

Hybrid static potentials in $SU(3)$ gauge theory on the lattice

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- Previous work

K. J. Juge, J. Kuti, and C. J. Morningstar. Gluon excitations of the static quark potential and the hybrid quarkonium spectrum. *Nucl. Phys. Proc. Suppl.*, 63:326–331, 1998, hep-lat/9709131.

K. Jimmy Juge, Julius Kuti, and Colin Morningstar. The heavy-quark hybrid meson spectrum in lattice qcd. 2003, arXiv:nucl-th/0307116.

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Outline

- Hybrid mesons
 - Static potential
 - On the lattice
- Lattice computation
 - Operators
- Results
- Outlook

Hybrid mesons

- Mesons with excited gluon fields
 - Quantum numbers J^{PC}
- On the lattice
 - Quantum numbers $|L|_{\mathcal{P}_x}^{\mathcal{P}_0\mathcal{C}}$
 - Angular momentum
 - Parity + charge conjugation (+ / - = g / u)
 - Spatial inversion along an axis perpendicular to separation axis (+ / -)

Static potential

- Correlator

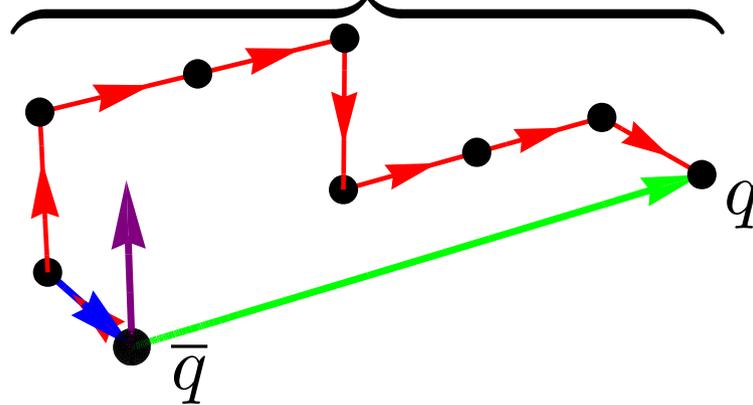
$$C(t) = \sum_n |\langle n | \mathcal{O}(0) | \Omega \rangle|^2 \exp(-(E_n - E_\Omega)t)$$

- Extract static potential

$$V = \lim_{t \rightarrow \infty} \frac{1}{a} \ln \left(\frac{C(t)}{C(t+a)} \right)$$

Hybrid mesons – on the lattice

- Construction of trial states for hybrid potentials
 - Choose some path I as spatial part of Wilson loop



- Build the sum

$$|\Psi_{\text{Hybrid}}\rangle = \sum_{k=0}^3 \exp(iLk \frac{\pi}{2}) \left[\hat{R}(k \frac{\pi}{2}) \mathcal{O} \right] |\Omega\rangle = S_L(I)$$

with angular momentum L

Hybrid mesons – on the lattice

- Project $S_L(I)$ onto the subspace of eigenstates to $\mathcal{P} \circ \mathcal{C}$, \mathcal{P}_x using the projectors

$$\mathbb{P}_{P_x} = \frac{1}{2}(1 + P_x \mathcal{P}_x) \quad \mathbb{P}_{PC} = \frac{1}{2}(1 + Q_{PC} \mathcal{P} \circ \mathcal{C})$$

$$\mathcal{P}_x \mathbb{P}_{P_x} = P_x \mathbb{P}_{P_x} \quad \mathcal{P} \circ \mathcal{C} \mathbb{P}_{PC} = Q_{PC} \mathbb{P}_{PC}$$

- Use the result in the Wilson loop

$$\mathbb{P}_{P_x} \mathbb{P}_{PC} S_L(I) =$$

$$\mathcal{N}(1 + P_x \mathcal{P}_x + Q_{PC} \mathcal{P} \circ \mathcal{C} + P_x Q_{PC} \mathcal{P}_x \mathcal{P} \circ \mathcal{C}) S_{|L|}(I)$$

Hybrid mesons – on the lattice

- Trial state, contains

$$|\Psi_{\text{Hybrid}}\rangle_I = \bar{q}(-R) I(-R, R) q(R) |\Omega\rangle$$

– \mathcal{P}_x

$$\mathcal{P}_x |\Psi_{\text{Hybrid}}\rangle_I = \bar{q}(-R) [\mathcal{P}_x I(-R, R)] q(R) |H\rangle$$

– Parity and charge conjugation

$$\mathcal{P} \circ \mathcal{C} |\Psi_{\text{Hybrid}}\rangle_I = \bar{q}(-R) I_{\mathcal{P},\mathcal{C}}^T(-R, R) q(R) |\Omega\rangle$$

Hybrid mesons – on the lattice

	$\varphi = 0$	$\varphi = \pi/2$	$\varphi = \pi$	$\varphi = 3\pi/2$
$\mathbb{1}$				
\mathcal{P}_x				
$\mathcal{P} \circ \mathcal{C}$				
$\mathcal{P}_x \times \mathcal{P} \circ \mathcal{C}$				

Lattice computation

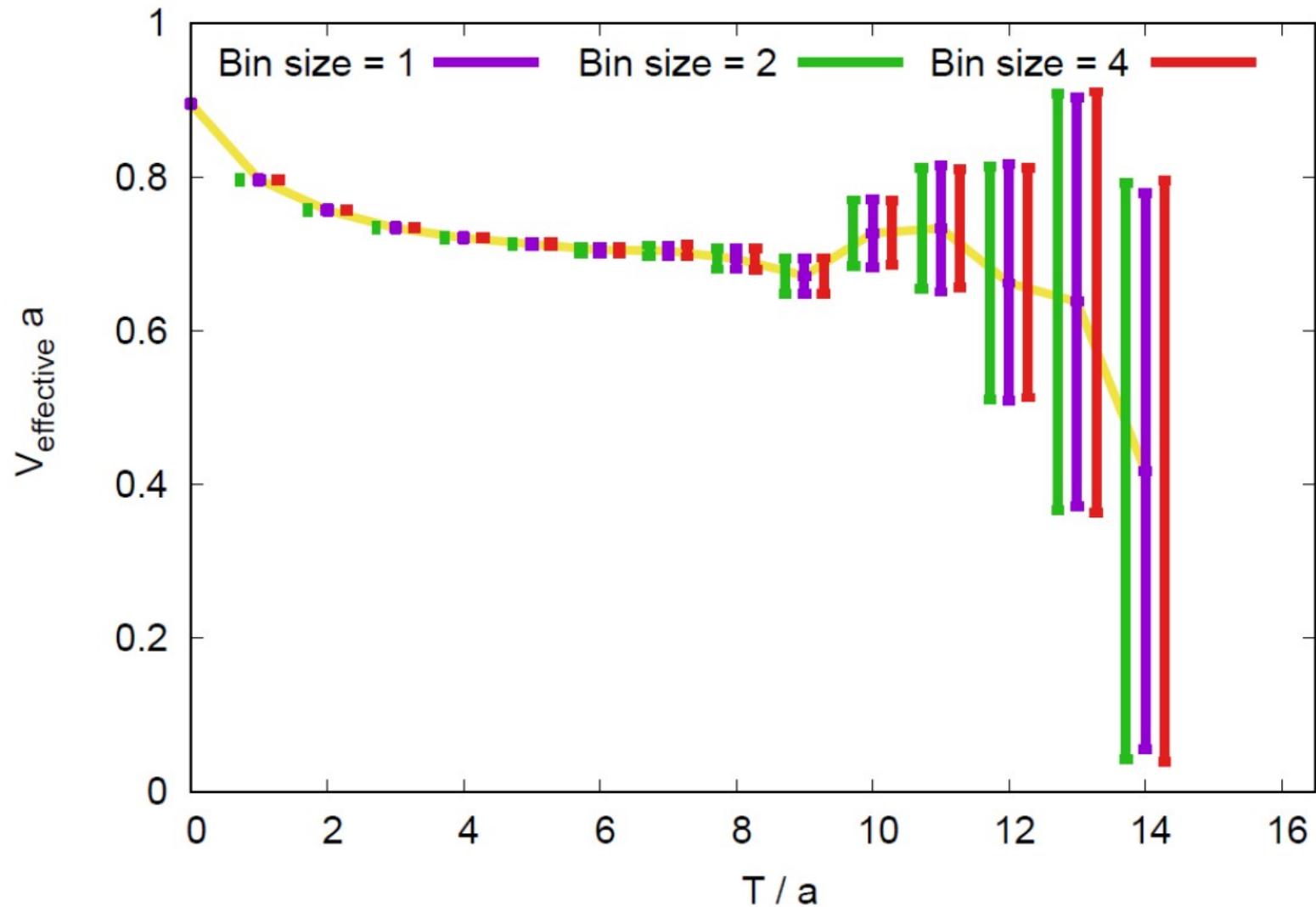
- Generate configurations with Chroma QCD using the Wilson plaquette action

$$S_g[U] = \frac{\beta}{3} \sum_n \sum_{\mu < \nu} \text{Re} \{ \text{Tr} [1 - U_{\mu\nu}(n)] \}$$

- $\beta = 6.0 \longrightarrow a \approx 0.093 \text{ fm}$
- Lattice size: $24^3 \times 48$

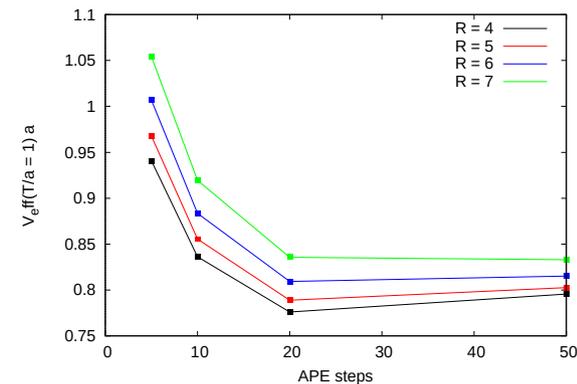
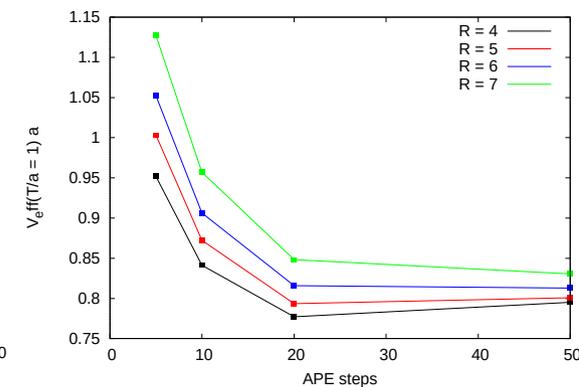
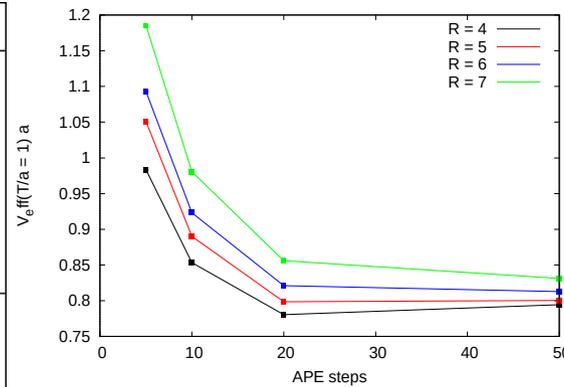
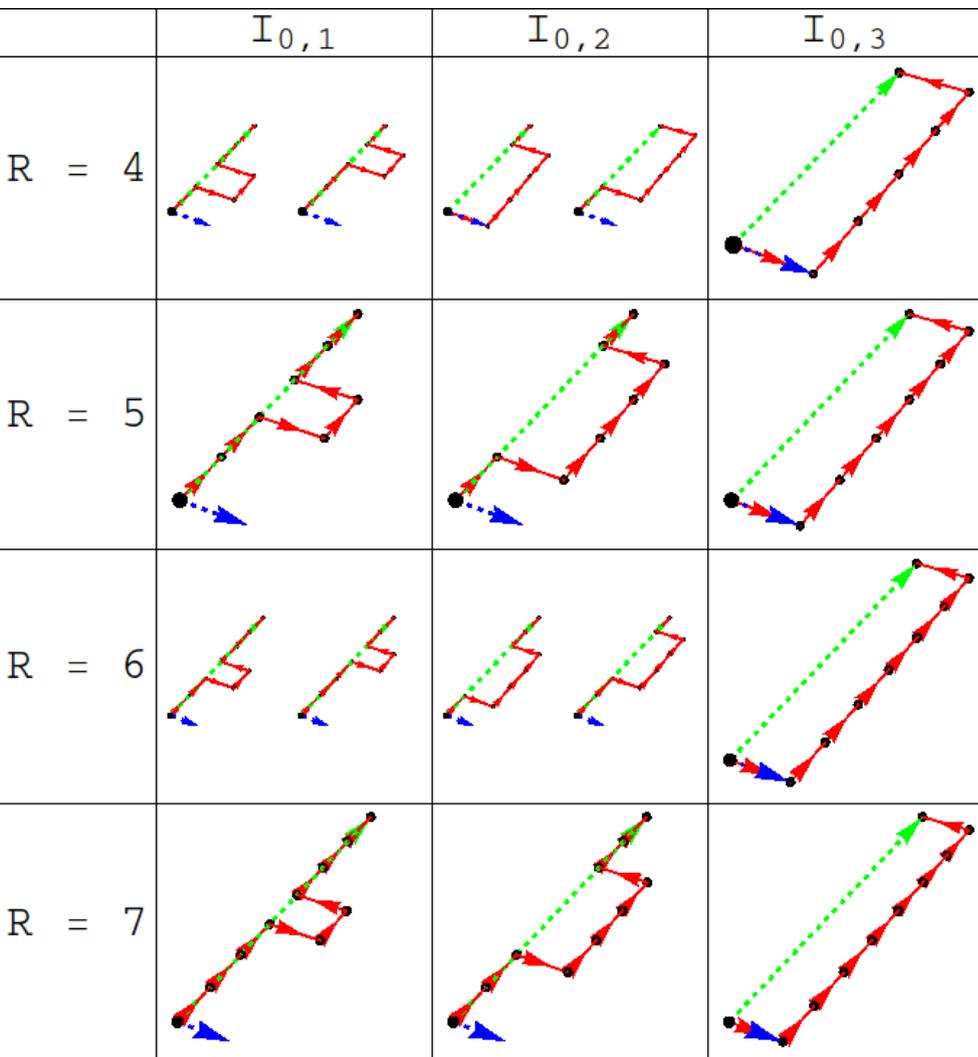
Lattice computation

- Check for autocorrelation



Lattice computation – APE smearing

- Optimize number of APE smearing steps
 - Investigate different shapes



V_{eff} for $N_{\text{APE}} = 5, 10, 20, 50$
for shapes $I_{0,1}, I_{0,2}, I_{0,3}$, state Π_u

Lattice computation – operators

- Our operators

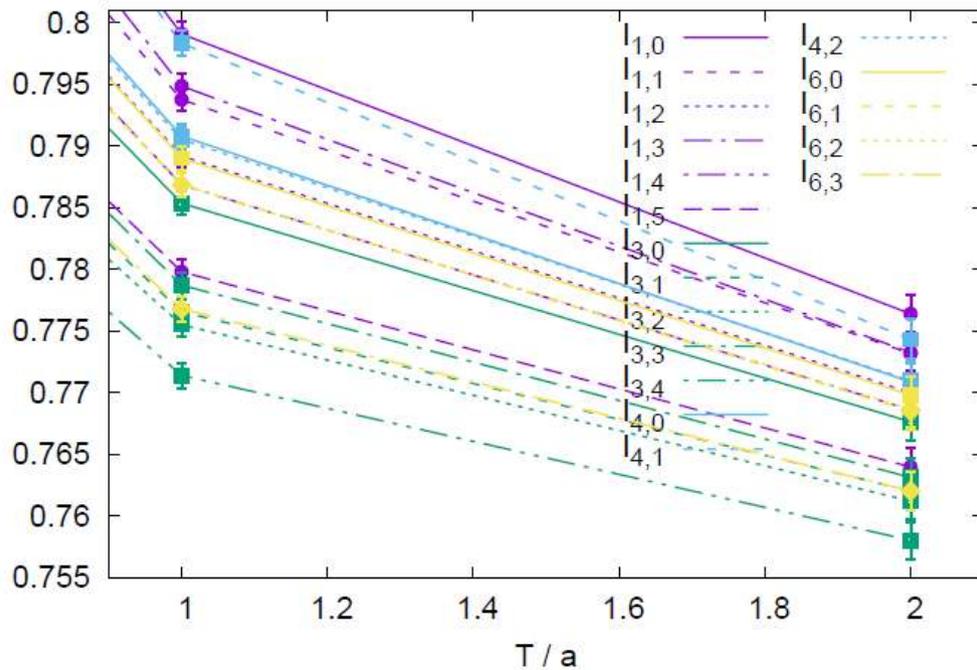
	I_1	I_2	I_3	I_4	I_5	I_6
$R = 2$						

- Investigate some variations for each shape
 - Length along the separation axis
 - Distance from the separation axis

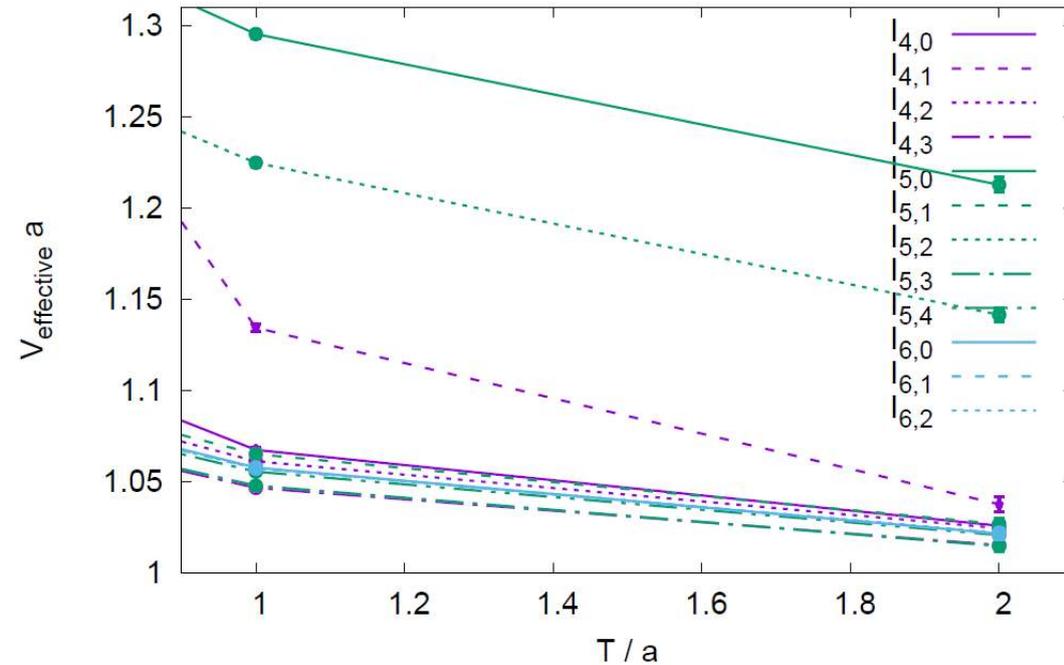
Lattice computation – operators

- Compare effective potentials at small temporal separations

$\Pi_U, R = 5$

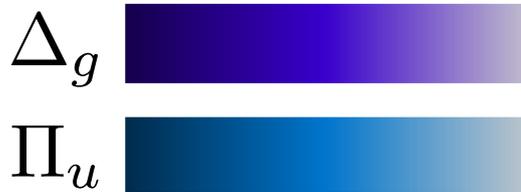
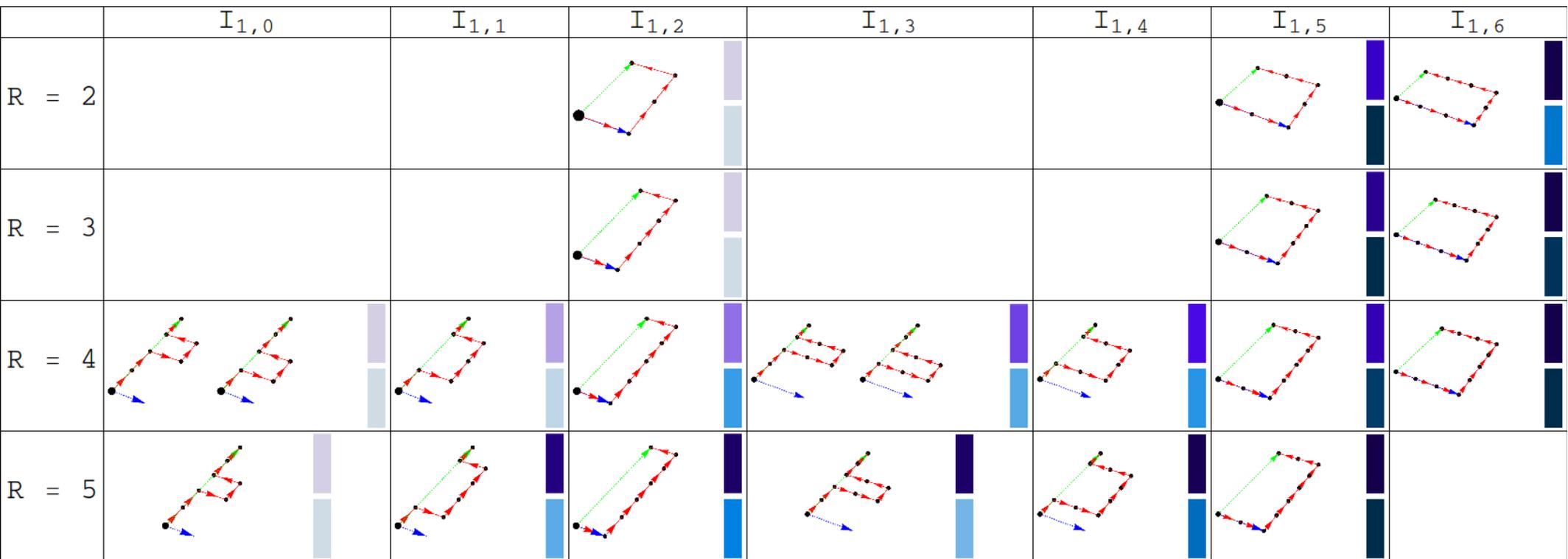


$\Delta_U, R = 4$



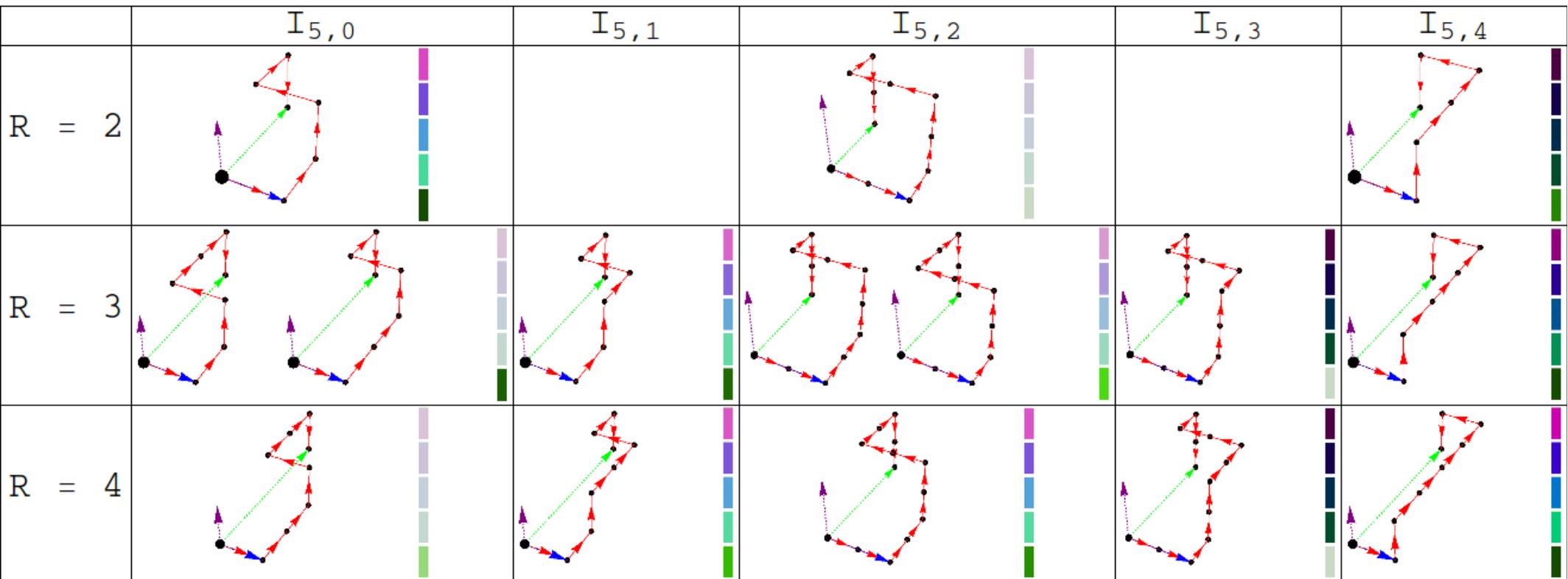
Lattice computation – operators

- Variations of I_1 .
 - Darker color \rightarrow higher ground state overlap



Lattice computation – operators

- Variations of I_5 .



Lattice computation – operators

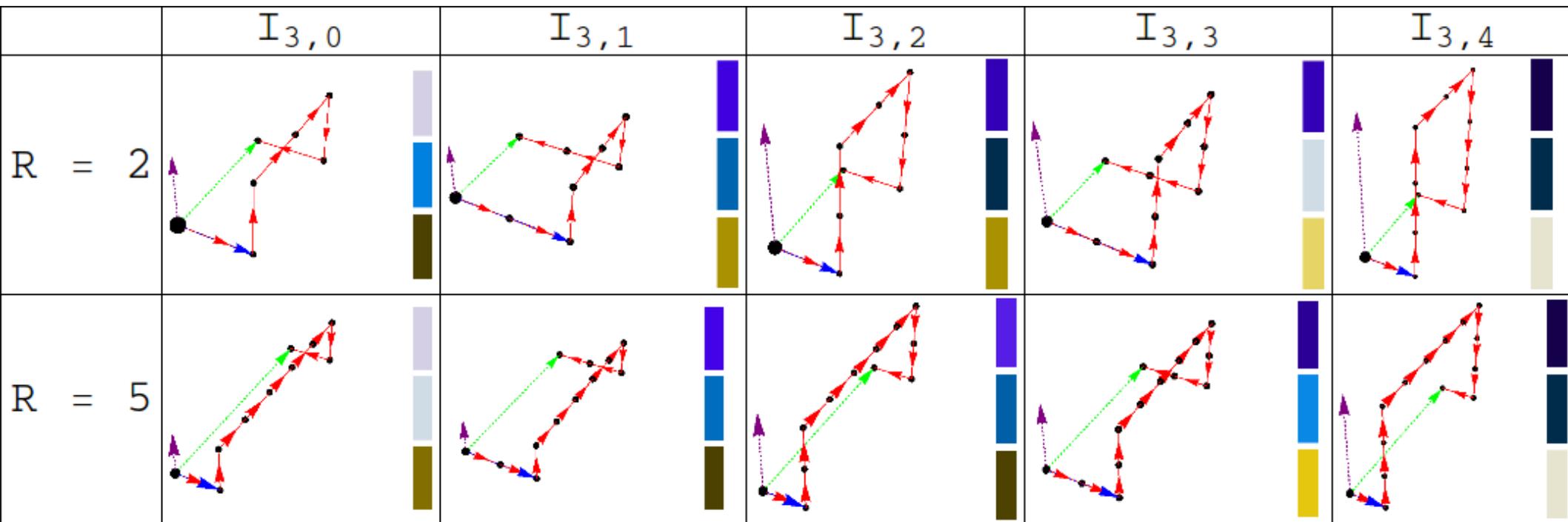
- Variations of I_2 .

	$I_{2,0}$	$I_{2,1}$	$I_{2,2}$	$I_{2,3}$	$I_{2,4}$	$I_{2,5}$	$I_{2,6}$	$I_{2,7}$	$I_{2,8}$	$I_{2,9}$	
R = 2											
R = 3											
R = 4											
R = 5											



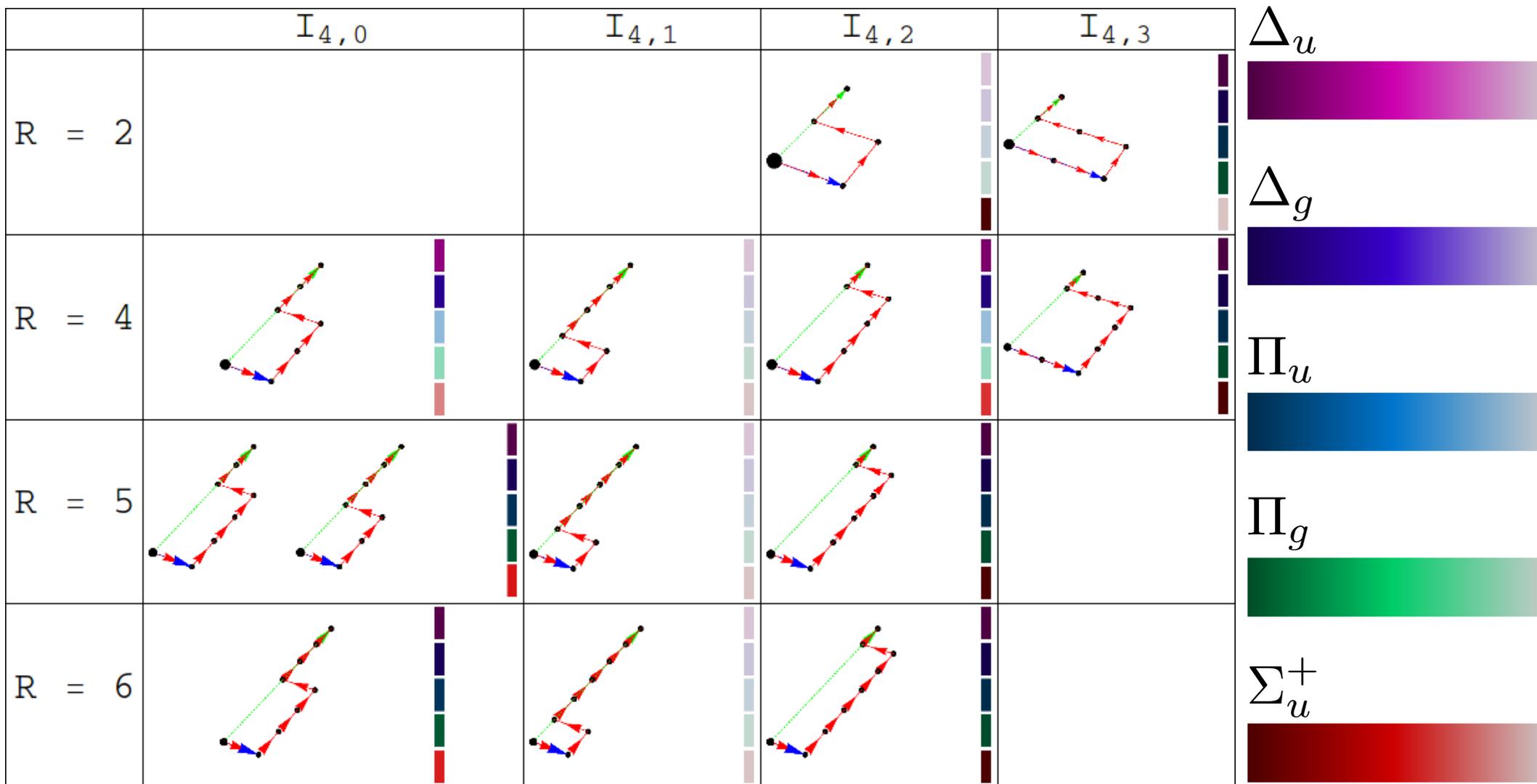
Lattice computation – operators

- Variations of I_3 .



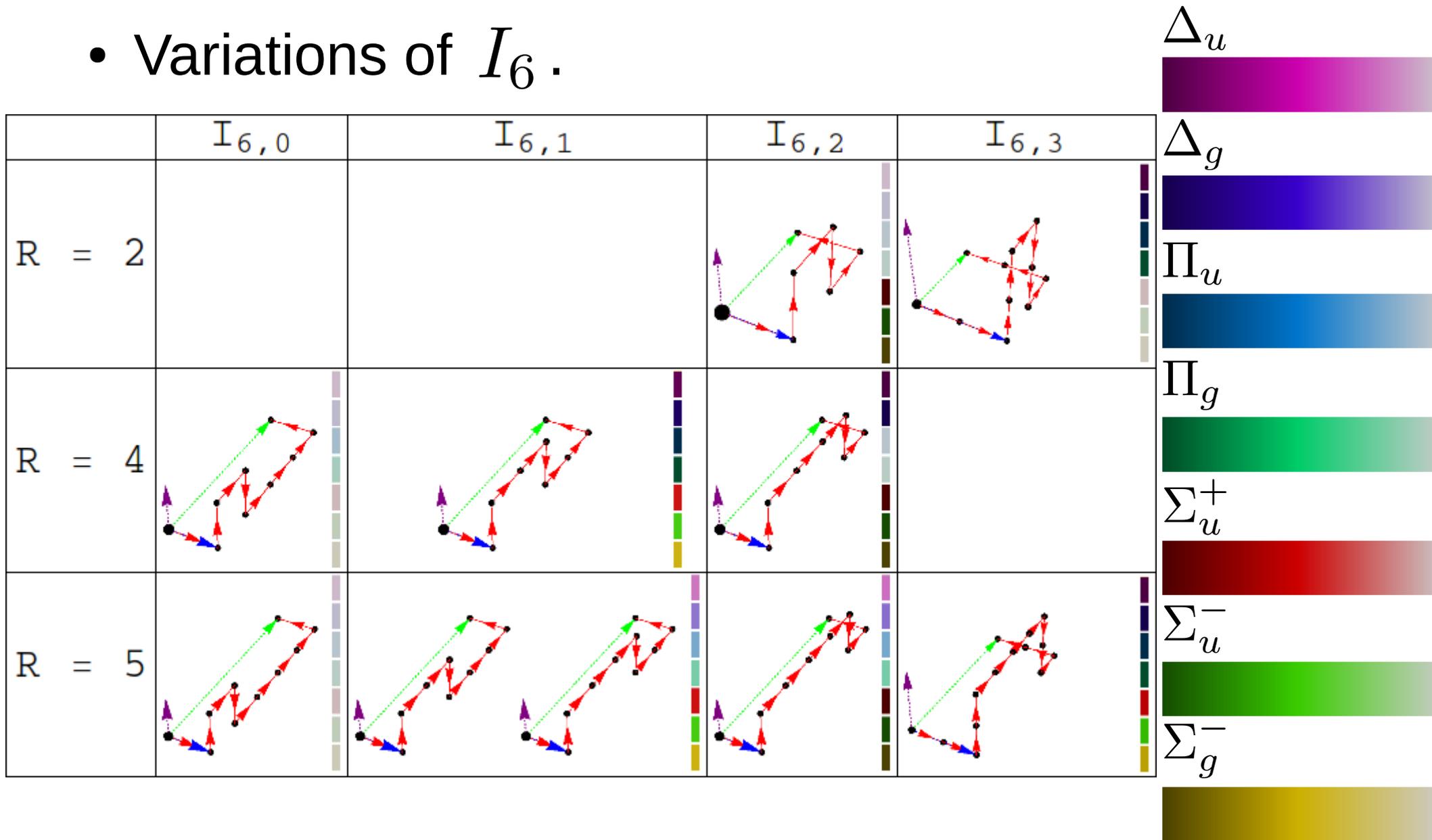
Lattice computation – operators

- Variations of I_4 .



Lattice computation – operators

- Variations of I_6 .



Lattice computation - observations

- Ground state overlap is largest for:
 - Long shapes
 - R increases or L increases \rightarrow distance from the separation axis increases
 - More structure close to the quarks
- Effects of shape more important at higher angular momentum

Lattice computation – potentials

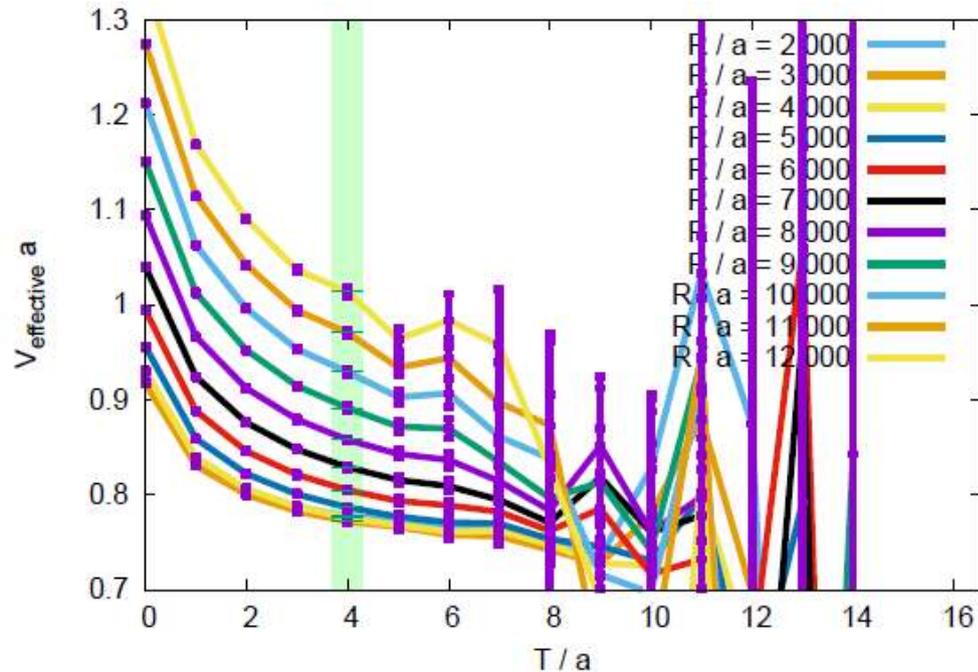
- To find an optimal combination of operators, compute the correlation matrix

$$C_{jk} = \text{Tr} \left[S_{\mathcal{I}(I_j)}^{t_0}(-R, R) T(t_0, t_1, \mathbf{r}_q) \right. \\ \left. \times \left\{ S_{\mathcal{I}(I_k)}^{t_1}(-R, R) \right\}^\dagger T^\dagger(t_0, t_1, \mathbf{r}_{\bar{q}}) \right]$$

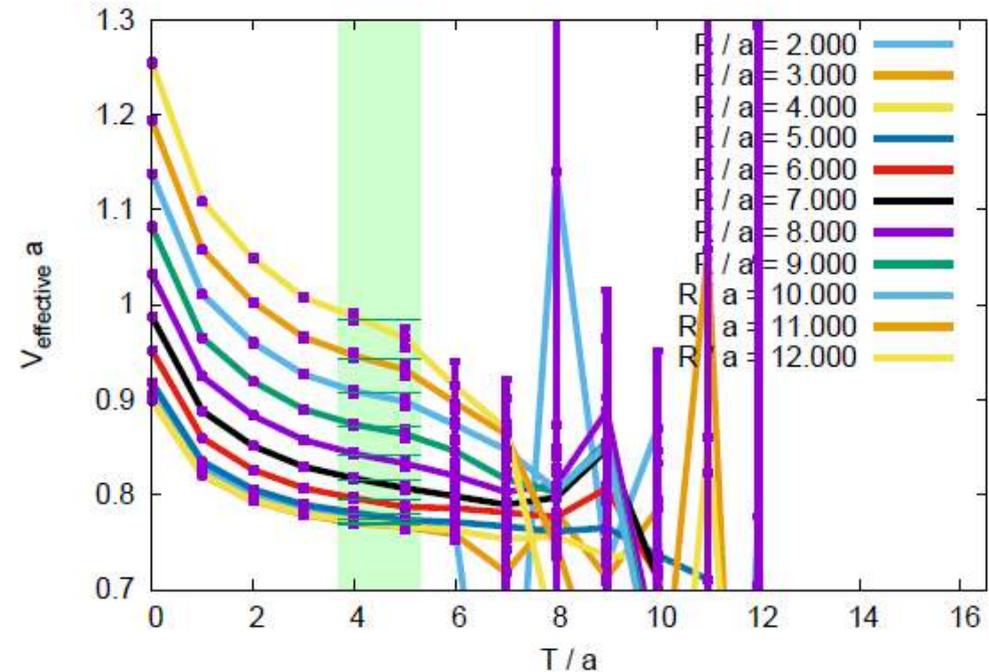
- Solve the GEVP

$$C(t) \mathbf{v}_n(t, \tau) = \lambda_n(t, \tau) C(\tau) \mathbf{v}_n(t, \tau)$$

Results – effective potentials



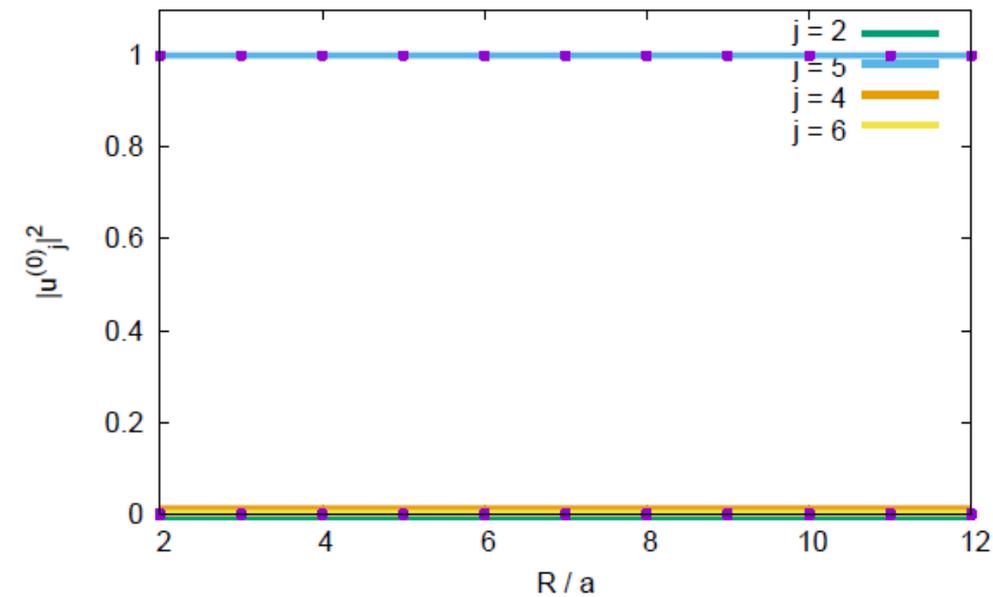
(a) Π_g , includes insertions I_2, I_4, I_5, I_6 .



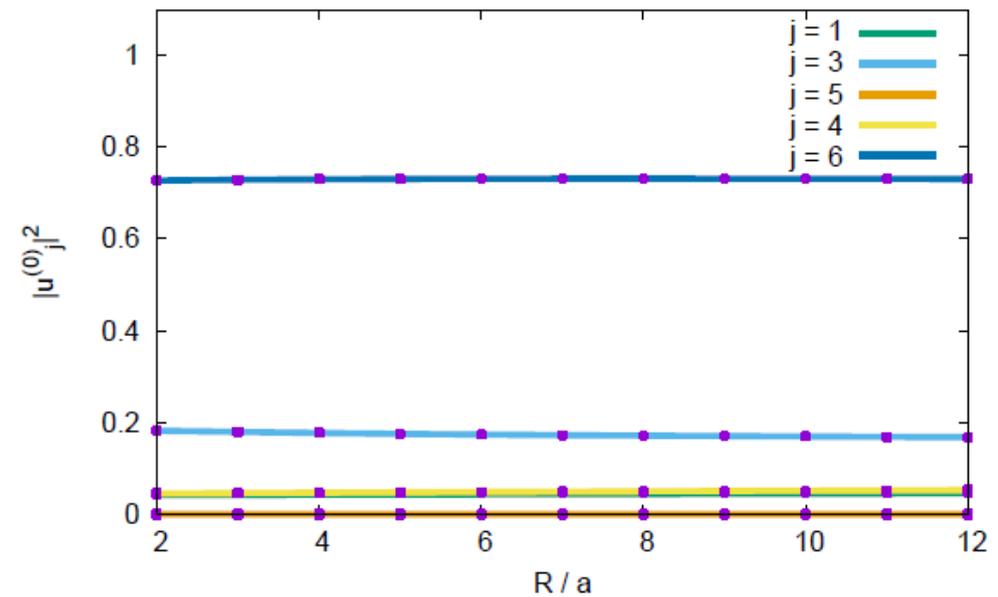
(b) Π_u , includes insertions I_1, I_3, I_4, I_5, I_6 .

Effective potentials for hybrid states with angular momentum $L = 1$.

Results – overlaps



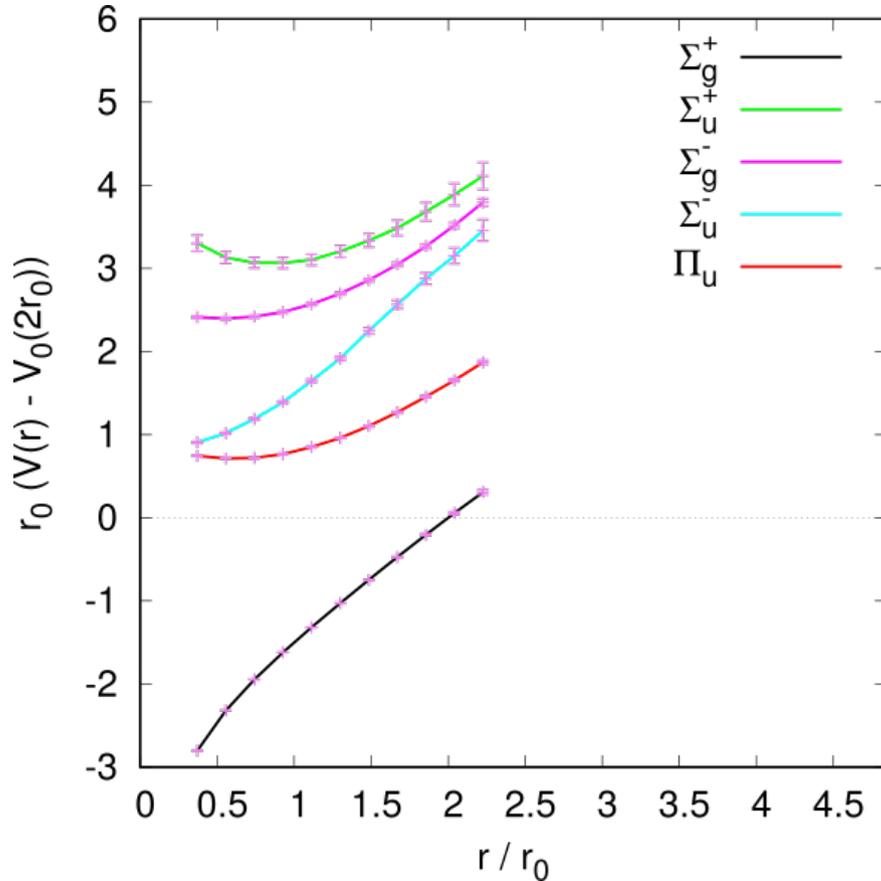
(a) Π_g , includes insertions I_2, I_4, I_5, I_6 .



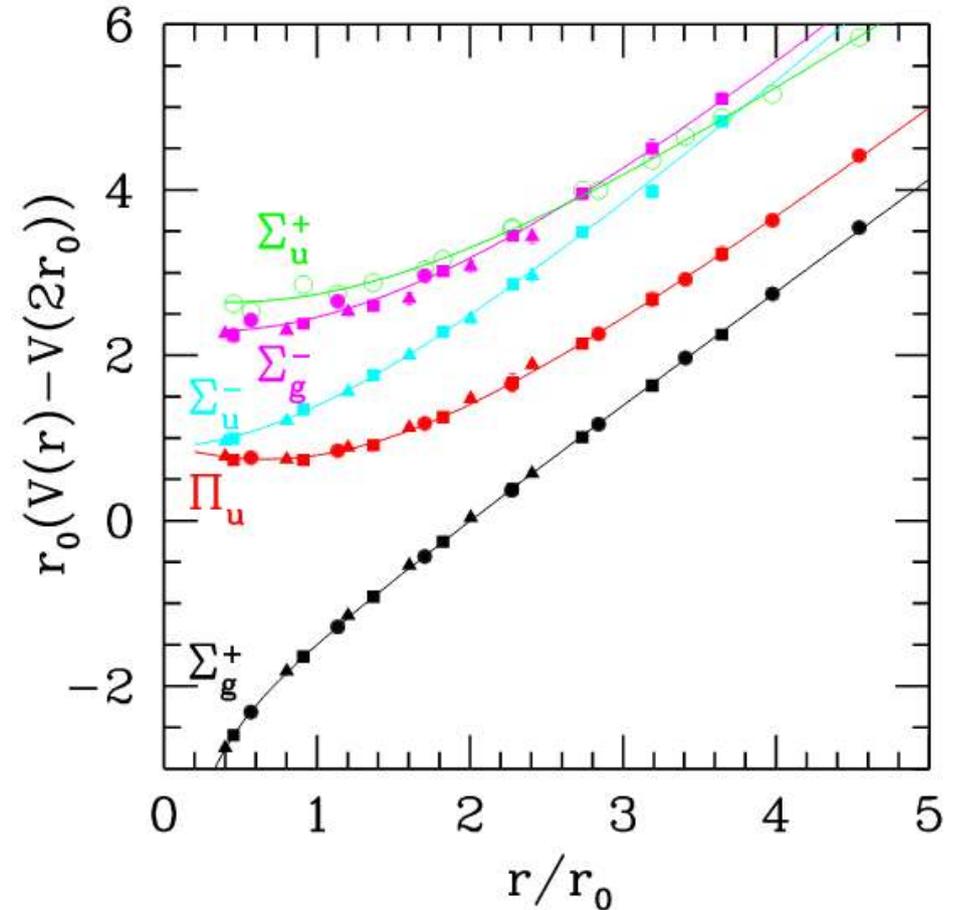
(b) Π_u , includes insertions I_1, I_3, I_4, I_5, I_6 .

Operator overlaps for hybrid states with angular momentum $L = 1$.

Results – static potentials



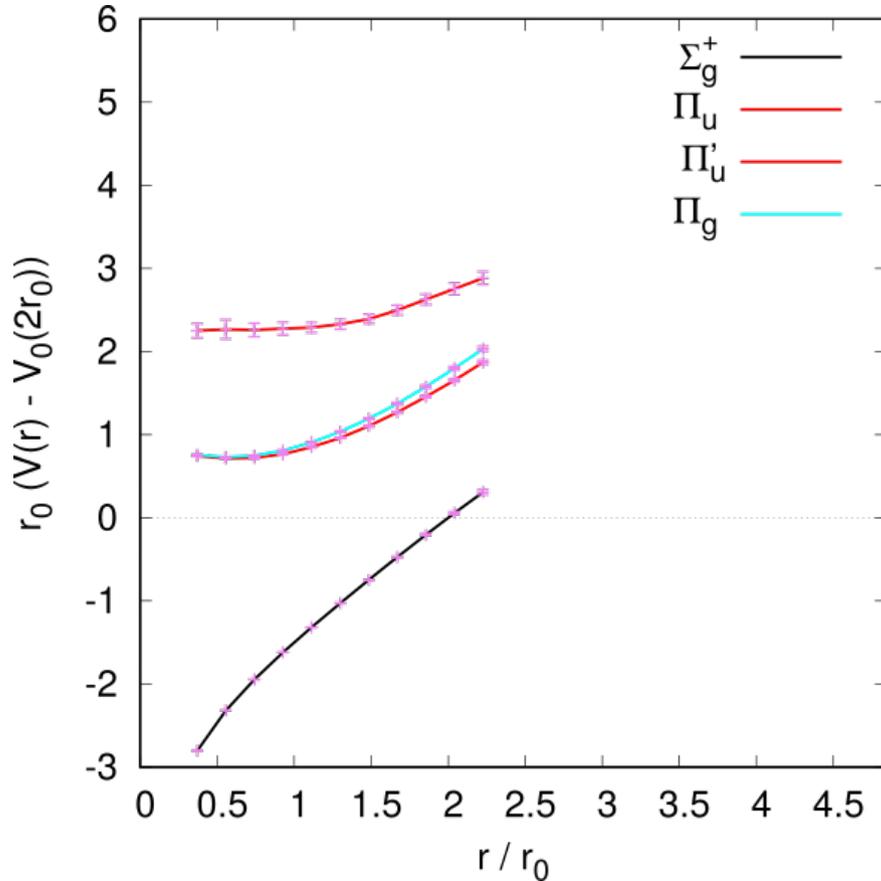
(a) Our results



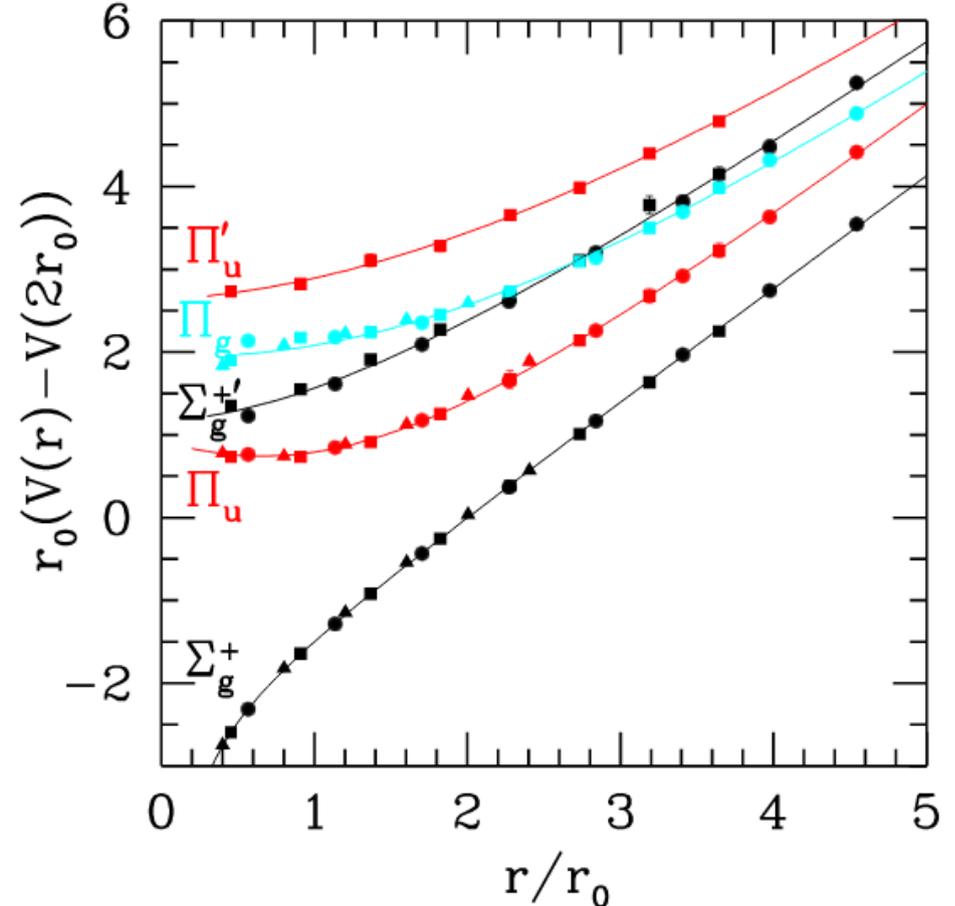
(b) Results from “K. Juge, J. Kuti, C. Morningstar. Gluon excitations of the static quark potential and the hybrid quarkonium spectrum. 1998, hep-lat/9709131”

Figure: Static hybrid potentials $\Sigma_g^+, \Pi_u, \Sigma_u^-, \Sigma_g^-, \Sigma_u^+$ (lowest to highest).

Results – static potentials



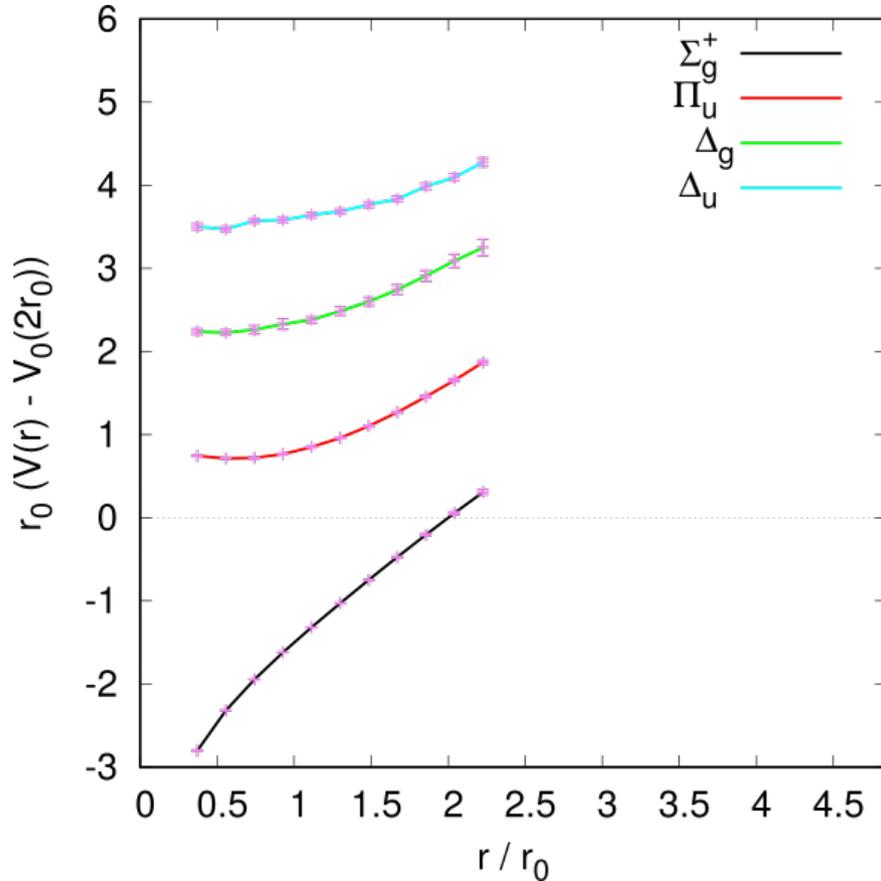
(a) Our results ($\Sigma_g^{+'}$ is missing).



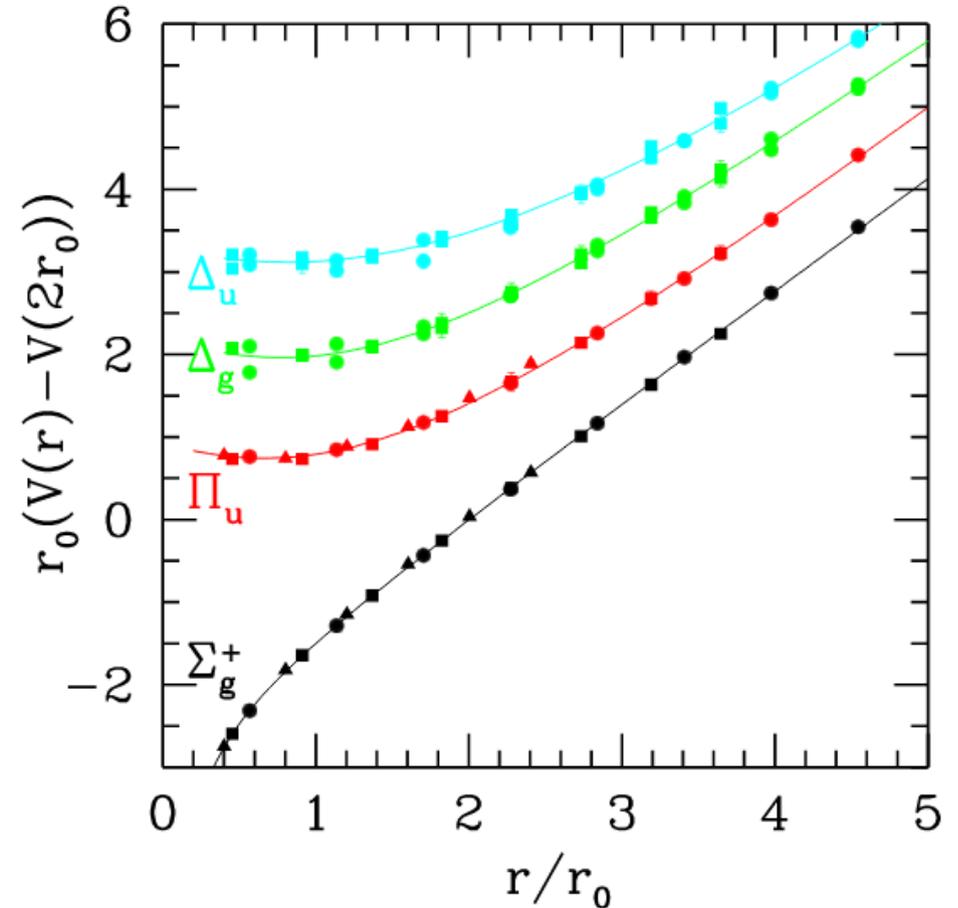
(b) Results from “K. Juge, J. Kuti, C. Morningstar. Gluon excitations of the static quark potential and the hybrid quarkonium spectrum. 1998, hep-lat/9709131”

Figure: Static hybrid potentials Σ_g^+ , Π_u , $\Sigma_g^{+'}$, Π_g , Π_u' (lowest to highest).

Results – static potentials



(a) Our results



(b) Results from “K. Juge, J. Kuti, C. Morningstar. Gluon excitations of the static quark potential and the hybrid quarkonium spectrum. 1998, hep-lat/9709131”

Figure: Static hybrid potentials Σ_g^+ , Π_u , Δ_g , Δ_u (lowest to highest).

Outlook

- Smaller lattice spacings
- Solve Schrödinger equations
- Spin effects