

Testing holographic principle using lattice simulations

Raghav G. Jha
Syracuse University

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with Simon Catterall, David Schaich and Toby Wiseman



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Motivation

Study maximally supersymmetric Yang-Mills (SYM) in $(p + 1)$ dimensions for $p < 3$. This is conjectured to be dual to Type IIA/B superstring theory containing stack of N D p -branes in the *decoupling limit*.

- ⇒ At low temperatures (strong coupling), there is a dual supergravity theory (as low-energy description of Type II string theory).
- ⇒ We want to use gauge/gravity duality to understand it from SYM theory.
- ⇒ In this case, the gauge theory is strongly coupled and we use *lattice* to study this system.

In this talk, I will focus on the $p=1$ case

Lattice construction of $\mathcal{N}=4$ SYM

SUSY on the lattice

Supersymmetry extends Poincaré symmetry adding spinorial generators Q and \bar{Q} to translations, rotations, boosts

The algebra includes $\{Q, \bar{Q}\} \sim P_\mu$

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Fortunately, there are certain constructions where we can exactly preserve a subset of SUSY algebra on the lattice based on *twisted* and orbifold constructions. See [0903.4881] for review.

Requirements : Enough supercharges in the continuum (at least 2^D , where D is space-time dimensions)

Unique features :

- ⇒ Single supercharge exactly preserved on the lattice in four dimensions.
- ⇒ Gauge symmetry, \mathbb{Q} nilpotent symmetry, S_5 point group symmetry.

$\mathcal{N} = 4$ SYM

—The only known 4d theory with a supersymmetric lattice formulation. Also the simplest non-trivial field theory in four dimensions without gravity.

—Context for development of AdS/CFT correspondence in large- N limit at strong couplings

$SU(N)$ gauge theory with four fermions Ψ^I and six scalars Φ^J , all massless and in adjoint rep.

Supersymmetric: 16 supercharges Q_α^I and $\bar{Q}_{\dot{\alpha}}^I$ with $I = 1, \dots, 4$. Fields and Q 's transform under global $SU(4) \simeq SO(6)$ R-symmetry

Conformal: β function is zero for any 't Hooft coupling
 $\lambda = g_{YM}^2 N$

Twisted construction

$$SO(4)_{tw} \equiv \text{diag} [SO(4)_{euc} \times SO(4)_R] \quad ; \quad SO(4)_R \subset SO(6)_R$$

The 16-real components of the spinors in $\mathcal{N} = 4$ SYM fill up the Dirac-Kähler multiplet :

$$\begin{pmatrix} Q_\alpha^1 & Q_\alpha^2 & Q_\alpha^3 & Q_\alpha^4 \\ \bar{Q}_{\dot{\alpha}}^1 & \bar{Q}_{\dot{\alpha}}^2 & \bar{Q}_{\dot{\alpha}}^3 & \bar{Q}_{\dot{\alpha}}^4 \end{pmatrix} = \mathbb{Q} + \gamma_\mu \mathbb{Q}_\mu + \gamma_\mu \gamma_\nu \mathbb{Q}_{\mu\nu} + \gamma_\mu \gamma_5 \mathbb{Q}_{\mu\nu\rho} + \gamma_5 \mathbb{Q}_{\mu\nu\rho\sigma} \\ \longrightarrow \mathbb{Q} + \gamma_a \mathbb{Q}_a + \gamma_a \gamma_b \mathbb{Q}_{ab} \\ \text{with } a, b = 1, \dots, 5$$

\mathbb{Q} 's transform with integer spin under the "twisted rotation group".

Twisting and repackaging gives a nilpotent, scalar supercharge \mathbb{Q} which can be exactly preserved on the lattice.

Twisted $\mathcal{N} = 4$ SYM fields

Start from a 5d setup

$$Q \text{ and } \bar{Q} \longrightarrow \mathbb{Q} + \mathbb{Q}_a + \mathbb{Q}_{ab}$$

$$\Psi \text{ and } \bar{\Psi} \longrightarrow \eta, \psi_a \text{ and } \chi_{ab}$$

$$A \text{ and } \Phi \longrightarrow \mathcal{A}_a \text{ and } \bar{\mathcal{A}}_a$$

Everything transforms with **integer spin** under $\text{SO}(4)_{tw}$ — **no spinors**. Then under dimensional reduction :

$$\mathbb{Q}, \mathbb{Q}_a \text{ and } \mathbb{Q}_{ab} \longrightarrow \mathbb{Q} + \gamma_\mu \mathbb{Q}_\mu + \gamma_\mu \gamma_\nu \mathbb{Q}_{\mu\nu} + \gamma_\mu \gamma_5 \mathbb{Q}_{\mu\nu\rho} + \gamma_5 \mathbb{Q}_{\mu\nu\rho\sigma}$$

$$\mathcal{A}_a \longrightarrow (A_\mu, \phi) + i(\Phi_\mu, \bar{\phi})$$

where, a b runs from $1 \cdots 5$ and μ from $1 \cdots 4$

Public code on GitHub

Code for supersymmetric construction of $\mathcal{N} = 4$ SYM evolved from MILC lattice QCD code and is hosted on GitHub.

Download, Fork, Contribute

<https://github.com/daschaich/susy>



Gauge/Gravity duality



Gravitational theory

Weakly coupled (low energy) string theory

Stack of N D $_p$ -branes, N units of charge at temperature T

Gauge theory

16 supercharge SYM theory in $D = p+1$ dimensions

SU(N) gauge group with large N , strongly coupled at temperature T

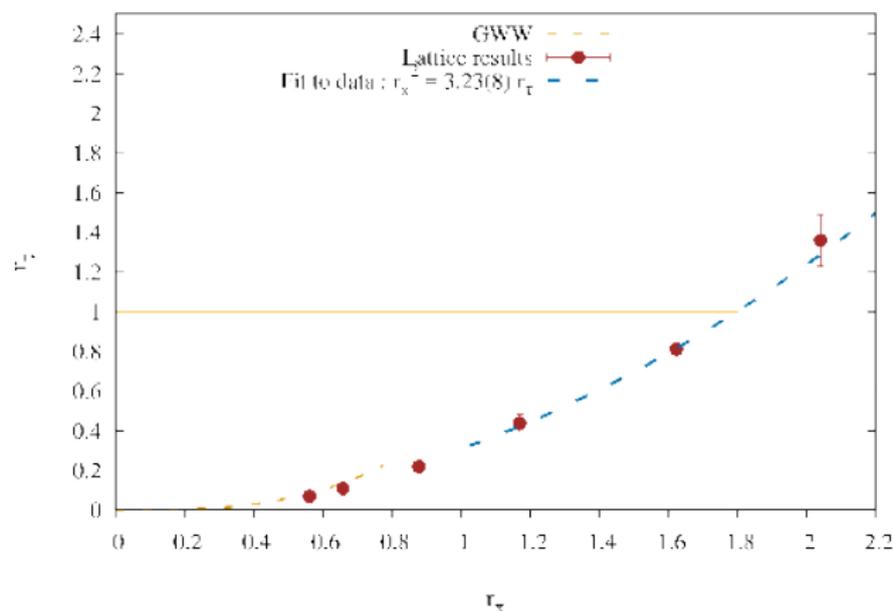
The map for $p=1$

- ⇒ Gravity has two different phases : Homogeneous black string & localized black hole with a first-order phase transition between them.
- ⇒ Gauge theory *should* have a deconfinement phase transition where deconfined phase is dual to - localized black hole phase *and* confined phase is dual to - homogeneous black string. Both phases have *different* thermodynamic behavior. Valid only at strong coupling and large-N.
- ⇒ Our aim : Confirm that the *map* is consistent through lattice calculations.

- ⇒ Dimensionally reduce the 4d theory to two dimensions.
- ⇒ We can construct dimensionless extents in two directions as : $r_x = \sqrt{\lambda}L$, $r_\tau = \sqrt{\lambda}\beta$. Dimensionless temperature, $t = 1/r_\tau$. Strong couplings implies low temperatures.
- ⇒ At high temperatures and $r_x \gg r_\tau$, there is a third (GWW) and closely separated second order phase transition. When coupling is increased to $r_\tau \gg 1$ and $\alpha = r_x/r_\tau = TL \approx \mathcal{O}(1)$, the gravity description kicks in and we have a first-order phase transition.
- ⇒ Gravity predicts the transition to occur across : $r_x^2 = c_{\text{grav}}r_\tau$.

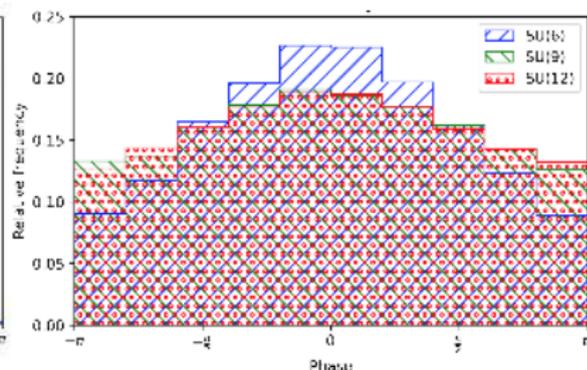
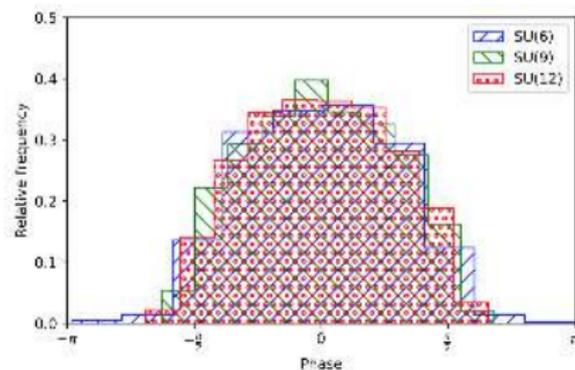
Results

Deconfinement transition

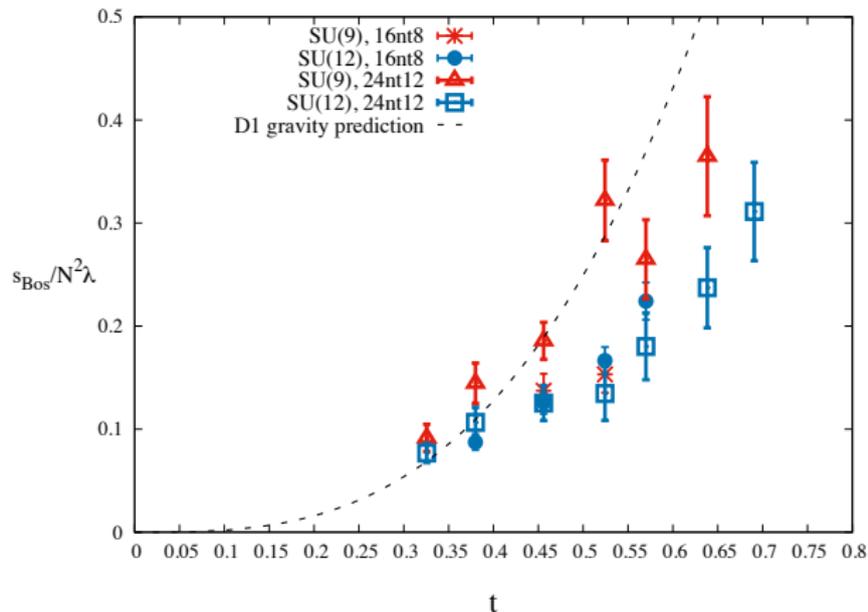


$r_x^2 > c_{\text{grav}} r_\tau$ corresponds to the homogeneous phase. The order parameter for the phase transition is the Wilson line.

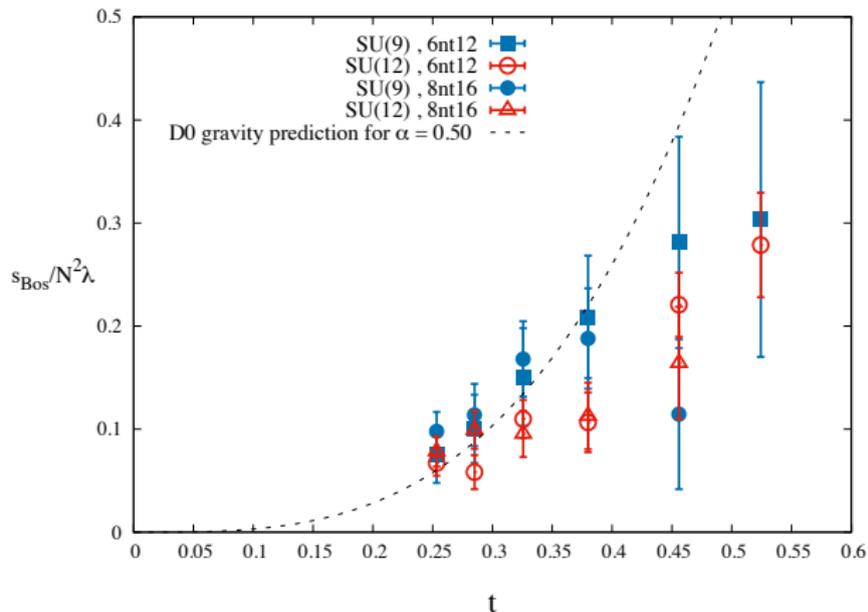
Unitarized Wilson line phases - localized and uniform



D1 gravity (homogeneous phase) - thermodynamics



D0 gravity (localized phase) - thermodynamics



Conclusions

- ⇒ Sixteen supercharge SYM theory is now possible to study at large N using twisted lattice construction in various dimensions.
- ⇒ Positive evidence from lattice simulations of strongly coupled SYM theory at large N that gauge/gravity duality might be correct.

Thank you.

Funding and computing resources



Other details (if needed)

- ⇒ No sign problem with anti-periodic boundary conditions for fermions (which we use here).
- ⇒ The $U(1)$ mode is truncated from the start, but, restored at sufficiently large N . See talk by Joel Giedt @ Lattice 2017.
- ⇒ To regulate $SU(N)$ flat directions, we added a small mass term μ . We extrapolated the energy density to the $\mu \rightarrow 0$ limit.
- ⇒ Soft-mass term added to ensure that center symmetry is completely broken along reduced directions.
- ⇒ The breaking of supersymmetry is within few % with the largest N we simulate.