Anderson localization in CP(N-1)

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Anderson model

- proposed by Anderson in 1958
- describes conductor $\rightarrow$ insulator transition
- (extended) Bloch states become localized (trapped) by impurities
- impurities modeled by randomized potential
- tight binding model

$$H = \sum_{<x,y>} h_{xy} |x\rangle\langle y| + \eta \sum_{x} v_x |x\rangle\langle x|,$$

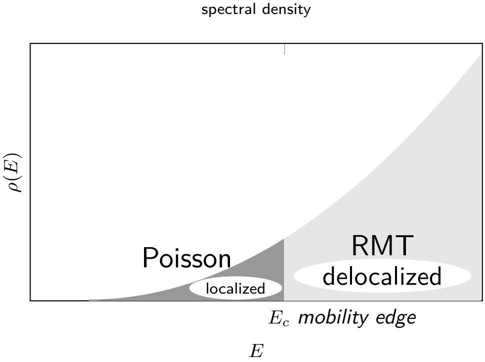
  with v_x random; simplest case: $h_{xy} = const.$
- η determines the strength of the disorder

Anderson transition

- for large disorder a transition in the spectrum occurs

$$H\psi = E\psi$$

- low eigenmodes ($E < E_c$) become localized (Poisson statistics)
- $E_c > E_{\text{Fermi}}$ no conduction possible



RMT observables

- how to distinguish localized and delocalized modes
- participation ratio: $PR(\lambda) = (V \sum_x |\psi_\lambda(x))|^4)^{-1}$
- fraction of space occupied by the mode

$\rightarrow$ localized modes: PR scales like $1/V$

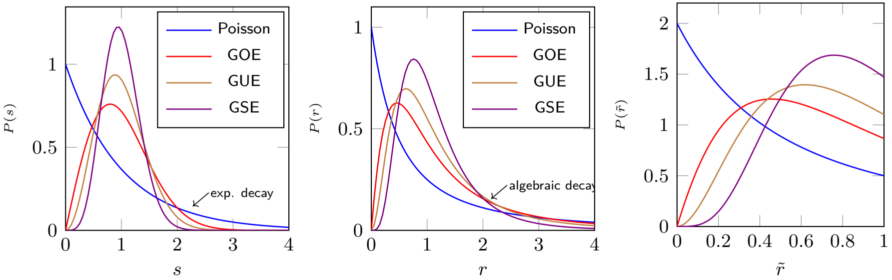
$\rightarrow$ delocalized modes: PR stays the same

RMT observables

- level spacing distribution $P_\lambda(s)$ of *unfolded* spectrum
- or distribution of ratio of consecutive levelspacings, $P_\lambda(r)$ or $P_\lambda(\tilde{r})$ (see, e.g., [1212.5611] Atas et al.):

$$s_n = \lambda_{n+1} - \lambda_n \qquad r = \frac{s_{n+1}}{s_n} \qquad \tilde{r} = \frac{\min(s_{n+1}, s_n)}{\max(s_{n+1}, s_n)}$$

- localized modes don't overlap → poissonian distributed
- delocalized modes obey RMT statistics (level repulsion)



Situation in QCD

- below T_c: chirally broken phase
- Banks-Casher relation: $\langle\bar{\psi}\psi\rangle = \int_0^\infty d\lambda\,\rho(\lambda)\frac{2m}{\lambda^2+m^2} \sim \rho(0)$

$\rightarrow$ $\langle\bar{\psi}\psi\rangle \neq 0$: no gap in the spectrum of $\not{D}$

- above T_c: $\langle\bar{\psi}\psi\rangle \rightarrow 0 \quad \Rightarrow$ gap emerges
- mobility egde $\lambda_c/m_{ud} \neq 0$ above deconfinement transition ($T_c \approx 160\,$MeV), [1208.3475] Kovács *et al.*, [1410.8308] Giordano *et al.*, [1604.00768] *Cossu et al.*
- also in pure gauge theory $\lambda_c \neq 0$ coincides with deconfinement transition, [1706.03562] Kovács *et al.*
- low modes get trapped by Polyakov loop defects $\rightarrow$ become localized, [1105.5336] Bruckmann *et al.*
- view gauge field as random disorder and tune strength of disorder with T

$CP(N-1)$

- nonlinear sigma model

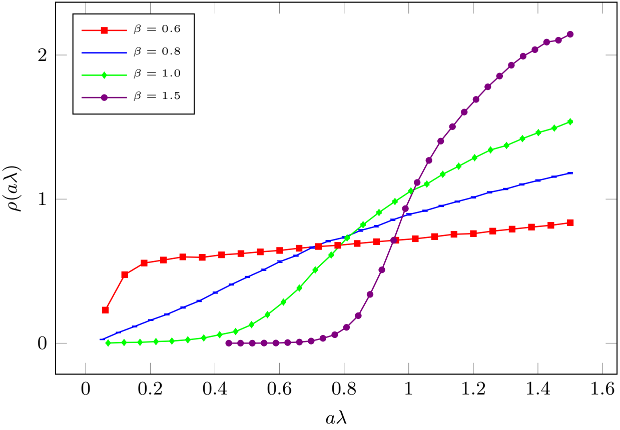
$$S = \frac{1}{g^2} \int d^2x \, (D_\nu n)^\dagger (D_\nu n), \qquad n^\dagger n = 1, \; n \in \mathbb{C}^N$$

- D_ν: U(1) gauge covariant derivative
- no kinetic term $\rightarrow$ auxiliary gauge field
- global U(N) symmetry
- asymptotic freedom
- dynamically generated mass gap
- on the lattice:

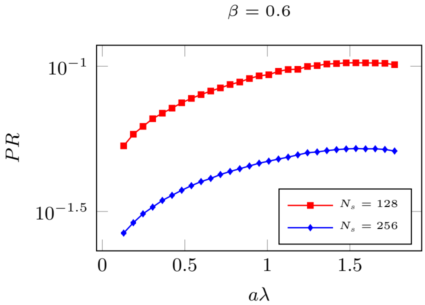
$$S = -N\beta \sum_{x,\nu} (n_x^\dagger U_{x,\nu} n_{x+\hat{\nu}} + n_{x+\hat{\nu}}^\dagger U_{x,\nu}^\dagger n_x)$$

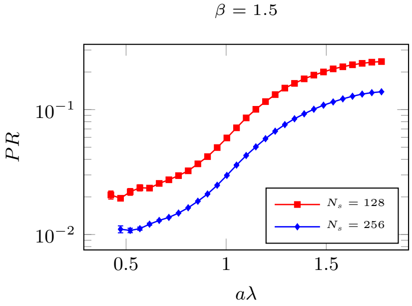
Spectral density

- quenched staggered fermions in CP(N-1) background, i.e., using the U-field configurations
- high T: gap emerges



Participation ratio

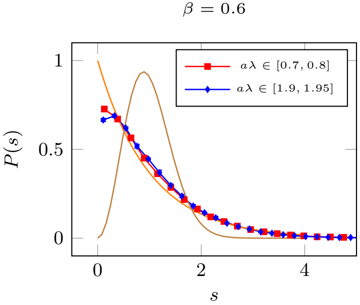


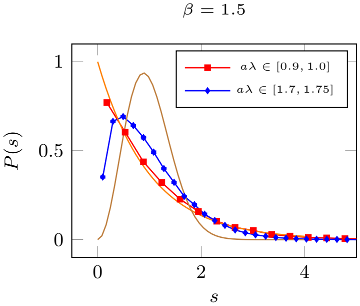


- always scales with volume

$\rightarrow$ modes always localized

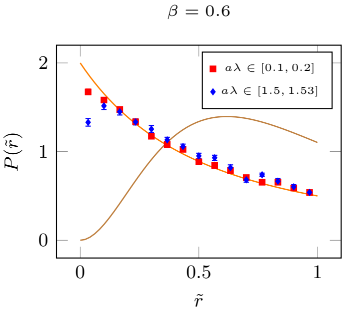
Level spacing

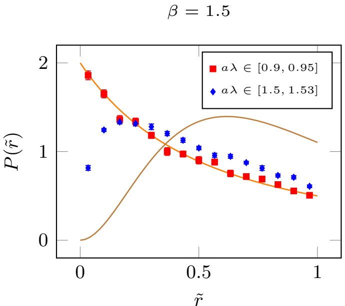




- Possonian in low end of the spectrum
- higher modes behave like mixed ensemble
- only $d > 2$ Anderson model shows transition $\rightarrow$ also here?

Ratio of level spacings





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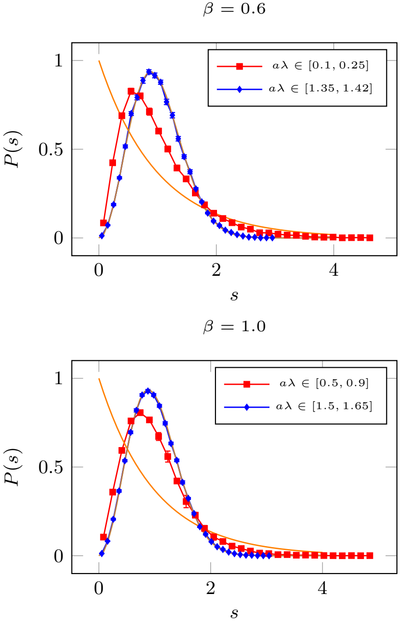
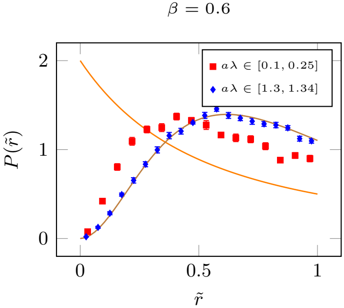
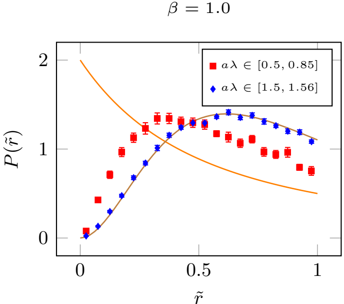
Comparison to 3D (level spacing)



- high end of spectrum: GUE
- low end: mixed ensemble (larger volume?)

Comparison to 3D (ratio of level spacings)





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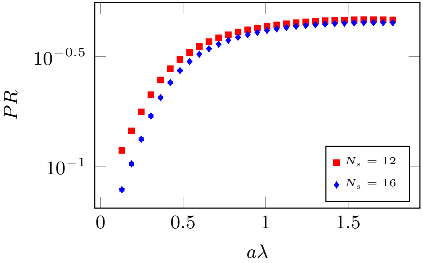
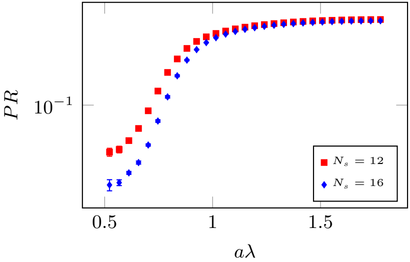
Comparison to 3D (PR)

$\beta = 0.6$



$\beta = 1.0$



- low end localized
- high modes extended

Conclusion

- transition present in QCD: mobility edge defines T_c compatible with deconfinement crossover
- no transition in 2D CP(N-1)
- transition (maybe) present in 3D CP(N-1) (study larger volume)

$\rightarrow$ (maybe) more general feature of lattice field theory

? any lattice model with gauged Dirac-like operator, $d > 2$, gap in the spectrum $\rightarrow$ Anderson transition (no onsite disorder, but in hopping terms)

- physics only affected if it survives continuum limit