Instanton effects on CP-violating gluonic correlators

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June 19, 2017

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Introduction

- At high temperature, T dependence of the topological susceptibility $\chi_t(T)$ is well described by dilute instanton gas approximation (DIGA).

$$\chi_t(T) \propto \int d\rho n(\rho) n_T(\rho) \propto \Lambda^b T^{4-b},$$
$$n_T(\rho) \propto \exp\left(-\frac{\pi^2}{3}\rho^2 T^2\right)$$

- However, at low T DIGA is invalid because large instantons are not negligible and the interaction between instantons must be taken into account.
- At zero T, the leading instanton contribution to χ_t has IR divergence in the instanton-size (ρ) integration.

Introduction

Questions

- The instanton picture exists in the local observable?
- The instanton picture exists at zero temperature?
- In the finite temperature regime, from which temperature instanton picture exists.

The leading instanton contribution to the gluonic two point correlation function

$$\langle F^2(x) F^2(0) \rangle$$

is

- IR divergent in SU(3) YM at zero T,
- Finite in SU(2) at zero T,
- Finite in SU(N) at non-zero T.

Introduction

- Considering gluonic operator

$$
\begin{aligned}
s(x) &= \frac{1}{2}\mathrm{tr}F_{\mu\nu}F_{\mu\nu}, \\
q(x) &= \frac{1}{16\pi^2}\mathrm{tr}F_{\mu\nu}\tilde{F}_{\mu\nu}.
\end{aligned}
$$

- Due to the self-duality of the instanton solution $FF = F\tilde{F}$, the x-dependence of the leading instanton contribution to three correlators

$$
\langle s(x)s(0)\rangle_\theta\,, \langle s(x)q(0)\rangle_\theta\,, \langle q(x)q(0)\rangle_\theta
$$

  is equivalent to that of $\langle F^2(x)F^2(0)\rangle_\theta$.
- Since $\langle s(x)q(0)\rangle_Q$ has no perturbative contribution, the instanton effects may dominate.

The CP-violating correlation function

At zero temperature [M. Dine, P. Draper and G. Festuccia '15]

There are two estimates on $\langle s(x)q(0)\rangle_\theta$ at zero T as

$$\begin{aligned}\langle s(x)q(0)\rangle_\theta^{OPE} &= k\frac{\alpha}{\pi}\frac{1}{x^4}\langle F\tilde{F}\rangle_\theta\,,\\ \langle s(x)q(0)\rangle_\theta^{Inst.} &= c\Lambda^b x^{-b+8}\sin\theta = c\Lambda^{22/3}x^{-2/3}\sin\theta.\ (\text{if SU(2)YM})\end{aligned}$$

- There are two facts. Which one is correct?
- While OPE is correct at short distance, instanton calculus may catch some feature of the correlation.
- The instanton picture is expected to be correct at high temperature, and we can study how the correlator is described by using instanton configuration.
- In this talk, we focus on the study at high temperature.

The CP-violating correlation function

At non-zero temperature

There are also two estimates on $\langle s(x)q(0)\rangle_\theta$ at non-zero T as

$$\begin{aligned}\langle s(x)q(0)\rangle_\theta^{OPE} &= k\frac{\alpha}{\pi}\frac{1}{x^4}\langle F\tilde{F}\rangle_\theta\,,\\ \langle s(x)q(0)\rangle_\theta^{Inst.} &= c\Lambda^b x^{-\gamma}T^{-b-\gamma+8}\sin\theta.\end{aligned}$$

- The thermal effects modify the power os x, γ, as a function of x.
- Since the explicit thermal instanton solution is known in the so-called *singular gauge*, we need to gauge transform it to regular gauge so as to be integrable by the instanton position.

$$\langle F^2(x)F^2(0)\rangle = \int d^4z d\rho n(\rho) n_T(\rho) F^2(x,z,\rho)F^2(0,z,\rho)$$

Gauge transformation of HS-caloron into regular gauge

- The HS-caloron solution is written as

$$\begin{aligned} A_\mu(x) &= i\tau^{(-)}_{\mu\nu}\partial_\nu \ln \Pi(x), \\ \Pi(x) &= 1 + \frac{2\lambda^2}{R}\frac{\mathrm{sh}R}{\mathrm{ch}R - \cos R_4} \approx 1 + \frac{4\lambda^2}{\mathcal{R}^2}, \\ R &= 2\pi T|\vec{x} - \vec{z}|,\ R_4 = 2\pi T(x_4 - z_4),\ \mathcal{R}^2 = R_\mu R_\mu \\ \lambda &= \pi\rho T. \end{aligned}$$

- We consider the periodic gauge transformation such as

$$g = \frac{i\tau^\dagger_\mu \bar{R}_\mu}{\sqrt{\bar{\mathcal{R}}^2}},$$

where $\bar{R}_\mu = (R_i, \sin R_4)$.

Gauge transformation of HS-caloron into regular gauge

- When $x_\mu \to z_\mu$,

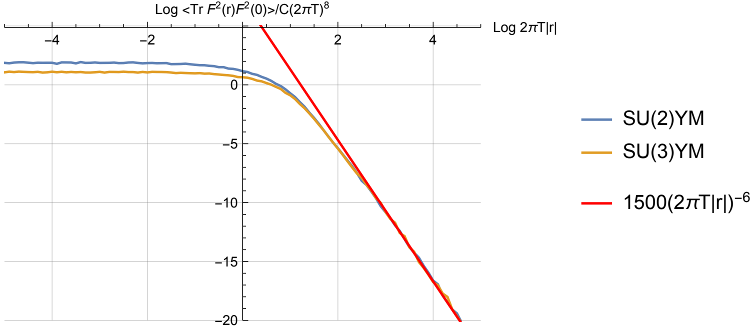
$$
\begin{aligned}
A_\mu(x) &\approx f(R_\mu) g^{-1}\partial^x_\mu g, \;\; f(R_\mu) \equiv \frac{4\lambda^2}{4\lambda^2+\mathcal{R}^2} \\
g^{-1}\partial^x_\mu g &\approx 2\pi T \frac{-2\tau^+_{\mu\nu} R_\nu}{\mathcal{R}^2}
\end{aligned} \tag{1}
$$

- We obtain regular caloron gauge solution as

$$
\begin{aligned}
A'_\mu(x) &= g A_\mu g^{-1} + g\partial^x_\mu g^{-1}, \\
&\approx (1 - f(R_\mu)) g \partial^x_\mu g^{-1}, \\
&= \frac{\mathcal{R}^2}{4\lambda^2+\mathcal{R}^2} \times 2\pi T \frac{-2\tau^+_{\mu\nu} R_\nu}{\mathcal{R}^2}, \\
&= -2\pi T \frac{2\tau^+_{\mu\nu} R_\nu}{4\lambda^2+\mathcal{R}^2}.
\end{aligned}
$$

Leading caloron contribution to the correlator

$$\ln \frac{\langle F^2(x) F^2(0) \rangle^{Inst.}_{Q=1}}{C_{inst}(2\pi T)^8} \text{ v.s. } \ln(2\pi T x)$$



We obtain $\langle F^2(x) F\tilde{F}(0) \rangle^{Inst.}_{I=1} \propto x^{-6}$ when $xT \gg 1$.

Setup

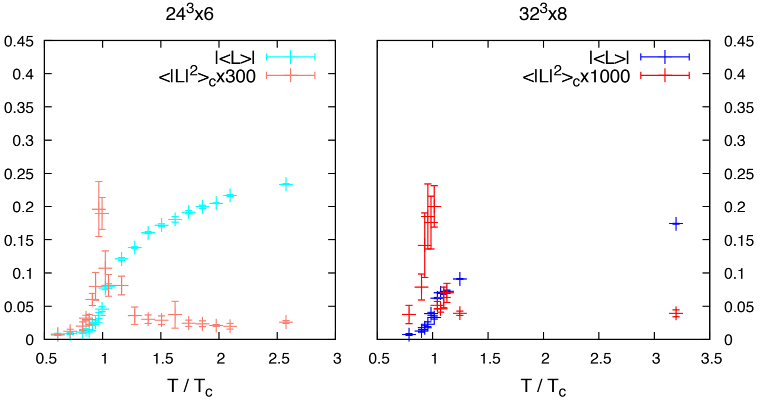
Ensembles

- SU(2) quenched simulation
- Wilson gauge action
- $24^3 \times 6,\ \beta = 2.3223 \sim 2.6187,\ T/T_c = 0.722 \sim 5.694$
  $32^3 \times 8,\ \beta = 2.4367 \sim 2.8986,\ T/T_c = 0.788 \sim 3.196$

Measurements

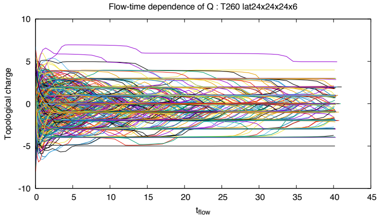
- Since we are interested in local structure of classical configuration, we use the configurations after Gradient flow with Wilson gauge action.
- $\mathcal{O}(a^4)$-improved action density, $s(x)$, and $\mathcal{O}(a^4)$-improved topological charge density, $q(x)$.
- CP-violating gluonic correlation function : $\langle s(x)q(0)\rangle$.

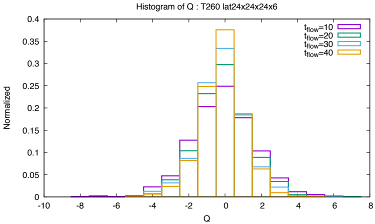
Polyakov loop as a function of T



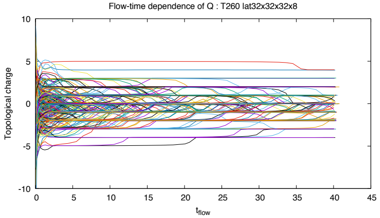
Gradient flow at large flow time

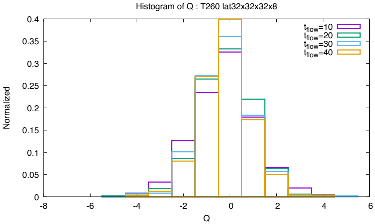
- $24^3 \times 6 \ T/T_c = 1.16$



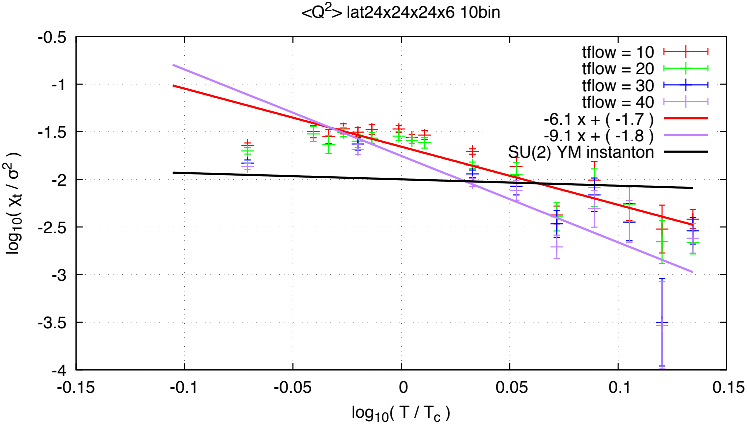


- $32^3 \times 8 \ T/T_c = 1.25$





- In SU(3) YM the T dependence of the topological susceptibility $\chi_t(T)$ at $T/T_c > 2$ is just same as what DIGA predicts.
- We will also investigate the T dependence of $\chi_t(T)$ in SU(2) YM.



Comparison between DIGA and Lattice:

$$\ln\left(\frac{\langle s(r)q(0)\rangle^{Inst.}_{Q=1}}{C_{inst}(2\pi T)^8}+B\right)+\ln A \text{ v.s. } \ln\left(\frac{\langle s(x)q(0)\rangle_{Lat.}}{(2\pi T)^8}\right)$$

in $\ln(2\pi x T)\in[0,1.8]$.

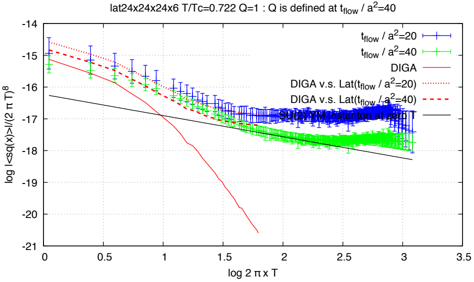


Figure: $T/T_c = 0.722$, $B = 0.34,\ 0.30$ for $\sqrt{8t_{flow}}\sigma = 4.2,\ 6.0$

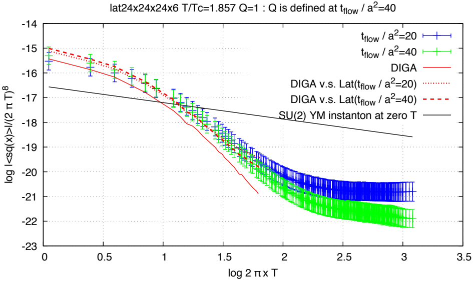


Figure: $T/T_c = 1.875$, $B = 0.029,\ 0.012$ for $\sqrt{8t_{flow}}\sigma = 1.6,\ 2.3$

Conclusion

- In order to study the validity of the instanton picture to the local observable at high temperature, we investigated the x-dependence of CP-violating gluonic correlator using lattice simulation.
- The two-point gluonic correlation function is calculated in the HS-caloron background with singular gauge transformation.
- After gradient flow, the x-dependence of CP-violating correlator approaches that of classical HS-caloron. Compared to the measurement at low temperature, the CP-violating correlator at high T approaches with much short flow time.

Thank you for attention!

Appendix

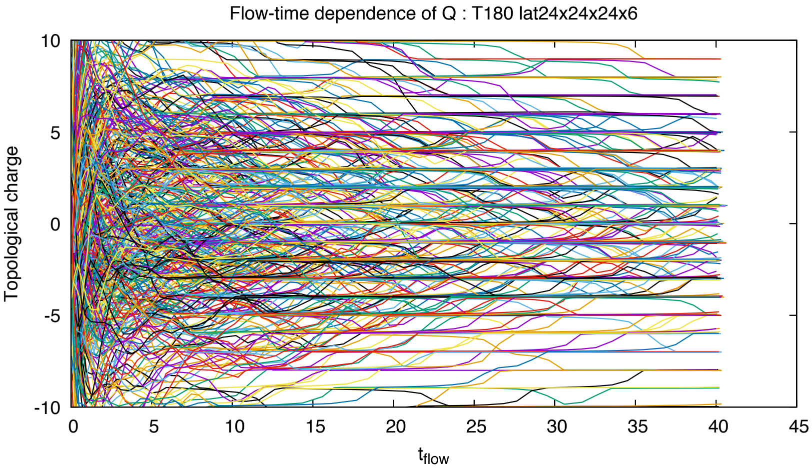


Figure: $T/T_c = 0.722$ Lvol $= 24^3 \times 6$