The Chiral Separation Effect in quenchend finite-density QCD

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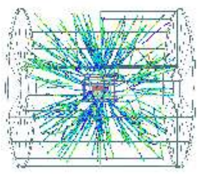
Introduction



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- Anomalous Transport Effects
  - medium with chiral fermions
  - related to the chiral anomaly
  - macroscopic current
- Chiral Separation/Magnetic Effect

$$j_i^{A/V} \propto \mu_{V/A} B_i, \qquad \mu_{V/A} = \frac{\mu_R \pm \mu_L}{2}$$

- Heavy-Ion Collisions
  - quark-gluon plasma
  - large magnetic fields ($B \sim m_\pi^2$)

Chiral Separation Effect

- CSE characterised by Chiral Separation Conductivity

$$j_i^A = \sigma_{CSE} B_i$$

- For one flavour of free chiral fermions with charge q

$$\sigma^0_{CSE} = \frac{N_c q}{2\pi^2}\mu$$

- Conductivity for free fermions fixed by the anomaly
- Corrections possible in interacting theories
  - current couples to dynamical gauge field
  - spontaneous breaking of chiral symmetry

Chiral Separation Effect

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  - current couples to dynamical gauge field
  - **spontaneous breaking of chiral symmetry**

Chiral Separation Effect

- Non-perturbative study of the CSE in QCD
  - Lattice QCD with external magnetic field
- CSE in phase with spontaneously broken chiral symmetry?
- Important “ingredients” for CSE:
  - finite μ
  - chiral symmetry
- To avoid sign problem at finite μ
  - work in quenched approximation
  - fermionic vacuum loops neglected

Chiral Symmetry on the Lattice

- In the continuum: chiral symmetry $\Leftrightarrow \gamma_5 D + D\gamma_5 = 0$
- On the lattice: Nielsen–Ninomiya "No-Go" theorem
- Ginsparg–Wilson equation:

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

- Lattice version of chiral symmetry:
  (M. Lüscher [hep-lat/9802011])

$$\delta\psi = i\varepsilon\gamma_5\left(1 - \frac{a}{2}D\right)\psi, \quad \delta\bar{\psi} = \bar{\psi}i\varepsilon\left(1 - \frac{a}{2}D\right)\gamma_5$$

Chiral Symmetry on the Lattice

- Overlap Dirac operator $(m_q = 0)$ at finite μ:

  (J. Bloch and T. Wettig [hep-lat/0604020])

$$D_{\text{ov}} = \frac{1}{a}\left(\mathbb{1} + \gamma_5 \, \text{sgn}[\gamma_5 D_{\text{W}}(\mu)]\right)$$

- $D_{\text{ov}}(\mu)$ respects Ginsparg–Wilson equation
- Wilson–Dirac operator:

$$\gamma_5 D_{\text{W}}(\mu)\gamma_5 = D_{\text{W}}^{\dagger}(-\mu)$$

- Sign function of non-Hermitian matrix numerically expensive

Conserved Lattice Current

- Continuum axial current

$$\langle j_\mu^A \rangle = \langle \bar{\psi} \gamma_\mu \gamma_5 \psi \rangle$$

- On the lattice
  - Noether current of lattice action
  - Correct transformation properties under lattice chiral symmetry
- For the overlap Dirac operator
  (Y. Kikukawa and A. Yamada [hep-lat/9810024])
  - $\gamma_\mu \gamma_5 \to \frac{1}{2}\left(\gamma_5 \frac{\partial D_{ov}}{\partial \Theta_\mu} - \frac{\partial D_{ov}}{\partial \Theta_\mu} \gamma_5 (\mathbb{1} - D_{ov})\right)$
  - with (external) $U(1)$ lattice gauge field $\Theta_\mu(x)$

Conserved Lattice Current

- Lattice axial current

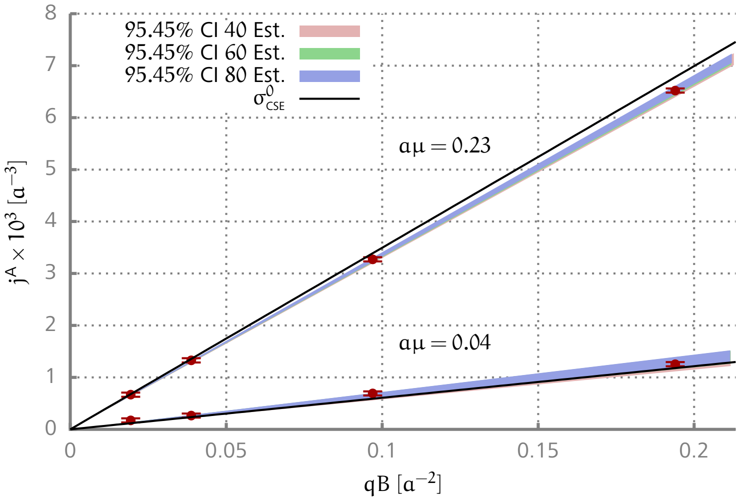
$$\langle j_\mu^A \rangle = \mathrm{tr}\left(D_{ov}^{-1} \frac{\partial D_{ov}}{\partial \Theta_{x,\mu}} \gamma_5 \right)$$

- Advantage
  - not renormalised ($Z_A = 1$)
  - results directly comparable to continuum σ_{CSE}
- Disadvantage
  - involves derivatives of the matrix sign function
  - numerically challenging
  - development of numerical tools
    (MP and P. Buividovich [1604.08057])

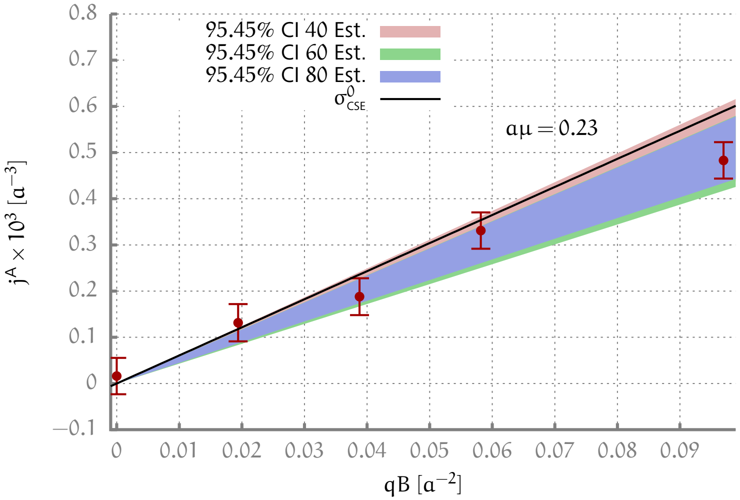
Lattice Setup

- Quenched $SU(3)$ configurations with Lüscher-Weisz action
  - $\{\beta = 8.45,\ V = 6 \times 18^3\}$, $\{\beta = 8.1,\ V = 14 \times 14^3\}$
  - Temperature T above and below $T_c \sim 300$ MeV
- D_{ov} can have exact zero modes for $m_q = 0$
  - Lattice index theorem:
    Topological charge $Q \leftrightarrow$ number of zero modes of D_{ov}
  - D_{ov} not invertible for quark mass $m_q = 0$ and $Q \neq 0$
  - For $m_q = 0$: Only configurations with $Q = 0$
  - Cross check: small finite m_q for $|Q| > 0$

Results: High temperature, $T > T_c$, $Q = 0$, $am_q = 0$



Results: High temperature, $T > T_c$, $|Q| = 1$, $am_q = 0.001$



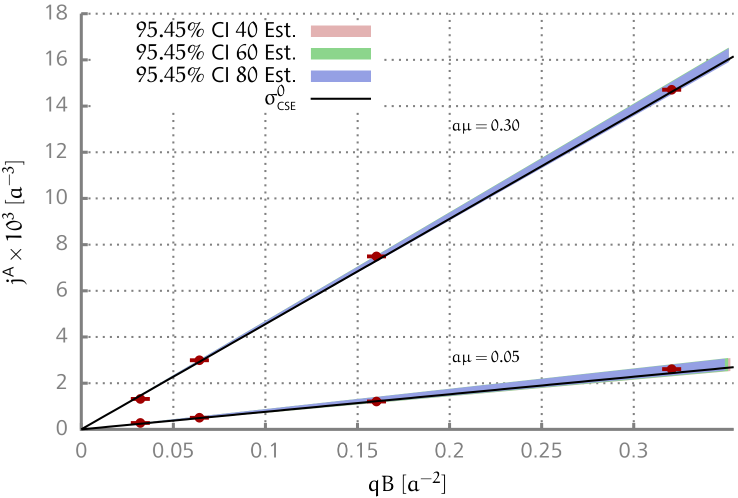
Low temperature

- Effect of spontaneous chiral symmetry breaking?
- Expectation :
  (G. M. Newman and D. T. Son [hep-ph/0510049])

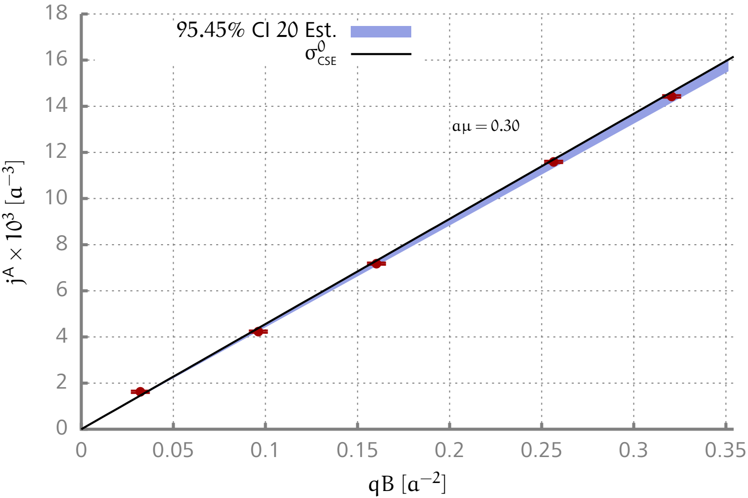
$$\sigma_{\text{CSE}} = \sigma^0_{\text{CSE}} \left(1 - g_{\pi^0\gamma\gamma}\right) \qquad g_{\pi^0\gamma\gamma} : \text{"}\pi^0 \to 2\gamma \text{ amplitude"}$$

- Estimated effect size
  - from linear sigma model $g_{\pi^0\gamma\gamma} = \frac{7\zeta(3)m^2}{4\pi^2T^2}$
  - $T \sim 100$ MeV and "constituent quark mass" $m \sim 300$ MeV
  - $g_{\pi^0\gamma\gamma} \sim \mathcal{O}(1)$
- Large corrections expected!

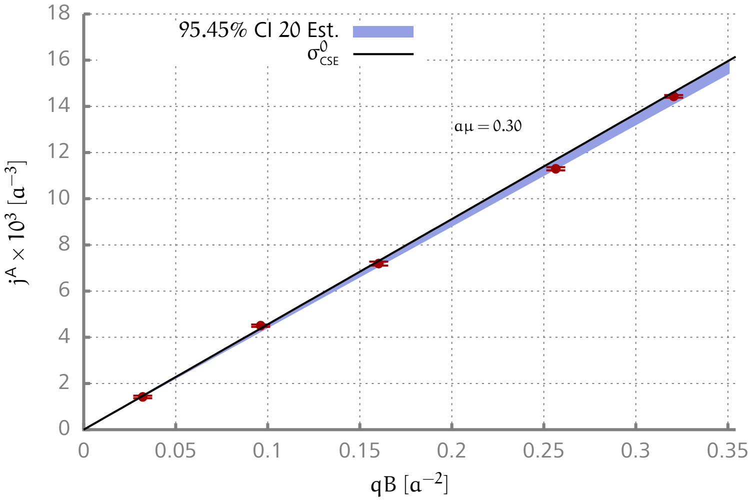
Results: Low temperature, $T < T_c$, $Q = 0$, $am_q = 0$



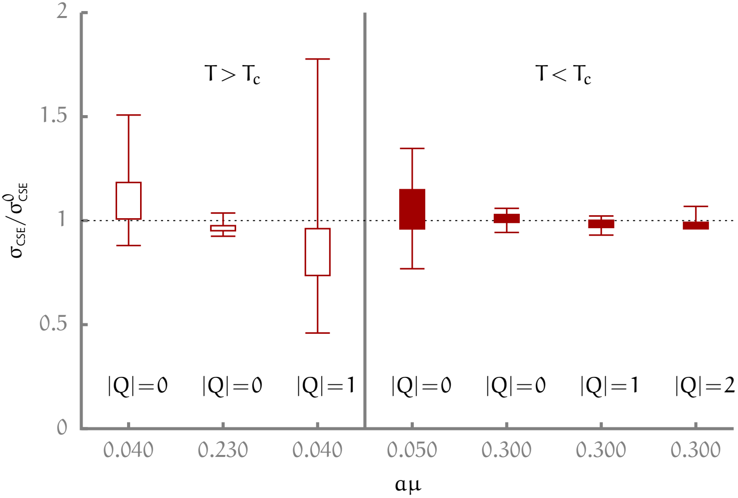
Results: Low temperature, $T < T_c$, $|Q| = 1$, $am_q = 0.001$

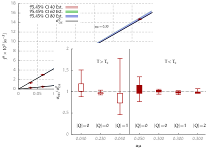


Results: Low temperature, $T < T_c$, $|Q| = 2$, $am_q = 0.001$



Results: Confidence intervals for σ_{CME}





Summary

- Study of the CSE in quenched QCD with overlap fermions
- σ_{CSE} in confined and deconfined phase
- Non-renormalisation of CSE in quenched lattice QCD
- Renormalisation constant Z_A can be calculated from CSE

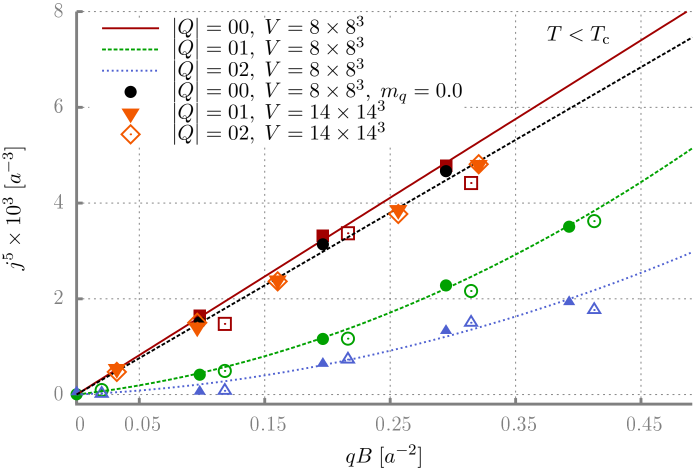
Outlook

- Effects of quenched approximation?
- Quenched QCD: chiral symmetry restored for $\mu \neq 0$ $(B = 0)$
  (M. A. Stephanov `[hep-lat/9604003]`)
- Small μ sufficient for CSE $\longrightarrow$ Reweighting?

Backup Slides

Small lattice results:

$T < T_c, a\mu = 0.10, am_q = 0.001, 0.002$



Results for $V = 8 \times 8^3$, $am_q = 0.002$ are shifted by $0.02/a^2$ in qB.

Numerics

- Stochastic estimators with Z_2–noise for trace
- $D_{ov}^{-1} = D_{ov}^\dagger (D_{ov} D_{ov}^\dagger)^{-1}$: Invert $D_{ov} D_{ov}^\dagger$ with CG
- Treat k low lying modes $|\psi_k\rangle$ of $D_{ov} D_{ov}^\dagger$ exactly
  - speedup CG
  - exact trace over space spanned by $|\psi_k\rangle$

$$\mathrm{tr}(A) = \underbrace{\sum_{i=1}^{k} \langle \psi_i | A | \psi_i \rangle}_{\text{exact}} + \underbrace{\sum_{i=k+1}^{n} \langle \psi_i | A | \psi_i \rangle}_{\text{stoch. est.}}$$

  - number of zero modes $\leftrightarrow$ top. charge $|Q|$ (Atiyah–Singer index theorem)