Testing a non-perturbative mechanism for elementary fermion mass generation: lattice setup

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(see also talk by P. Dimopoulos: *"Testing a non-perturbative mechanism for elementary fermion mass generation: numerical result"*)

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- Frezzotti and Rossi in [Phys. Rev. D92 (2015) 054505] conjectured a new non-perturbative mechanism for the elementary particle mass generation
- We are testing this conjecture in the "simplest" appropriate $d = 4$ "toy model"

$$\mathcal{L}_{\rm toy}(Q, A, \Phi) = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} {\rm Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \frac{\mu_0^2}{2} {\rm Tr}\left[\Phi^\dagger \Phi\right] + \frac{\lambda_0}{4} \left({\rm Tr}\left[\Phi^\dagger \Phi\right]\right)^2$$
$$+ \overline{Q}_L \gamma_\mu \mathcal{D}_\mu Q_L + \overline{Q}_R \gamma_\mu \mathcal{D}_\mu\, Q_R + \frac{b^2}{2} \rho \left(\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + h.c.\right)$$
$$+ \eta \left(\overline{Q}_L \Phi Q_R + h.c.\right), \qquad \Phi \equiv \varphi_0 1\!\!1 + i \tau_i \varphi_i$$

- "Wilson-like" $\propto \rho$ (naively irrelevant)
- UV cutoff $\sim b^{-1}$
- Fermionic chiral transformations $\tilde{\chi}$ are not a symmetry if $(\rho, \eta) \neq (0, 0)$

$$\mathcal{L}_{\rm toy}(Q,A,\Phi) = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2}{\rm Tr}\big[\partial_\mu\Phi^\dagger\partial_\mu\Phi\big] + \frac{\mu_0^2}{2}{\rm Tr}\big[\Phi^\dagger\Phi\big] + \frac{\lambda_0}{4}\big({\rm Tr}\big[\Phi^\dagger\Phi\big]\big)^2$$
$$+\overline{Q}_L\gamma_\mu\mathcal{D}_\mu Q_L + \overline{Q}_R\gamma_\mu\mathcal{D}_\mu\, Q_R + \frac{b^2}{2}\rho\big(\overline{Q}_L\overleftarrow{\mathcal{D}}_\mu\Phi\mathcal{D}_\mu Q_R + h.c.\big)$$
$$+\eta\,\big(\overline{Q}_L\Phi Q_R + h.c.\big), \qquad \Phi \equiv [\varphi| - i\tau^2\varphi^*]$$

- Symmetries & power counting $\begin{pmatrix}\text{in suitable}\\ \text{UV-regul.}\end{pmatrix} \Longrightarrow$ Renormalizability
- Invariant under χ (global) $SU(2)_L \times SU(2)_R$ transformations

$$\bullet\, \chi_{L,R}: \quad \tilde\chi_L \otimes (\Phi \to \Omega_L\Phi) \quad \text{and/or} \quad \tilde\chi_R \otimes (\Phi \to \Phi\Omega_R^\dagger)$$
$$\tilde\chi_{L,R}: \begin{cases} Q_{L,R} \to \Omega_{L,R}Q_{L,R} & \Omega_{L,R} \in SU(2)_{L,R} \\ \overline{Q}_{L,R} \to \overline{Q}_{L,R}\Omega^\dagger_{L,R} \end{cases}$$

- Not invariant under purely fermionic transformations $\tilde\chi$
- χ invariance forbids $\frac{1}{b}\overline{Q}Q$ terms and softens power like U.V. divergences

- Purely fermionic $\tilde{\chi}$ tranformations yield bare Schwinger Dyson Eq.s (SDEs)

$$\partial_\mu \langle \tilde{J}^{L,i}_\mu(x)\, \hat{\mathcal{O}}(0)\rangle = \langle \tilde{\Delta}^i_L \hat{\mathcal{O}}(0)\rangle \delta(x) - \eta\, \langle O^{L,i}_{Yuk}(x)\, \hat{\mathcal{O}}(0)\rangle - b^2 \langle O^{L,i}_{Wil}(x)\, \hat{\mathcal{O}}(0)\rangle$$

$$\tilde{J}^{L,i}_\mu = \overline{Q}_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{b^2}{2}\rho\Big(\overline{Q}_L \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} Q_L\Big)$$

$$O^{L,i}_{Yuk} = \Big[\overline{Q}_L \frac{\tau^i}{2}\Phi Q_R - \text{h.c.}\Big] \qquad O^{L,i}_{Wil} = \frac{\rho}{2}\Big[\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \text{h.c.}\Big]$$

- Mixing of $O^{L,i}_{Wil}$ under renormalization

$$b^2 O^{L,i}_{Wil} = (Z_{\tilde{\partial} J} - 1)\partial_\mu \tilde{J}^{L,i}_\mu - \overline{\eta}(\eta; g_s^2, \rho, \lambda_0)\, O^{L,i}_{Yuk} + \underline{\ldots} + \mathrm{O}(b^2)$$

- Renormalized SDEs read

$$Z_{\partial\tilde{J}} \partial_\mu \langle \tilde{J}^{L,i}_\mu(x)\, \hat{\mathcal{O}}(0)\rangle = \langle \tilde{\Delta}^i_L \hat{\mathcal{O}}(0)\rangle \delta(x) - (\eta - \overline{\eta}(\eta))\, \langle O^{L,i}_{Yuk}(x)\, \hat{\mathcal{O}}(0)\rangle + \underline{\ldots} + \mathrm{O}(b^2)$$

where the ellipses ($\underline{\ldots}$) stand for possible NP mixing contributions

- At the critical $\eta_{cr}(g_s^2, \rho, \lambda_0)$ s.t. $\eta_{cr} = \overline{\eta}(\eta_{cr}; g_s^2, \rho, \lambda_0)$ the SDEs become WTIs

$$Z_{\partial\tilde{J}}\partial_\mu\langle\tilde{J}_\mu^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} = \langle\tilde{\Delta}_L^i\hat{\mathcal{O}}(0)\rangle_{\eta_{cr}}\delta(x) + \underline{\ldots} + \mathrm{O}(b^2)$$

- In Wigner phase $\big(\langle\Phi\rangle = 0\big)$ the Wilson-like term is uneffective for $\tilde{\chi}^{SSB}$

$$Z_{\partial\tilde{J}}\partial_\mu\langle\tilde{J}_\mu^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} = \langle\tilde{\Delta}_L^i\hat{\mathcal{O}}(0)\rangle_{\eta_{cr}}\delta(x) + \mathrm{O}(b^2)$$

- In Nambu-Goldstone $\big(\langle\Phi\rangle = v\mathbb{1}_{2\times 2}\big)$ expect (conjecture)

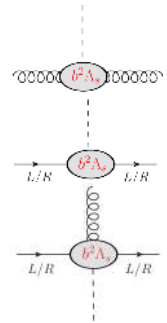
$$Z_{\partial\tilde{J}}\partial_\mu\langle\tilde{J}_\mu^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} = \langle\tilde{\Delta}_L^i\hat{\mathcal{O}}(0)\rangle_{\eta_{cr}}\delta(x) + \langle C_1\Lambda_s[\overline{Q}_L\frac{\tau^i}{2}\mathcal{U}Q_R + \mathrm{hc}]\hat{\mathcal{O}}(0)\rangle + \mathrm{O}(b^2)$$

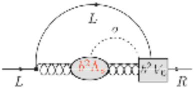
- The term $\propto C_1\Lambda_s$ can exist only in the NG phase where

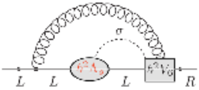
$$\mathcal{U} = \frac{\Phi}{\sqrt{\Phi^\dagger\Phi}} = \frac{(v + \sigma + i\vec{\tau}\vec{\varphi})}{\sqrt{(v+\sigma)^2 + \vec{\varphi}\vec{\varphi}}} = \mathbb{1} + i\frac{\vec{\tau}\vec{\varphi}}{v} + \ldots$$

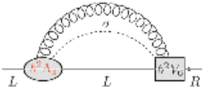
In Γ_{loc}^{NG} NP term $C_1\Lambda_s[\overline{Q}_L\mathcal{U}Q_R + \mathrm{hc}] \supset C_1\Lambda_s\overline{Q}Q_R$
$\begin{cases} \text{Natural mass} \\ \neq \text{Yukawa term} \\ C_1 = O(\alpha_s^2) \to \text{Hierarchy} \end{cases}$

- Intuitive idea of the NP mass generation mechanism $O(b^2)$ NP corrections to ($\tilde{\chi}$-preserving) effective vertices combined in loop "diagrams" with $O(b^2)$ ($\tilde{\chi}$-breaking) vertices from the Wilson-like term

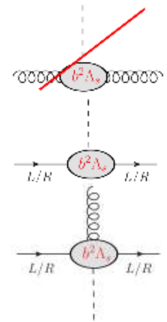








- b^{-4} loop divergency $\implies$ $O(b^0)C_1\Lambda_s$ mass term



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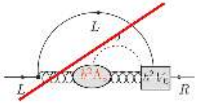

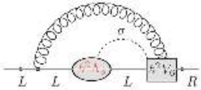

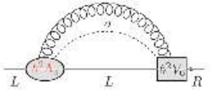

- b^{-4} loop divergency $\implies$ $O(b^0)C_1\Lambda_s$ mass term
- Phenomenon occurring even in the quenched fermion approximation

Choose a cheap lattice regularization of $\int d^4x \mathcal{L}_{toy}$

- First NP study of a theory with gauge, $\Psi = \begin{pmatrix} u \\ d \end{pmatrix}$ & $(\varphi_0, \vec{\varphi})$
- "Naive" fermions (good for quenched approximation only)

$$\mathcal{S}_{lat} = b^4 \sum_x \left\{ \mathcal{L}_{kin}^{YM}[U] + \mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) + \sum_g \overline{\Psi}_g D_{lat}[U,\Phi]\Psi_g \right\}$$

$\mathcal{L}_{kin}^{YM}[U]$: SU(3) plaquette action

$$\mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) = \tfrac{1}{2}\,\mathrm{tr}\,[\Phi^\dagger(-\partial_\mu^*\partial_\mu)\Phi] + \tfrac{\mu_0^2}{2}\,\mathrm{tr}\left[\Phi^\dagger\Phi\right] + \tfrac{\lambda_0}{4}\left(\mathrm{tr}\left[\Phi^\dagger\Phi\right]\right)^2$$

where $\Phi = \varphi_0 1\!\!1 + i\varphi_j\tau^j$ and $F(x) \equiv [\varphi_0 1\!\!1 + i\gamma_5\tau^j\varphi_j](x)$

$$(D_{lat}[U,\Phi]\Psi)(x) = \gamma_\mu\widetilde{\nabla}_\mu\Psi(x) + \eta F(x)\Psi(x) - b^2\rho\tfrac{1}{2}F(x)\widetilde{\nabla}_\mu\widetilde{\nabla}_\mu\Psi(x) +$$
$$- b^2\rho\tfrac{1}{4}\Big[(\partial_\mu F)(x)U_\mu(x)\widetilde{\nabla}_\mu\Psi(x+\hat{\mu}) + (\partial_\mu^* F)(x)U_\mu^\dagger(x-\hat{\mu})\widetilde{\nabla}_\mu\Psi(x-\hat{\mu})\Big]$$

- Yukawa ($d=4$) term $\propto \eta$, Wilson-like ($d=6$) term $\propto \rho$
- Unquenched studies will require overlap, DW or staggered fermions
- Extension to two generations of fermions $\overline{\Psi}_\ell D_{latt}\Psi_\ell + \overline{\Psi}_h D_{latt}\Psi_h$ in order to have valence correlators involving no fermionic disconnected diagrams

- Using staggered formalism to analyze naive valence fermions
- $\Psi(x)$ contain 4 replicas $B = 1, .., 4$
  $\Psi(x) = \mathcal{A}_x \chi(x), \;\; \mathcal{A}_x = \gamma_1^{x_1}\gamma_2^{x_2}\gamma_2^{x_2}\gamma_2^{x_2},$ Spin diagonalization of S_{latt} in χ^B basis
- $\chi(x)$ contain 4 tastes a=1,...,4
  $q^B_{\alpha,a}(y) = \frac{1}{8}\sum_\xi \overline{U}(2y, 2y+\xi)[\Gamma_\xi]_{\alpha,a}(1 - b\sum_\mu \xi_\mu \widetilde{\nabla}_\mu)\chi^B(2y+\xi),$
  $q^B_{\alpha,a}(y)$ taste basis, $\quad x_\mu = 2y_\mu + \xi_\mu, \quad \xi_\mu = 0, 1$
- Flavour content: $\underbrace{(4 \text{ replicas:} B) \times (4 \text{ tastes:} a)\times}_{16 \text{ doublers}}$ (2 isospin)$\times$ generations
- Following Kluberg-Stern et al. ('83),..., Sharpe et al. ('93), Luo ('96) and adding scalars we get the small b expansion of S^{fer}_{lat} on smooth U,Φ configuration

$$S^{fer}_{lat} = \sum_{y,B} \bar{q}^B(y)\left\{\sum_\mu (\gamma_\mu \otimes 1\!\!1) D_\mu + (\eta - \bar{\eta})\mathcal{F}(y)\right\} q^B(y) + O(b^2)$$

$$\mathcal{F}(y) = \varphi_0(2y)(1\!\!1 \otimes 1\!\!1) + S^B i\tau^i \varphi_i(2y)(\gamma_5 \otimes t_5),\;\; S^A = \pm 1,\; \text{taste matrices } t_\mu = \gamma^*_\mu$$

- Quark bilinear in Ψ basis that have the classical continuum limit in q^B basis
- Point split vector current

$$\widetilde{J}_\mu^{V^i}(x) = \overline{\Psi}(x-\hat{\mu})\gamma_\mu \frac{\tau^i}{2} U_\mu(x-\hat{\mu})\Psi(x) + \overline{\Psi}(x)\gamma_\mu \frac{\tau^i}{2} U_\mu^\dagger(x-\hat{\mu})\Psi(x-\hat{\mu})$$

$$\sum_\xi \widetilde{J}_\mu^{V^i}(2y+\xi) = \sum_{B=1}^{4} \overline{q}^B(y)(\gamma_\mu \otimes \mathbb{1})\frac{\tau^i}{2} q^B(y) + O(b^2)$$

- Point split axial current

$$\widetilde{J}_\mu^{A^i}(x) = \overline{\Psi}(x-\hat{\mu})\gamma_\mu\gamma_5 \frac{\tau^i}{2} U_\mu(x-\hat{\mu})\Psi(x) + \overline{\Psi}(x)\gamma_\mu\gamma_5 \frac{\tau^i}{2} U_\mu^\dagger(x-\hat{\mu})\Psi(x-\hat{\mu})$$

$$\sum_\xi \widetilde{J}_\mu^{A^i}(2y+\xi) = \sum_{B=1}^{4} \overline{q}^B(y)(\gamma_\mu\gamma_5 \otimes t_5)\frac{\tau^i}{2} q^B(y) + O(b^2)$$

- Correlators with generation off-diagonal operator $\Rightarrow$ no disconnected diagrams

$$\text{e.g.} \quad \langle \overline{\Psi}_\ell(x)\Gamma\tau\Psi_h(x)\overline{\Psi}_h(y)\Gamma\tau\Psi_\ell(y)\rangle \qquad \ell = (u,d) \quad h = (c,s)$$

- Loop effects do not generate $d \leq 4$ operator besides $F_{\mu\nu}F_{\mu\nu}$, $\partial_\mu \Phi^\dagger \partial_\mu \Phi$ $q^B(\gamma_\mu \otimes 1\!\!1)\tilde{\nabla} q^B$, $\Phi^\dagger \Phi$, $(\Phi^\dagger \Phi)^2$, $\eta \overline{q}^B(y)\mathcal{F}^B(y)q^B(y)$ (all in S_{lat})

- Argument: In S_{latt} there are only $\widetilde{\nabla}_\mu$ acting on fermions $\Longrightarrow$ Spectrum Doubling Symmetry :
  $\Psi \to e^{-ix\cdot\pi_H} M_H \Psi$ $\overline{\Psi} \to \overline{\Psi} M_H^\dagger e^{+ix\cdot\pi_H}$, $H = \{\mu_1, ..., \mu_h\}$ ordered,
  16 vectors $\pi_H(\pi_{H,\mu} = \pi$ if $\mu \in H)$ with $M_H = (i\gamma_5\gamma_1)...(i\gamma_5\gamma_{\mu_h})$

- It is a symmetry of S_{latt}, thus also of the $\Gamma_{lat}[U, \Phi, \Psi]$. So the latter can only have terms with symmetric covariant derivatives $\widetilde{\nabla}_\mu$ acting on Ψ.
  Close to the continuum limit among the local terms of Γ_{lat} only those with no or one $\widetilde{\nabla}_\mu$ fermionic derivative are relevant

- At $\eta = \eta_{cr}$ $\widetilde{\chi}$ gets simultaneously restored for all tastes up to cutoff effects

- Quenched $(U, \Phi) \implies$ exceptional configurations: at large $|\eta|$ and $|\rho|$ enhanced by Φ fluctuations
- Add twisted mass term: $S_{lat}^{toy+tm} = S_{lat} + i\mu b^4 \sum_x \overline{\Psi}\gamma_5\tau_3\Psi$ control over exceptional confs. at the price of harmless breaking of $\chi_{L,R}$ (and $\widetilde{\chi}_{L,R}$ when restored)
- A convenient (generation off diagonal) axial $\widetilde{\chi}$ SDE

$$Z_A \partial_\mu \widetilde{J}_\mu^{A^\pm} = 2(\eta - \eta_{cr})\widetilde{D}^{P\pm} + \delta_{ph,NG}\, C_1 \Lambda_s \mathcal{P}^\pm$$

$$\widetilde{J}_\mu^{A^\pm} = \overline{\Psi}\gamma_\mu\gamma_5 \frac{\tau^\pm}{2}\Psi(x)\Big|_{\text{point split}}$$

$$\widetilde{D}^{P\pm} = \overline{\Psi}_L \left\{\Phi, \frac{\tau^\pm}{2}\right\}\Psi_R - h.c. \qquad \mathcal{P}^\pm = \overline{\Psi}_L \left\{\mathcal{U}, \frac{\tau^\pm}{2}\right\}\Psi_R - h.c.$$

- ... at $\eta = \eta_{cr}$ in NG phase takes the form of a $\tilde{\chi}$ WTI with NP breaking

$$Z_A \langle 0|\partial_\mu \widetilde{J}_\mu^{A^\pm}|M_{PS\pm}\rangle = C_1 \Lambda_s \langle 0|\mathcal{P}^\pm|M_{PS\pm}\rangle$$

$$Z_A\langle 0|\partial_\mu \widetilde{J}_\mu^{A^\pm}|M_{PS\pm}\rangle = C_1\Lambda_s\langle 0|\mathcal{P}^\pm|M_{PS\pm}\rangle$$

- $\mathcal{U} = \mathbb{1} + i\frac{\vec{\tau}\cdot\vec{\varphi}}{v} + O(\frac{\sigma^2}{v^2}, \frac{\pi^2}{v^2})$
- $\mathcal{P}^\pm = \overline{\Psi}_L\left\{\mathcal{U}, \frac{\tau^\pm}{2}\right\}\Psi_R - h.c. = \overline{\Psi}_L\left\{\mathbb{1} + i\frac{\vec{\tau}\cdot\vec{\varphi}}{v} + ..., \frac{\tau^\pm}{2}\right\}\Psi_R - h.c. = P^\pm + ...$
- χ invariance $\implies$ $\mathcal{P}^\pm$ renormalizes as $P^\pm = \overline{\Psi}\gamma_5\tau^\pm\Psi$
- At $\eta = \eta_{cr}$ a renormalized measure of the NP $\widetilde{\chi}$ breaking is given by

$$m_{ren}^{WTI} \equiv \frac{Z_A}{Z_P}\frac{\langle 0|\partial_\mu \widetilde{J}_\mu^{A^\pm}|M_{PS\pm}\rangle}{\langle 0|P^\pm|M_{PS\pm}\rangle} = C_{1,ren}\Lambda_s(1+...) \qquad C_{1,ren} = \frac{C_1}{Z_P}$$

- Generation off diagonal operators $P^{\pm}_{h,\ell}(x) = \overline{\Psi}_h(x)\gamma_5\tau^{\pm}\Psi_\ell(x)$
  $\widetilde{J}_\mu^{A^{\pm}_{\ell,h}}(x) = \overline{\Psi}_\ell(x-\hat{\mu})\gamma_\mu\gamma_5\frac{\tau^i}{2}U_\mu(x-\hat{\mu})\Psi_h(x) + \overline{\Psi}_\ell(x)\gamma_\mu\gamma_5\frac{\tau^i}{2}U^\dagger_\mu(x-\hat{\mu})\Psi_h(x-\hat{\mu})$

  $\Longrightarrow$ multiplicative renormalization of $\widetilde{J}_\mu^{A^{\pm}_{\ell,h}}$ and $P^{\pm}_{\ell,h}$
- at $\eta = \eta_{cr}$ Z_A does not have anomalous dimension $\mu\frac{\partial Z_A}{\partial\mu} = O(b^2)$
- Suitable renormalization scheme to compute the ratio Z_A/Z_P

$$\frac{\sum_{\bar{y}}\langle \widetilde{J}_\mu^{A^{\pm}_{\ell,h}}(0)P^{\pm}_{h,\ell}(y_0,\bar{y})\rangle Z_A}{\sum_{\bar{y}}\langle P^{\pm}_{\ell,h}(0)P^{\pm}_{h,\ell}(y_0,\bar{y})\rangle Z_P} = \textit{tree level} \quad \text{at} \quad \eta = \eta_{cr}$$

- Renormalization scale $\mu = \frac{1}{y_0}$ e.g. $y_0 = 2r_0 \simeq 1fm$ (if scale taken from QCD)
- Error on Z_A/Z_P is $\sim 3\%$ with small statistics (30 gauges x 8 scalars)

- Quenched lattice study: independent M.C. update of U and Φ

$$Z = \int \mathcal{D}\Phi\mathcal{D}U\mathcal{D}\Psi\mathcal{D}\overline{\Psi}e^{S[U]+S[\phi]}\cancel{\det\left(D_{latt}[U,\phi]\right)} = Z[\Phi]Z[U]$$

- Action parameter renormalization can be carried out separately for gauge and scalars:
  - In the fermionic sector effective Yukawa coupling vanishes at $\eta = \eta_{cr}$
  - Gauge coupling renormalized keeping $r_0 \sim 0.5fm$ fixed
    M. Guagnelli, R. Sommer and H. Wittig (1998) S. Necco, R. Sommer (2001)
  - Scalars parameters μ_0, λ_0 fixed by the renormalization condition

$$m_\sigma^2 r_0^2 = const. \qquad \lambda_R = \frac{m_\sigma^2}{2v_R^2} = const.$$

- Wilson-like coupling ρ: free parameter as long as we are only interested to see if the mechanism exists, relevant for the magnitude of the NP mass (if any)

- For numerical result on the model see talk P. Dimopoulos: *"Testing a non-perturbative mechanism for elementary fermion mass generation: numerical result"*

Thank you for your attention