymbol{n}\in\mathbb{Z}^3}\frac{1}{|\boldsymbol{n}|^2-(kL/2\pi)^2}$$

2. **HAL QCD method**

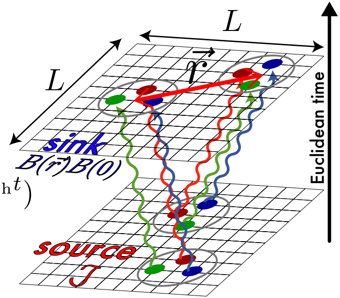
use "spatial correlations" for the information of the interaction

⟹ This talk, we demonstrate the usefulness of this approach.



1. Systematics in HAL QCD and Consistency between Lüscher's formula
2. Diagnosis of Fake Plateau in the Direct Method
3. Summary

Time-dependent HAL QCD Method



$$R(\vec{r},t) \equiv \frac{\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\overline{\mathcal{J}}(0)|0\rangle}{\{G_B(t)\}^2}$$

$$= \sum_n A_n \psi_{W_n}(\vec{r}) e^{-(W_n - 2m_B)t} + \mathcal{O}(e^{-\Delta W_{\rm th} t})$$

■ **Nambu-Bethe-Salpeter wave function**

□ asymptotic region ⟹ $\delta(k)$ by Lüscher's formula

$$\psi_k(r) \simeq C \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

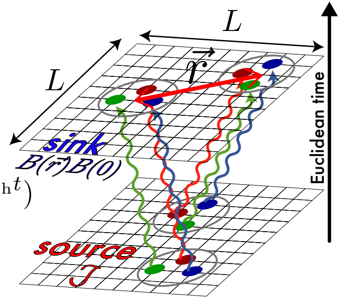
■ interacting region ⟹ ground state & scattering states satisfy

$$[E_{W_0} - H_0]\, \psi_{W_0}(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'}) \psi_{W_0}(\vec{r'})$$

$$[E_{W_1} - H_0]\, \psi_{W_1}(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'}) \psi_{W_1}(\vec{r'})$$

$$[E_{W_2} - H_0]\, \psi_{W_2}(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'}) \psi_{W_2}(\vec{r'}) \quad \cdots$$

Time-dependent HAL QCD Method



$$R(\vec{r},t) \equiv \frac{\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\overline{\mathcal{J}}(0)|0\rangle}{\{G_B(t)\}^2}$$
$$= \sum_n A_n \psi_{W_n}(\vec{r}) e^{-(W_n-2m_B)t} + \mathcal{O}(e^{-\Delta W_{\rm th} t})$$

■ **Nambu-Bethe-Salpeter wave function**

■ $R(r,t)$ satisfies

$$\left[\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\vec{r},t) = \int d\vec{r'} U(\vec{r},\vec{r'}) R(\vec{r'},t)$$

with **elastic** saturation — exponentially easier than g.s. saturation

This method does not require the ground state saturation.

▶ **"potential"** by velocity expansion of $U(r,r') \simeq V(r)\delta(r-r')$

$$V(\vec{r}) = \frac{1}{4m_B}\frac{(\partial/\partial t)^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} - \frac{H_0 R(\vec{r},t)}{R(\vec{r},t)}$$

▶ **CHECK** quark source dependence — different mixiture

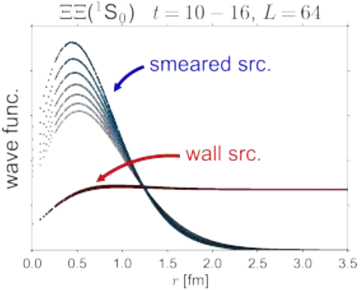
HAL: Potential of $\Xi\Xi(^1S_0)$ Smeared Src. vs Wall Src.

- we focus on $\Xi\Xi(^1S_0)$ (27 multiplet) and volume $L = 64, 48, 40$

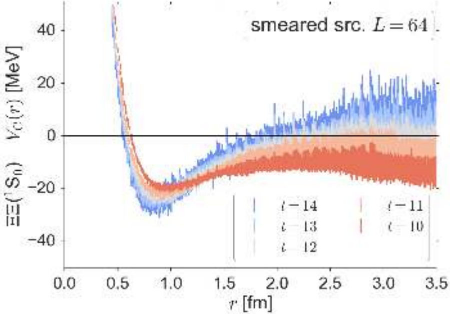
@ $m_\pi = 0.51$GeV, $a = 0.09$fm

wavefunction: $R^{\text{smear}}(r,t)$ or $R^{\text{wall}}(r,t)$

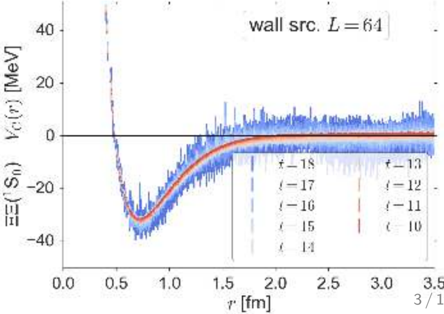
$$V_c(r) = \frac{1}{4m}\frac{(\partial^2/\partial t^2)R}{R} - \frac{(\partial/\partial t)R}{R} - \frac{H_0 R}{R}$$



■ **smeared src.** t-dependent

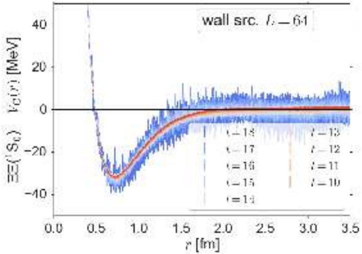


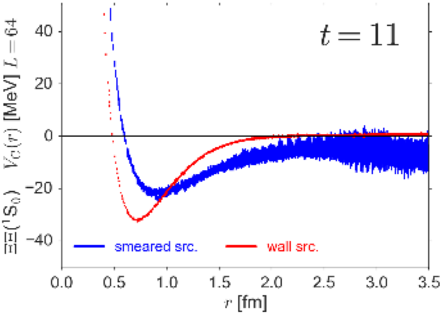
■ **wall src.** t-independent

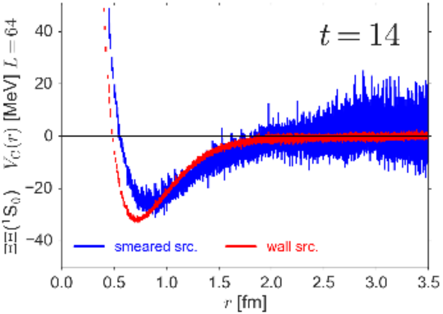


HAL: Potential of $\Xi\Xi(^1\mathrm{S}_0)$ Smeared Src. vs Wall Src.

- **Wall src.** — stable
- **Smeared src.** — t-dep. $\longrightarrow$ **wall src.**
- What is the origin of discrepancy?







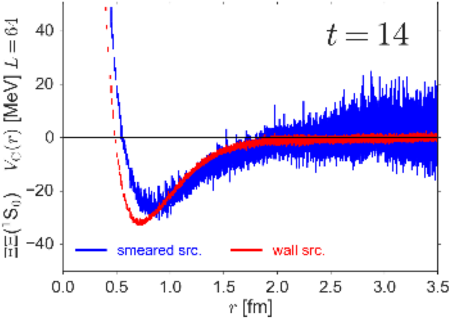
Next Leading Order Potential

- effective LO $\Rightarrow$ $U(r,r') = [V_{\text{eff}}(r)]\delta(r-r')$
- LO + NLO $\Rightarrow$ $U(r,r') = [V_{\text{LO}}(r) + V_{\text{NLO}}(r)\nabla^2]\delta(r-r')$

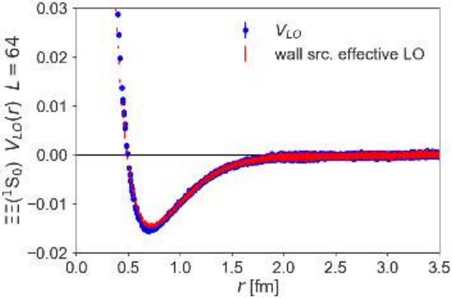
NLO correction is negligible in **wall src.**, which appears in **smeared src.**

▶ non-locality in $U(r,r')$ is under control

☐ Leading order approximation



☐ LO potential



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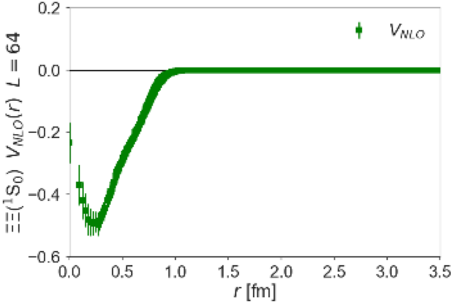
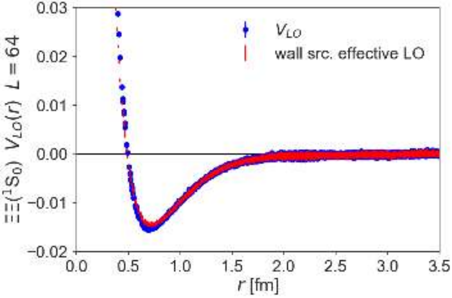
□ NLO potential



□ LO potential

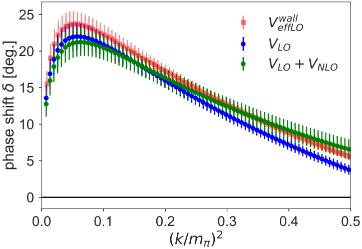


Systematics in HAL QCD Method

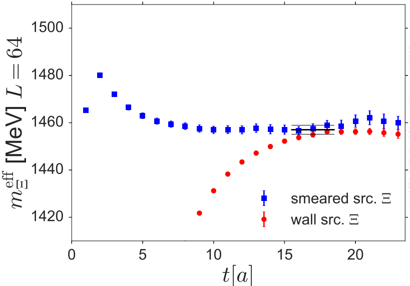
☐ convergence of derivative expansion — **Passed**

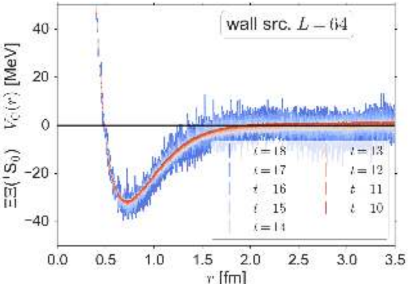
scattering phase shift δ

- At low energy, **wall src.** is consistent with LO potential
- NLO correction at high energy



☐ saturation of single baryon in **wall src.** — **Passed**





HAL and Lüscher: Energy Shift from Potential

- HAL QCD works well **w/o g.s. saturation problem**

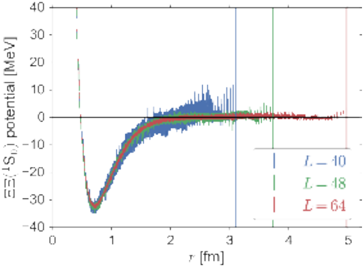
$\Longrightarrow$ correct **"energy shift"** in finite volume

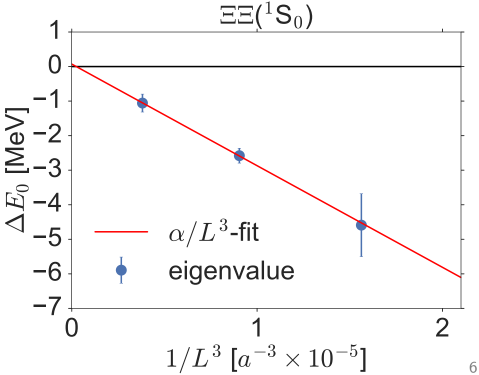
▶ Eigenequation in finite volume L^3 with HAL QCD potential $V(\vec{r})$

$$[H_0 + V]\,\psi = \Delta E \psi$$

☐ eigenvalue $\Delta E_0 \propto 1/L^3 \longrightarrow 0 \Longrightarrow$ **scattering** by **Lüscher's formula**

- potential $V(r)$





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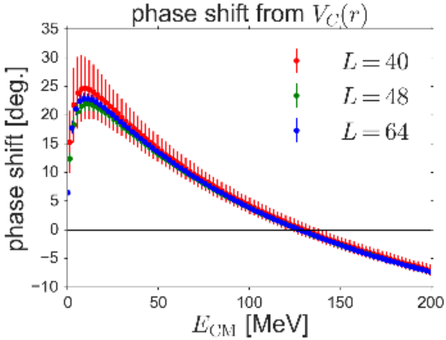
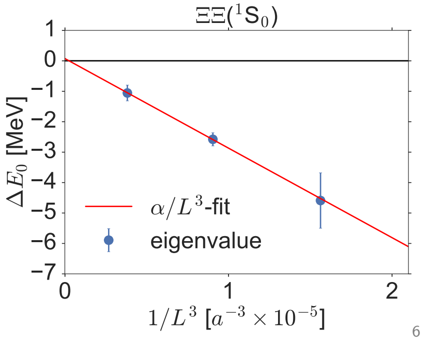
$$[H_0 + V]\,\psi = \Delta E \psi$$

☐ eigenvalue $\Delta E_0 \propto 1/L^3 \longrightarrow 0 \Longrightarrow$ **scattering** by **Lüscher's formula**
$\Rightarrow$ consistent with **potential analysis** — $\Xi\Xi(^1S_0)$ unbound (at $m_\pi = 0.51$GeV)

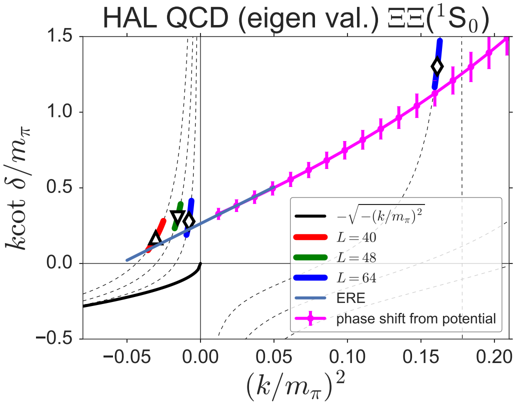




Sanity Check of ERE

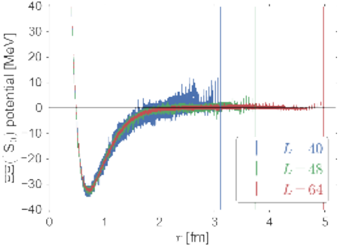
✓ phase shift from Lüscher's formula shows **reasonable behavior**

- finite volume energy shift
  $\Delta E_L \to \underline{k \cot \delta(k)}$
  at $L = 40, 48, 64$
- potential $V(r)$
  $\longrightarrow$ phase shift δ
- Effective Range Expansion
  $k \cot \delta = \frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} k^2$



Wavefunc. → Potential → Eigenenergies and Eigenfuncs.

— ▶ 1. Potential

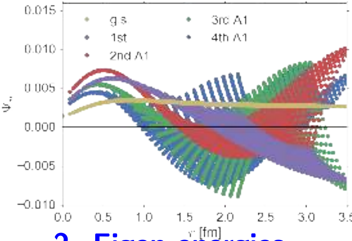


■ **Solve**

Schrödinger eq.
in Finite Volume

———— ▶

2. Eigen-wave functions

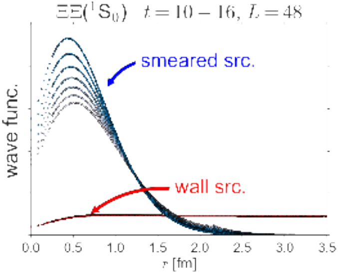


2. Eigen-energies

n	ΔE_n [MeV]
g.s.	-2.58(1)
1st	52.49(2)
2nd	112.08(2)
3rd	169.78(2)
4th	224.73(2)

■ **HAL QCD method** ⇑

0. wavefunction



■ **Feedback**

◀ ————

decompose
by eigenmodes

Contaminations of Excited States in Correlator

HAL pot. ▶ eigenfunc/value Ψ_n, ΔE_n ▶ eigenmode decomposition

$$R^{\rm wall/smear}(\vec{r},t) = \sum_n a_n^{\rm wall/smear}\Psi_n(\vec{r})\exp\left(-\Delta E_n t\right)$$

$$\therefore R(t) \equiv R(\vec{p}=0,t) = \sum_r R(\vec{r},t) = \sum_n b_n^{\rm wall/smear} e^{-\Delta E_n t}$$

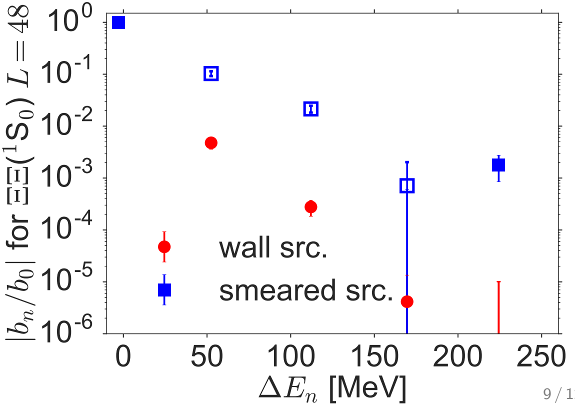
☐ ex. **1st excited state**

- **wall source**
  $b_1/b_0 \ll 1$ %
- **smeared source**†
  $b_1/b_0 \simeq -10$ %

- with energy gap
  $E_1 - E_0 \simeq 50$ MeV
  for $L^3 = 48^3$

†unfilled symbols: $b_n/b_0 < 0$

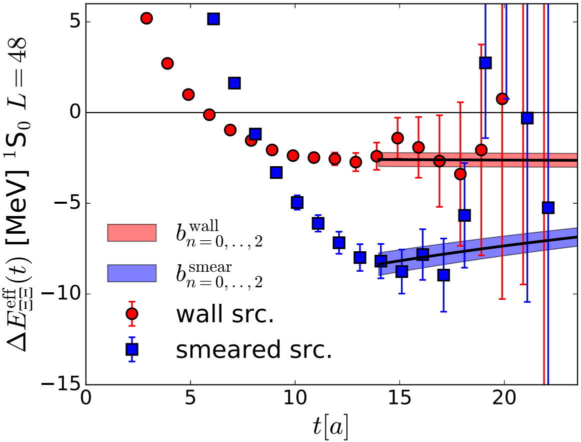
"contamination" of excited states b_n/b_0



Diagnosis of Fake Plateau

$$\Delta E_{\mathrm{eff}}^{\mathrm{wall/smear}}(t) \equiv \log \frac{R(t)}{R(t+1)} = \log \frac{\sum_n b_n^{\mathrm{wall/smear}} \exp\left(-\Delta E_n t\right)}{\sum_n b_n^{\mathrm{wall/smear}} \exp\left(-\Delta E_n (t+1)\right)}$$

- "direct measurement" — reproduced by low-lying states



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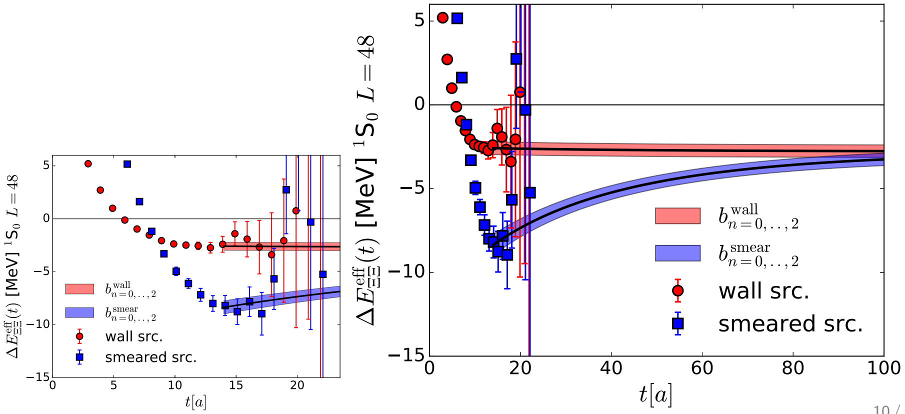
■ "direct measurement" — reproduced by low-lying states

□ **g.s. saturation** of smeared source — **100 lattice units ∼ 10 fm !!!**

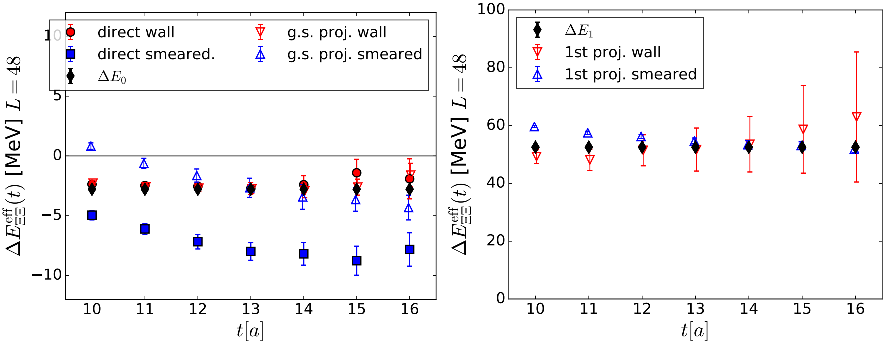


Eigenmode Projected Direct Method

$$R_{\mathrm{proj}}(t) = \sum_{\vec{r}} \Psi_n^\dagger(\vec{r}) R^{\mathrm{wall/smear}}(\vec{r}, t)$$

$$\Delta E_{\mathrm{eff}}(t) = \log \frac{R_{\mathrm{proj}}(t)}{R_{\mathrm{proj}}(t+1)}$$

plateaux are reproduced by correct sink op., i.e., eigenfunction Ψ_n



Summary: Two Baryons in Lattice QCD

- **HAL QCD method** can control the scattering states contamination.

- Some systematics in HAL QCD method are also under control.
  - potential with single baryon saturation scale
  - the derivative expansion is converged at low energy

- **HAL QCD method** and **Lüscher's method** give the same results.
  Lüscher's formula is theoretically corret, but the inputs by direct method are wrong, cf. sanity check in arXiv:1703.07210.

- **NBS wavefunc.** + **"potential"** $\Longrightarrow$ diagnosis of contaminations
  and the oigin of **fake plateau** in the direct method

4 Appendix

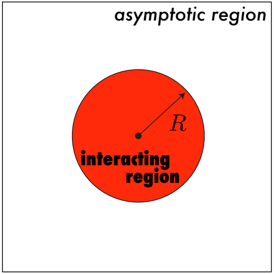
(Original) HAL QCD Method

■ **Nambu-Bethe-Salpeter wave function**

$\psi_k(\vec{r}) = \langle 0|B(\vec{x}+\vec{r},0)B(\vec{x},0)|BB, W_k\rangle$

- asymptotic region — $r > R$

$$\psi_k(\vec{r}) \simeq C\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$



- interacting region — $r < R$

$$[E_k - H_0]\,\psi_k(\vec{r}) = \int d\vec{r'}U(\vec{r},\vec{r'})\psi_k(\vec{r'})$$

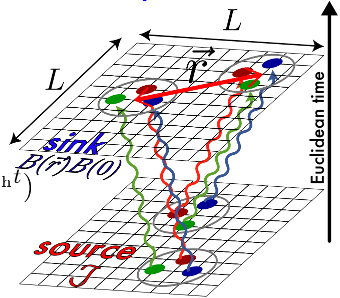
$U(r,r')$: E-independent potential, which is faithful to **the phase shift**

□ we calculate **4-pt function**

$$R(\vec{r},t) \equiv \frac{\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\overline{\mathcal{J}}(0)|0\rangle}{\{G_B(t)\}^2}$$

$$= \sum_n A_n\psi_{W_n}(\vec{r})e^{-(W_n-2m_B)t} + \mathcal{O}(e^{-\Delta W_{\rm th}t})$$

$$\longrightarrow A_0\psi_{W_0}(\vec{r})e^{-(W_0-2m_B)t}$$



⇒ g.s. saturation is required !!

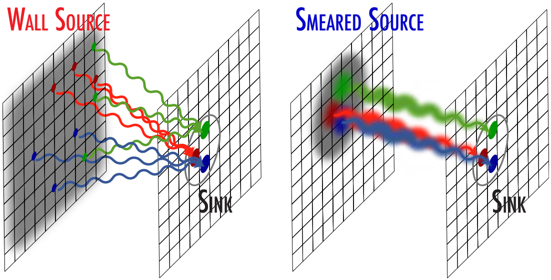
Lattice Setup: Wall Source and Smeared Source

☐ ex. $\Xi\Xi(^1S_0)$ **interaction** from HAL QCD methods

27 multiplet — the same rep. as $NN(^1S_0)$

☐ CHECK **2 quark sources** — mixture of excited states are different

- **wall source**
  standard of HAL QCD
- **smeared source**
  standard of direct method†



■ setup — $2+1$ improved Wilson + Iwasaki gauge†

- lattice spacing: $a = 0.08995(40)$ fm, $a^{-1} = 2.194(10)$ GeV
- lattice volume: $32^3 \times 48$, $40^3 \times 48$, $48^3 \times 48$, and $64^3 \times 64$

$m_\pi = 0.51$ GeV, $m_N = 1.32$ GeV, $m_K = 0.62$ GeV, $m_\Xi = 1.46$ GeV

† Yamazaki-Ishikawa-Kuramashi-Ukawa, arXiv:1207.4277.

NLO potential

$R^{W/S}(r,t)$: wall(smeared) src. wavefunction

$$V_{\rm eff}^{W} = \frac{1}{4m}\frac{(\partial^2/\partial t^2)R^W}{R^W} - \frac{(\partial/\partial t)R^W}{R^W} - \frac{H_0 R^W}{R^W} = V_{\rm LO} + V_{\rm NLO}\frac{\nabla^2 R^W}{R^W}$$

$$V_{\rm eff}^{S} = \frac{1}{4m}\frac{(\partial^2/\partial t^2)R^S}{R^S} - \frac{(\partial/\partial t)R^S}{R^S} - \frac{H_0 R^S}{R^S} = V_{\rm LO} + V_{\rm NLO}\frac{\nabla^2 R^S}{R^S}$$

- $V_{\rm LO}(r)$ and $V_{\rm NLO}(r)$