Rare Kaon Decays $K^+ \to \pi^+ \ell^+ \ell^-$ with 3 Flavours

A. Lawson

School of Physics and Astronomy
University of Southampton

Lattice 2017

June 21, 2017

RBC & UKQCD Collaborations

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Luchang Jin
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
David Murphy
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang

University of Connecticut

Tom Blum
Dan Hoying
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Tobias Tsang
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

Peking University

Xu Feng

University of Southampton

Jonathan Flynn
Vera Gülpers
James Harrison
Andreas Jüttner
Andrew Lawson
Edwin Lizarazo
Christopher Sachrajda

York University (Toronto)

Renwick Hudspith

RBC & UKQCD Collaborations

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Luchang Jin
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
David Murphy
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang

University of Connecticut

Tom Blum
Dan Hoying
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Tobias Tsang
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

Peking University

Xu Feng

University of Southampton

Jonathan Flynn
Vera Gülpers
James Harrison
Andreas Jüttner
Andrew Lawson
Edwin Lizarazo
Christopher Sachrajda

York University (Toronto)

Renwick Hudspith

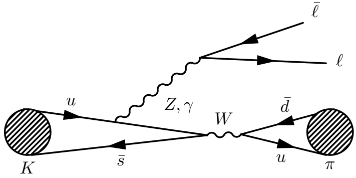
Papers

- "Prospects for a lattice computation of rare kaon decay amplitudes: I $K \to \pi\ell^+\ell^-$ decays", N. H. Christ, X. Feng., A. Portelli and C. T. Sachrajda. arXiv:hep-lat\1507.03094
- "Prospects for a lattice computation of rare kaon decay amplitudes: II $K \to \pi\nu\bar{\nu}$ decays", N. H. Christ, X. Feng., A. Portelli and C. T. Sachrajda. arXiv:hep-lat\1605.04442
- "First exploratory calculation of the long distance contributions to the rare kaon decay $K \to \pi\ell^+\ell^-$" N. H. Christ, X. Feng., A. Juttner, A. Lawson, A. Portelli and C. T. Sachrajda. arXiv:hep-lat\1608.07585.
- "Exploratory Lattice QCD Study of the Rare Kaon Decay $K^+ \to \pi^+\nu\bar{\nu}$" Z. Bai, N. H. Christ, X. Feng., A. Lawson, A. Portelli and C. T. Sachrajda. arXiv:hep-lat\1701.02858.

Outline

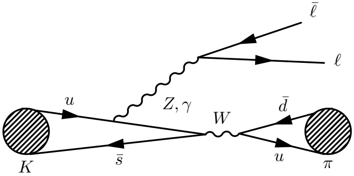
Rare Kaon Decays

- The processes $K \to \pi\ell\bar{\ell}$ proceed via a flavour changing neutral current (FCNC).
  - $s \to d\ell\bar{\ell}$ transition.
- FCNCs are forbidden at tree level in the SM.
- Ideal probes for new physics!



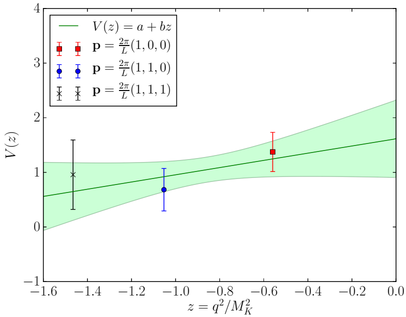
Rare Kaon Decays

- The processes $K \to \pi\ell\bar{\ell}$ proceed via a flavour changing neutral current (FCNC).
  - $s \to d\ell\bar{\ell}$ transition.
- FCNCs are forbidden at tree level in the SM.
- Ideal probes for new physics!
- Six different decays with very different phenomology:
  - $K_L \to \pi^0\nu\bar{\nu}$
  - $K_S \to \pi^0\nu\bar{\nu}$
  - $K^+ \to \pi^+\nu\bar{\nu}$
  - $K_L \to \pi^0\ell^+\ell^-$
  - $K_S \to \pi^0\ell^+\ell^-$
  - $K^+ \to \pi^+\ell^+\ell^-$
- $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$ long distance dominated.
  - Ideal target for a lattice calculation!



Exploratory lattice results

- Studied $K \to \pi\ell^+\ell^-$ on coarse lattice ($a^{-1} = 1.78\,\mathrm{GeV}$) with $M_\pi \simeq 430\,\mathrm{MeV}$, $M_K \simeq 625\,\mathrm{MeV}$, with a charm quark of mass $m_c \simeq 530\,\mathrm{MeV}$.
- Obtained approximate q^2 dependence of form factor, $V(z)$.

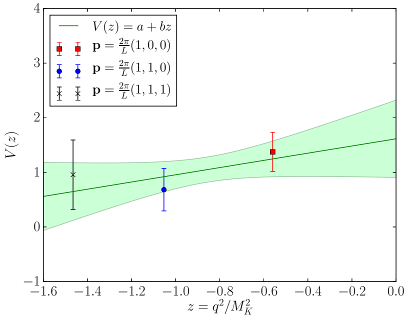


- Now wish to try physical calculation - computationally much more expensive!
  - Physical charm requires a fine lattice to avoid discretisation effects - integrate out?

[1] N. H. Christ et al., Phys. Rev. D94 (2016) 114516. arXiv:hep-lat/1608.07585.

Exploratory lattice results

- Studied $K \to \pi\ell^+\ell^-$ on coarse lattice ($a^{-1} = 1.78\,\mathrm{GeV}$) with $M_\pi \simeq 430\,\mathrm{MeV}$, $M_K \simeq 625\,\mathrm{MeV}$, with a charm quark of mass $m_c \simeq 530\,\mathrm{MeV}$.
- Obtained approximate q^2 dependence of form factor, $V(z)$.



- Now wish to try physical calculation - computationally much more expensive!
  - Physical charm requires a fine lattice to avoid discretisation effects - integrate out?

[1] N. H. Christ et al., Phys. Rev. D94 (2016) 114516. arXiv:hep-lat/1608.07585.

Lattice Outlook

- Four flavour simulation:
  - Requires fine lattice to simulate physical charm (expensive!)
  - Lattice must be large enough to allow pion to propagate
    - $64^3 \times 128$ lattice available with $a^{-1} = 2.36\,\mathrm{GeV}$
- Three flavour simulation:
  - Renormalisation more complicated!
  - Perturbation theory may not be completely reliable around $\mu \sim m_c$.
  - Computationally much cheaper for "physical" calculation.
    - $48^3 \times 96$ lattice with $a^{-1} = 1.78\,\mathrm{GeV}$
- Aim to generate new data using new simulation framework: Grid.

Lattice Outlook

- Four flavour simulation:
  - Requires fine lattice to simulate physical charm (expensive!)
  - Lattice must be large enough to allow pion to propagate
    - $64^3 \times 128$ lattice available with $a^{-1} = 2.36\,\mathrm{GeV}$
- Three flavour simulation:
  - Renormalisation more complicated!
  - Perturbation theory may not be completely reliable around $\mu \sim m_c$.
  - Computationally much cheaper for "physical" calculation.
    - $48^3 \times 96$ lattice with $a^{-1} = 1.78\,\mathrm{GeV}$
- Aim to generate new data using new simulation framework: Grid.

Lattice Methodology

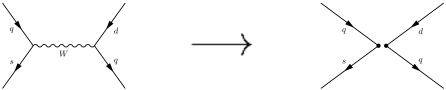
$K \to \pi\ell^+\ell^-$ Amplitude

- We compute the amplitude of $K \to \pi\gamma^*$,

$$A_\mu\left(q^2\right) = \int d^4x \left\langle \pi(p) \left| T\left[J^\mu(0) H_W(x)\right] \right| K(k) \right\rangle + \left\langle \pi(p) \left| C_{7V} Q^\mu_{7V} \right| K(k) \right\rangle$$

- The effective Weak Hamiltonian for a $|\Delta S| = 1$ transition is:

$$H_W = \frac{G_F}{\sqrt{2}} V^*_{us} V_{ud} \left(\sum_{i=1}^{2} C_i \left(Q^u_i - Q^c_i\right) + \sum_{j=3}^{10} C_j Q_j + \mathcal{O}\left(\frac{V^*_{ts} V_{td}}{V^*_{us} V_{ud}}\right) \right).$$



[2] G. Isidori et al., Phys. Lett. B633 (2005) 75-83. arXiv:hep-lat/0506026
[3] N. H. Christ et al., Phys. Rev. D92 (2015) 094512. arXiv:hep-lat/1507.03094.

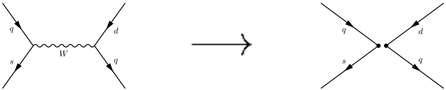
$K \to \pi \ell^+ \ell^-$ Amplitude

- We compute the amplitude of $K \to \pi\gamma^*$,

$$A_\mu\left(q^2\right) = \int d^4x \, \langle \pi(p) | T\left[J^\mu(0) H_W(x)\right] | K(k) \rangle + \langle \pi(p) | C_{7V} Q_{7V}^\mu | K(k) \rangle$$

- The effective Weak Hamiltonian for a $|\Delta S| = 1$ transition is:

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \left(\sum_{i=1}^{2} C_i \left(Q_i^u - Q_i^c\right) + \sum_{j=3}^{10} C_j Q_j + \mathcal{O}\left(\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}\right) \right).$$



[2] G. Isidori et al., Phys. Lett. B633 (2005) 75-83. arXiv:hep-lat/0506026
[3] N. H. Christ et al., Phys. Rev. D92 (2015) 094512. arXiv:hep-lat/1507.03094.

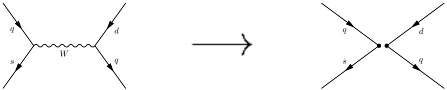
$K \to \pi\ell^+\ell^-$ Amplitude

- We compute the amplitude of $K \to \pi\gamma^*$,

$$A_\mu\left(q^2\right) = \int d^4x \left\langle \pi\left(p\right)\right| T\left[J^\mu\left(0\right) H_W\left(x\right)\right]\left|K\left(k\right)\right\rangle + \left\langle \pi\left(p\right)\right| C_{7V} Q^\mu_{7V} \left|K\left(k\right)\right\rangle$$

- The effective Weak Hamiltonian for a $|\Delta S| = 1$ transition is:

$$H_W = \frac{G_F}{\sqrt{2}} V^*_{us} V_{ud} \left(\sum_{i=1}^{2} C_i \left(Q_i^u - Q_i^c\right) + \sum_{j=3}^{10} C_j Q_j + \mathcal{O}\left(\frac{V^*_{ts} V_{td}}{V^*_{us} V_{ud}}\right)\right).$$



- Also a small contribution from local operator

$$Q^\mu_{7V} = \left(\delta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right) \bar{s}\gamma_\nu \left(1 - \gamma_5\right) d$$

- Negligible contribution from magnetic operator

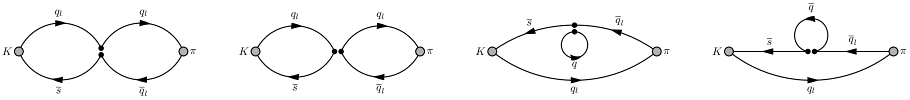
$$Q^\mu_m = i\partial_\nu \bar{s}\sigma^{\nu\mu} \left(m_s \left(1 - \gamma_5\right) + m_d \left(1 + \gamma_5\right)\right) d$$

[2] G. Isidori et al., Phys. Lett. B633 (2005) 75-83. arXiv:hep-lat/0506026
[3] N. H. Christ et al., Phys. Rev. D92 (2015) 094512. arXiv:hep-lat/1507.03094.

Wick Contractions

- Performing Wick contractions with just H_W, we obtain 4 different classes of diagrams:

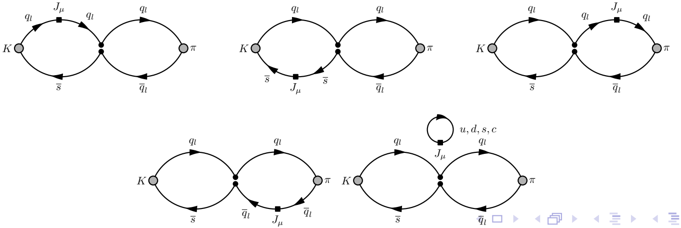
$$\langle \pi | H_W | K \rangle$$



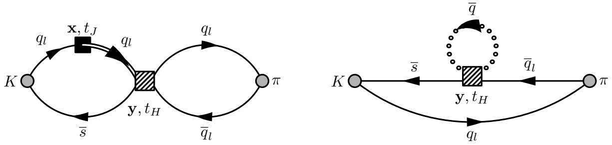
C W E S

- Adding in the current, we obtain 5 diagrams per H_W class:

$$\int d^4x \, \langle \pi(p) | T[J^\mu(0) H_W(x)] | K(k) \rangle$$



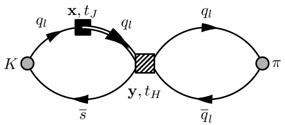
Simulation Cost

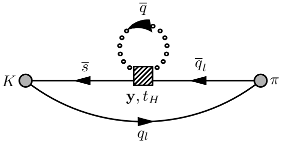


- Current: Sequential insertion (fixes t_J).
- Loops: stochastically approximate diagonal of $D^{-1}(x, y)$.
  - Use low mode deflation.
  - Stochastic approximation necessary for high modes.
- Exploratory studies, used 128 configurations on $24^3 \times 64$ lattice, $a^{-1} = 1.78\,\mathrm{GeV}$:
  - 254 light inversions / config
  - 36 strange inversions / config
  - 182 charm inversions / config
- Prospects for physical point calculation very expensive!

[1] N. H. Christ et al., Phys. Rev. D94 (2016) 114516, arXiv:hep-lat/1608.07585

Simulation Cost

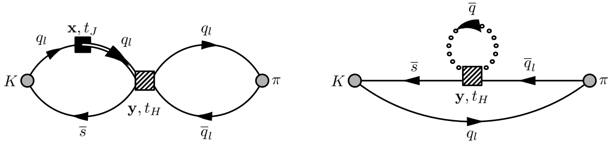




- Current: Sequential insertion (fixes t_J).
- Loops: stochastically approximate diagonal of $D^{-1}(x, y)$.
  - Use low mode deflation.
  - Stochastic approximation necessary for high modes.
- Exploratory studies, used 128 configurations on $24^3 \times 64$ lattice, $a^{-1} = 1.78\,\mathrm{GeV}$:
  - 254 light inversions / config
  - 36 strange inversions / config
  - 182 charm inversions / config
- Prospects for physical point calculation very expensive!

[1] N. H. Christ et al., Phys. Rev. D94 (2016) 114516, arXiv:hep-lat/1608.07585

Simulation Cost



- Current: Sequential insertion (fixes t_J).
- Loops: stochastically approximate diagonal of $D^{-1}(x, y)$.
  - Use low mode deflation.
  - Stochastic approximation necessary for high modes.
- Exploratory studies, used 128 configurations on $24^3 \times 64$ lattice, $a^{-1} = 1.78\,\mathrm{GeV}$:
  - 254 light inversions / config
  - 36 strange inversions / config
  - 182 charm inversions / config
- Prospects for physical point calculation very expensive!

[1] N. H. Christ et al., Phys. Rev. D94 (2016) 114516, arXiv:hep-lat/1608.07585

Renormalisation

Operator Renormalisation

$$A_\mu\left(q^2\right) = \int d^4x \,\langle\pi(p)|\,T\left[J_\mu(0)\,H_W(x)\right]|K(k)\rangle + \langle\pi(p)|\,C_{7V}\,Q_{7V}^{\mu}|K(k)\rangle$$

- If we choose J_μ to be conserved - no renormalisation is required.
- For H_W we renormalise the bare operators Q_i non-perturbatively, e.g. using RI-SMOM.
- The Wilson coefficients C_i known at NLO in $\overline{\text{MS}}$ scheme; we can use continuum perturbation theory to match to RI-SMOM.

$$Q_i \xrightarrow{\text{NPR}} Q_i^{\text{RI-SMOM}} \xrightarrow{\text{PT matching}} Q_i^{\overline{\text{MS}}}$$

[4] RBC-UKQCD Collaboration, Phys. Rev. D88 (2013) 014508. arxiv:hep-lat/1212.5931

Operator Renormalisation

$$A_\mu\left(q^2\right) = \int d^4x \left\langle \pi\left(p\right)\right| T\left[J_\mu\left(0\right) H_W\left(x\right)\right] \left|K\left(k\right)\right\rangle + \left\langle \pi\left(p\right)\right| C_{7V} Q_{7V}^{\mu} \left|K\left(k\right)\right\rangle$$

- If we choose J_μ to be conserved - no renormalisation is required.
- For H_W we renormalise the bare operators Q_i non-perturbatively, e.g. using RI-SMOM.

- The Wilson coefficients C_i known at NLO in $\overline{\text{MS}}$ scheme; we can use continuum perturbation theory to match to RI-SMOM.

$$Q_i \xrightarrow{\text{NPR}} Q_i^{\text{RI-SMOM}} \xrightarrow{\text{PT matching}} Q_i^{\overline{\text{MS}}}$$

[4] RBC-UKQCD Collaboration, Phys. Rev. D88 (2013) 014508. arxiv:hep-lat/1212.5931

Operator Renormalisation

$$A_\mu\left(q^2\right) = \int d^4x \left\langle \pi\left(p\right) \right| T\left[J_\mu\left(0\right) H_W\left(x\right)\right] \left| K\left(k\right) \right\rangle + \left\langle \pi\left(p\right) \right| C_{7V} Q_{7V}^{\mu} \left| K\left(k\right) \right\rangle$$

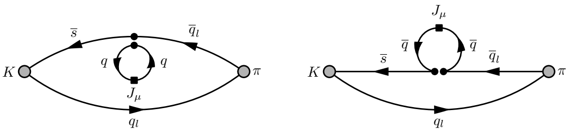
- If we choose J_μ to be conserved - no renormalisation is required.
- For H_W we renormalise the bare operators Q_i non-perturbatively, e.g. using RI-SMOM.
- The Wilson coefficients C_i known at NLO in $\overline{\text{MS}}$ scheme; we can use continuum perturbation theory to match to RI-SMOM.

$$Q_i \xrightarrow{\text{NPR}} Q_i^{\text{RI-SMOM}} \xrightarrow{\text{PT matching}} Q_i^{\overline{\text{MS}}}$$

[4] RBC-UKQCD Collaboration, Phys. Rev. D88 (2013) 014508. arxiv:hep-lat/1212.5931

Divergences

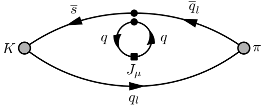
- Divergences may arise from the contact of H_W and J_μ.
- Consider the Wick contractions:

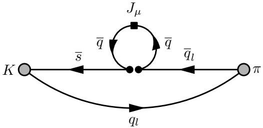


- These diagrams naively appear quadratically divergent!
- With conserved current electromagnetic gauge invariance implies divergence is logarithmic.
  - Cancelled by GIM mechanism.
- How to handle 3 flavour case?

Divergences

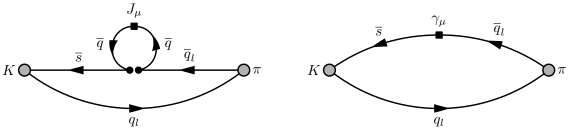
- Divergences may arise from the contact of H_W and J_μ.
- Consider the Wick contractions:





- These diagrams naively appear quadratically divergent!
- With conserved current electromagnetic gauge invariance implies divergence is logarithmic.
  - Cancelled by GIM mechanism.
- How to handle 3 flavour case?

3 Flavour Renormalisation



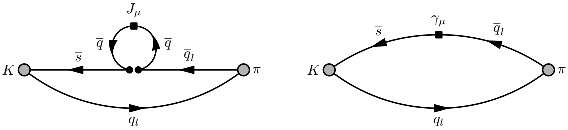
- With 3 flavours, charm contribution moved to local piece.
  - Must match the divergence from the lattice to the $\overline{\text{MS}}$ scheme.
- Divergence in bilocal matrix element regulated by local operator:

$$\langle \pi | T[Q_i J^\mu] | K \rangle^{\overline{\text{MS}}}(\mu) = Z_{ij}^{\text{RI}\to\overline{\text{MS}}} \Big(Z_{jk}^{\text{RI}}(\mu_{\text{RI}}, a) \langle \pi | T[Q_k J^\mu] | K \rangle^{lat} - X_j(\mu_{\text{RI}}, a) \langle \pi | Q_{7V}^\mu | K \rangle^{\text{RI}} \Big) + Y_i(\mu, \mu_{\text{RI}}) \langle \pi | Q_{7V}^\mu | K \rangle^{\text{RI}}$$

- Note the bilocal matrix element is still divergent! Charm contribution not yet included.

[5] N. H. Christ et al., Phys. Rev. D93 (2016) 114517, arXiv:hep-lat/1605.04442

3 Flavour Renormalisation



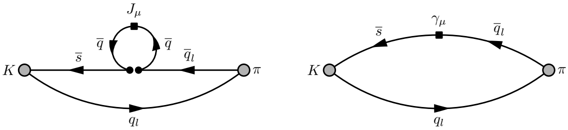
- With 3 flavours, charm contribution moved to local piece.
  - Must match the divergence from the lattice to the $\overline{\text{MS}}$ scheme.
- Divergence in bilocal matrix element regulated by local operator:

$$\begin{aligned}\langle\pi|T\left[Q_i J^\mu\right]|K\rangle^{\overline{\text{MS}}}(\mu) \quad = \quad & Z_{ij}^{\text{RI}\to\overline{\text{MS}}}\left(Z_{jk}^{\text{RI}}(\mu_{\text{RI}},a)\langle\pi|T\left[Q_k J^\mu\right]|K\rangle^{lat}\right. \\ & \left. -X_j(\mu_{\text{RI}},a)\left\langle\pi|Q_{7V}^\mu|K\right\rangle^{\text{RI}}\right) \\ & +Y_i(\mu,\mu_{\text{RI}})\left\langle\pi|Q_{7V}^\mu|K\right\rangle^{\text{RI}}\end{aligned}$$

- Note the bilocal matrix element is still divergent! Charm contribution not yet included.

[5] N. H. Christ et al., Phys. Rev. D93 (2016) 114517, arXiv:hep-lat/1605.04442

3 Flavour Renormalisation



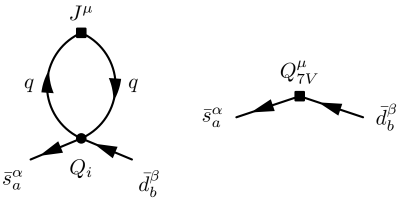
- With 3 flavours, charm contribution moved to local piece.
  - Must match the divergence from the lattice to the $\overline{\text{MS}}$ scheme.
- Divergence in bilocal matrix element regulated by local operator:

$$\begin{aligned}\langle\pi|T\left[Q_i J^\mu\right]|K\rangle^{\overline{\text{MS}}}(\mu) \quad = \quad & Z_{ij}^{\text{RI}\to\overline{\text{MS}}}\Big(Z_{jk}^{\text{RI}}(\mu_{\text{RI}},a)\,\langle\pi|T\left[Q_k J^\mu\right]|K\rangle^{lat} \\ & -X_j(\mu_{\text{RI}},a)\,\langle\pi|Q_{7V}^\mu|K\rangle^{\text{RI}}\Big) \\ & +Y_i(\mu,\mu_{\text{RI}})\,\langle\pi|Q_{7V}^\mu|K\rangle^{\text{RI}}\end{aligned}$$

- Note the bilocal matrix element is still divergent! Charm contribution not yet included.

[5] N. H. Christ et al., Phys. Rev. D93 (2016) 114517, arXiv:hep-lat/1605.04442

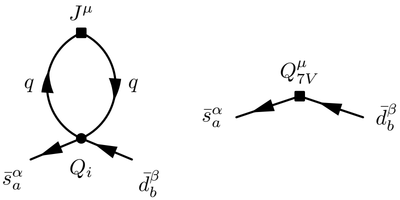
3 Flavour Renormalisation



- Step 1: convert divergence from lattice to RI-SMOM scheme:

$$\begin{aligned} \langle d | T\left[Q_i J^\mu\right] | s \rangle^{\mathrm{RI}} \Big|_{p_i^2=\mu_{\mathrm{RI}}^2} &= Z_{ij}^{\mathrm{RI}}\left(\mu_{\mathrm{RI}}, a\right) \langle d | T\left[Q_j J^\mu\right] | s \rangle^{lat} \Big|_{p_i^2=\mu_{\mathrm{RI}}^2} \\ &\quad - X_i\left(\mu_{\mathrm{RI}}, a\right) \langle d | Q_{7V}^\mu | s \rangle^{\mathrm{RI}} \Big|_{p_i^2=\mu_{\mathrm{RI}}^2} \\ &= 0. \end{aligned}$$

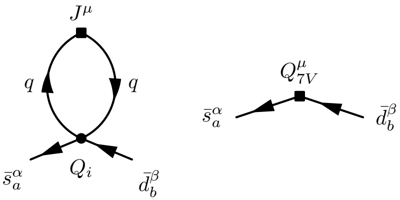
3 Flavour Renormalisation



- Step 2: Perturbative calculation to calculate Y_i to match to $\overline{\text{MS}}$ scheme.

$$\langle d | \, T \left[Q_i J^\mu \right] |s\rangle^{\overline{\rm MS}} \Big|_{p_i^2=\mu_{\rm RI}^2} = Y_i \left(\mu_{\rm RI}, \mu\right) \langle d | Q_{7V}^\mu | s \rangle^{\rm RI} \Big|_{p_i^2=\mu_{\rm RI}^2} .$$

3 Flavour Renormalisation



- Step 3: Divergences cancel between bilocal matrix element and Wilson coefficient of local operator

$$\mathcal{A}\left(K^+ \to \pi^+\ell^+\ell^-\right) = \sum_i C_i(\mu)\,\langle\pi|\,T\left[Q_i J^\mu\right]|K\rangle^{\overline{\mathrm{MS}}}(\mu) + C_{7V}(\mu)\,\langle\pi|Q^\mu_{7V}|K\rangle\,.$$

[6] Buchalla et al., Rev. Mod. Phys. 68 (1996) 1125. arXiv:hep-ph/9512380.

Summary

- We have developed theoretical techniques for calculating the long distance contributions to $K^+ \to \pi^+\ell^+\ell^-$.
- Our exploratory calculations show that we can successfully extract the matrix elements in practice.
- Physical point simulation very expensive
  - 3-flavour calculation good intermediate step before 4-flavour physical lattice calculation is feasible.
- Development of Grid ongoing
  - Implementation of $K \to \pi\ell^+\ell^-$ contractions with Grid complete.
  - NPR code in progress.

Thank you!