Leptonic decay constants for D-mesons from 3-flavour CLS ensembles

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Graduiertenkolleg 2149 Research Training Group

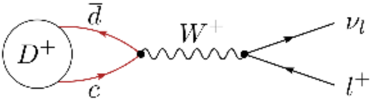
ALPHA Collaboration

RQCD

SFB TRR55

Outline

- aim of project
- utilized CLS ensembles
- analysis techniques
- preliminary results
- summary/outlook



Branching ratio for leptonic decay of $D_{(s)}$:

$$\mathcal{B}(D_{(s)} \to l\nu) = \frac{G_F^2\, |V_{cq}|^2}{8\pi} f_{D_{(s)}}^2 m_l^2 m_{D_{(s)}} \left(1 - \frac{m_l^2}{m_{D_{(s)}}^2}\right)^2$$

Experimentally accessible: product of $|V_{cq}| \times f_{D_{(s)}}$

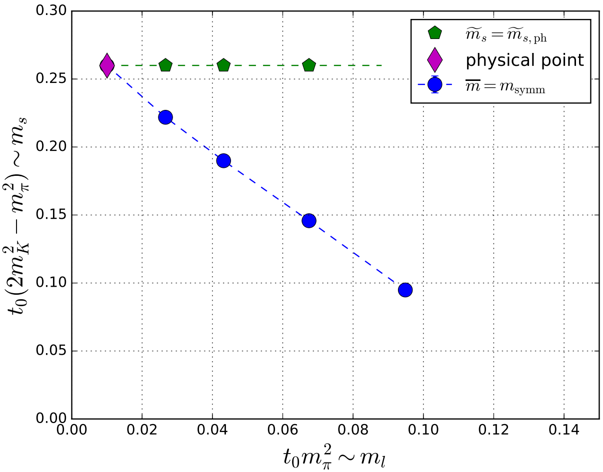
$\Rightarrow$ aim: precision determination of $f_{D_{(s)}}$ on the lattice

($\rightarrow$ constrain CKM matrix elements)

Ensembles were generated within Coordinated Lattice Simulations (CLS) effort. Properties:

- $N_f = 2 + 1$ non-perturbatively $\mathcal{O}(a)$ improved Wilson-Sheikholeslami-Wohlert fermions
- tree-level improved Lüscher-Weisz gauge action
- open boundary conditions in temporal direction to avoid topological freezing
- lattice spacings from $a \approx 0.085$fm to $a \approx 0.039$fm (corresponding to $\beta = 3.4$ to $\beta = 3.85$)
- use of twisted-mass reweighting for light quarks to prevent instabilities resulting from accidental near-zero eigenmodes of the Dirac-operator and rational approximation for the strange quark
- pion masses varied from 422MeV to 223MeV

We follow two RG trajectories in the quark mass plane (here for $\beta = 3.4$):



- $\overline{m} = (2m_l + m_s)/3 = m_{sym}$ (sum of ren. quark masses const. up to $\mathcal{O}(a)$)
- renormalized strange quark mass $\tilde{m}_s = \tilde{m}_s^{phys} = const.$ (const. up to $\mathcal{O}(a)$)

In order to set κ_{charm} we simulate at two different values, allowing the lattice spacing to vary by $\pm 2\%$ and interpolate

- most analyses so far based on estimation via $\overline{M}(1S) = \left(m_{\eta_c} + 3m_{J/\Psi}\right)/4$
- issues: neglected disconnected diagrams and possible flavour mixing introduce uncertainty
- alternative: setting via mass combinations along RG trajectories
- Spin-flavour average $M_X = \left(6m_{D^*} + 2m_D + 3m_{D_s^*} + m_{D_s}\right)/12$ along $\overline{m} = const.$ line
- Spin-flavour average $M_X = \left(3m_{D_s^*} + m_{D_s}\right)/4$ along $\hat{m}_s = const.$ line
- results for interpolated decay constants so far consistent within one standard deviation

Leptonic decay constants defined via

$$\langle 0|A_\mu^{\mathrm{lc}}|\mathrm{D}(p)\rangle = \mathrm{i} f_{\mathrm{D}} p_\mu, \qquad \left\langle 0 \left| A_\mu^{\mathrm{sc}} \right| \mathrm{D_s}(p) \right\rangle = \mathrm{i} f_{\mathrm{D_s}} p_\mu$$

with the axial vector current $A_\mu^{\mathrm{lc}} = \overline{q}\gamma_\mu\gamma_5 c$ (with $q = l, s$). At zero spatial momentum this becomes:

$$\left\langle 0 \left| A_0^{\mathrm{qc}} \right| \mathrm{D}_{(s)} \right\rangle = \mathrm{i} f_{D_{(s)}} m_{D_{(s)}}$$

Remove $\mathcal{O}(a)$ discretization artifacts using the pseudoscalar current $P^{qc} = \overline{q}\gamma_5 c$

$$A_\mu^{\mathrm{qc,I}} = A_\mu^{\mathrm{qc}} + a c_{\mathrm{A}} \frac{1}{2} \left(\partial_\mu + \partial_\mu^*\right) P^{\mathrm{qc}}$$

Renormalization via

$$\left(A_\mu^{qc}\right)^{\mathrm{R}} = Z_{\mathrm{A}} \left[1 + a \left(b_{\mathrm{A}} m_{\mathrm{qc}} + 3\tilde{b}_{\mathrm{A}} \overline{m}\right)\right] A_\mu^{\mathrm{qc,I}} + \mathcal{O}(a^2)$$

c_A, Z_A and b_{A} have been determined non-perturbatively by Bulava, Della Morte, Heitger, Wittemeier in [1502.04999], [1604.05287] and Korcyl, Bali in [1607.07090];
$\tilde{b}_{\mathrm{A}} = 0$ in one-loop perturbative calculation

Extract matrix elements from two-point functions

$$C_{\mathrm{A,I}}(x_0, y_0) = -\frac{a^6}{L^3} \sum_{\vec{x},\vec{y}} \langle A_0^{\mathrm{qc,I}}(x)\,(P^{\mathrm{qc}}(y))^\dagger \rangle,$$

$$C_{\mathrm{P}}(x_0, y_0) = -\frac{a^6}{L^3} \sum_{\vec{x},\vec{y}} \langle P^{\mathrm{qc}}(x)\,(P^{\mathrm{qc}}(y))^\dagger \rangle$$

$\rightarrow$ for large separations of source y_0 and sink x_0 these reduce to

$$C_{\mathrm{A,I}}(x_0, y_0) \approx \frac{\langle 0|A_0^{\mathrm{qc,I}}|\mathrm{D_q}\rangle\langle \mathrm{D_q}\,|P^{\mathrm{qc}}|\,0\rangle}{2m_{D_\mathrm{q}}}\,\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)} = \frac{f_{\mathrm{qc}}^{\mathrm{bare}}}{2}\,A(y_0)\,\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)}$$

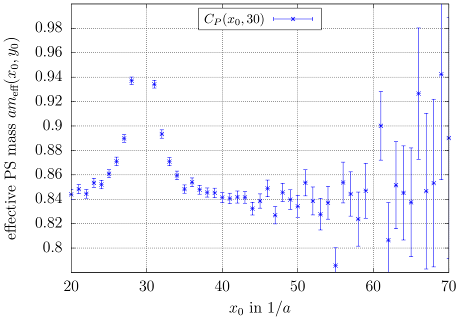
$$\equiv A_{A,I}\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)},$$

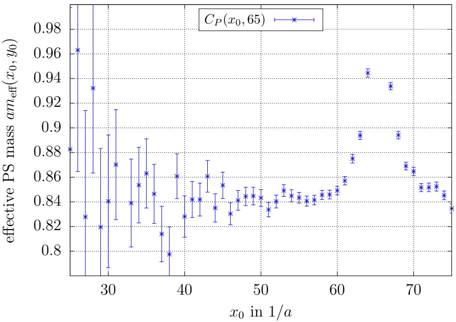
$$C_{\mathrm{P}}(x_0, y_0) \approx \frac{|\langle 0\,|P^{\mathrm{qc}}|\,\mathrm{D_q}\rangle|^2}{2m_{D_\mathrm{q}}}\,\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)} = \frac{|A(y_0)|^2}{2m_{D_\mathrm{q}}}\,\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)}$$

$$\equiv A_P\mathrm{e}^{-m_{D_\mathrm{q}}(x_0-y_0)}$$

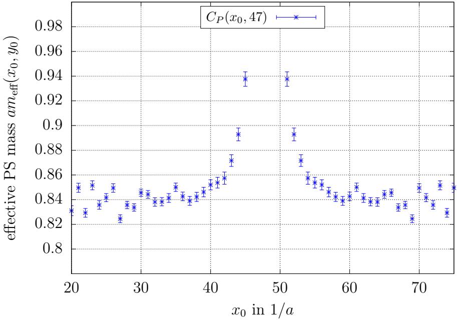
Several approaches so far, e.g. stochastic noise sources. Favored setup at the moment: Point-to-all propagators with smeared sources, placed at three different time-slices.

→ Example with H105 ensemble ($T = 96$ lattice points in temporal direction) with source positions of $x_0/a = 30, 47, 65$: Average over forward and backward propagating parts of correlators (30+65)





Average over forward and backward propagating parts of correlator with source at $y_0 = 47$



Repeat same procedure for axial correlator, in total four correlators. Pseudoscalar and axial correlators are separately fitted to the functional form

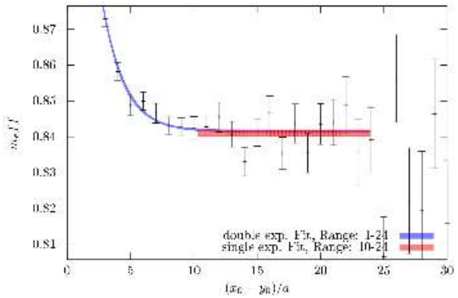
$$Ae^{-m(x_0-y_0)} + Be^{-M(x_0-y_0)}$$

with the second term representing the first exited state.

Find the range where exited state contributions are negligibly small via

$$\frac{|B|^2 \, \mathrm{e}^{-M(x_0^{\min}-y_0)}}{2M} < \frac{1}{4}\,\Delta C_{\mathrm{P}}(x_0^{\min}, y_0)$$

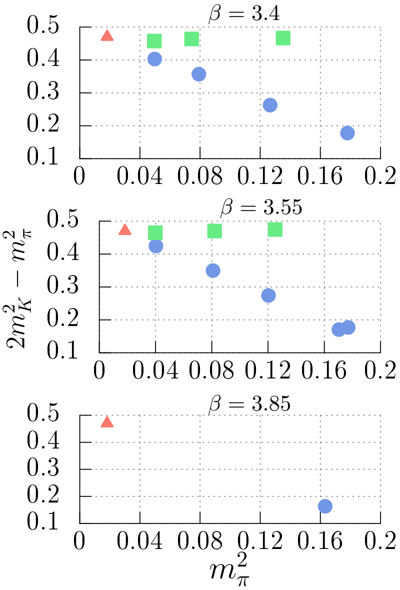
(Example on the right for D-meson on H105 with $y_0 = 30$)



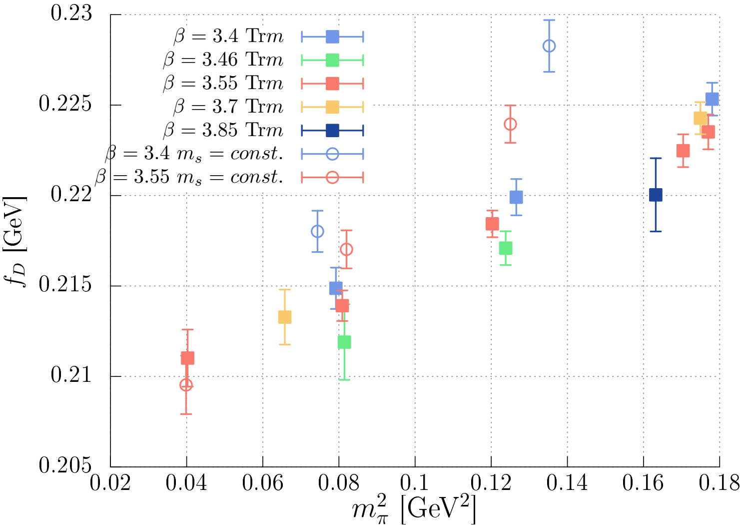
- $x_0^{\min}$ determines fit-range for combined one-state fit of all four correlators
- allow different amplitudes for axial and pseudoscalar correlators, but enforce same amplitude for different source positions
- extract decay constant from fitted amplitudes $A_{A,I}$ and A_P

$$\Rightarrow f_{D_{(s)}}^{bare} = \frac{A_{A,I}}{\sqrt{A_P}}\sqrt{\frac{2}{m_{D_{\mathrm{q}}}}}$$

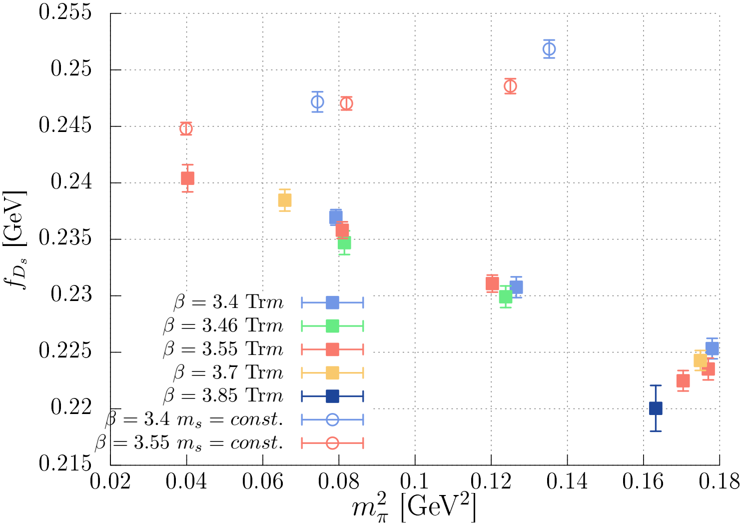
Ensembles analysed so far:

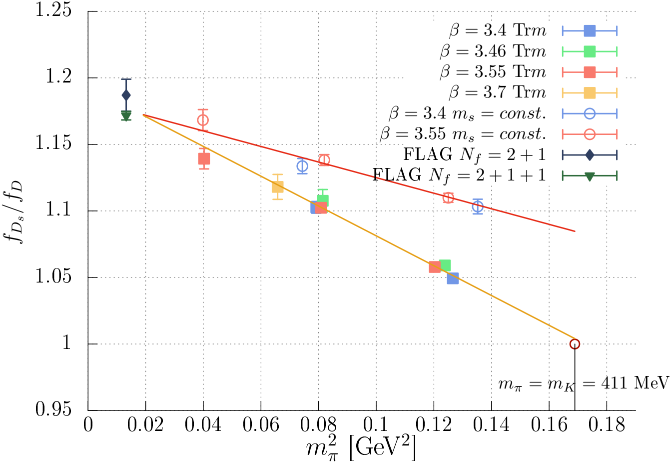


Results for the decay constant f_D on the ensembles analysed so far:



Results for f_{D_s}:





careful continuum/chiral extrapolation as soon as fitting and statistical error estimation are under full control

→ data consistent with FLAG results, no strong discretization effects, ratio compatible with one at symmetric point

Summary:

- multitude of ensembles at different β and quark masses already analysed
- combined two- and one-state-fits show good stability
- preliminary results of $f_{D_{(s)}}$ promising
- discretization artifacts and finite size effects small and well under control

Outlook:

- inclusion of further ensembles at small lattice spacings
- increase of statistics for several ensembles already included in analysis
- preparation of chiral/continuum extrapolation with full control over statistical and systematic uncertainties

Thank you for your attention!

Literature

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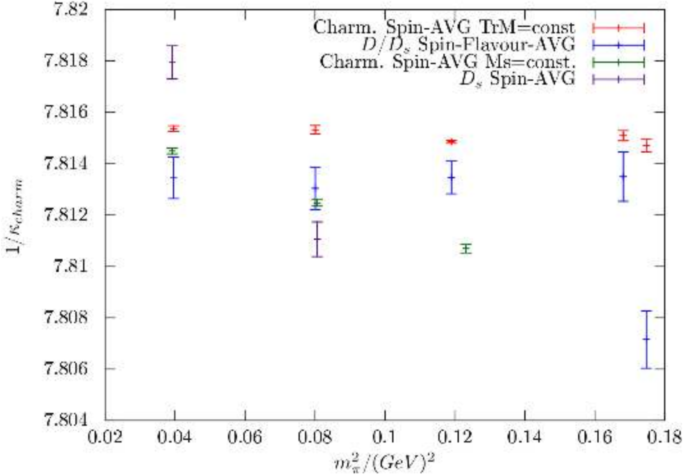
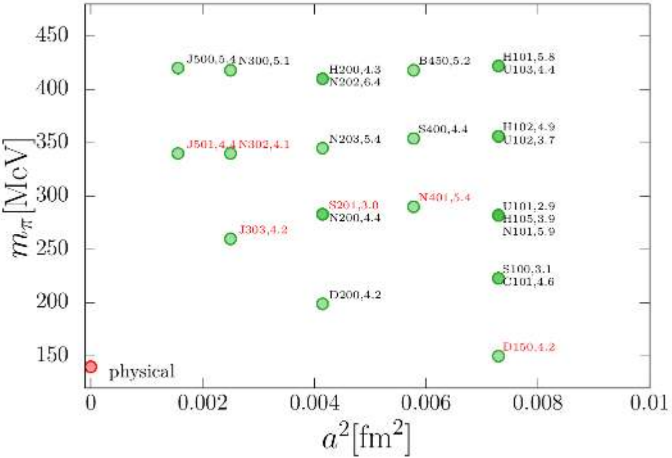
- **ALPHA** Collaboration, J. Bulava, M. Della Morte, J. Heitger, C. Wittemeier, *Non-perturbative improvement of the axial current in $N_f = 3$ lattice QCD with Wilson fermions and tree-level improved gauge action*, *Nucl.Phys.* **B896** (2015) 555 [1502.04999]
- **ALPHA** Collaboration, J. Bulava, M. Della Morte, J. Heitger, C. Wittemeier, *Non-perturbative renormalization of the axial current in $Nf = 3$ lattice QCD with Wilson fermions and tree-level improved gauge action*, *Phys.Rev* **D93** (2016) no.11, 114513 [1604.05827]
- P. Korcyl, G. S. Bali, *Non-perturbative determination of improvement coefficients using coordinate space correlators in $Nf = 2+1$ lattice QCD*, *Phys.Rev.* **D95** (2017) no.1, 014505 [1607.07090]
- M. Bruno, T. Korzec, S. Schaefer, *Setting the scale for the CLS $2+1$ flavor ensembles*, *Phys.Rev.* **D95** (2017) no.7, 074504 [1608.08900]

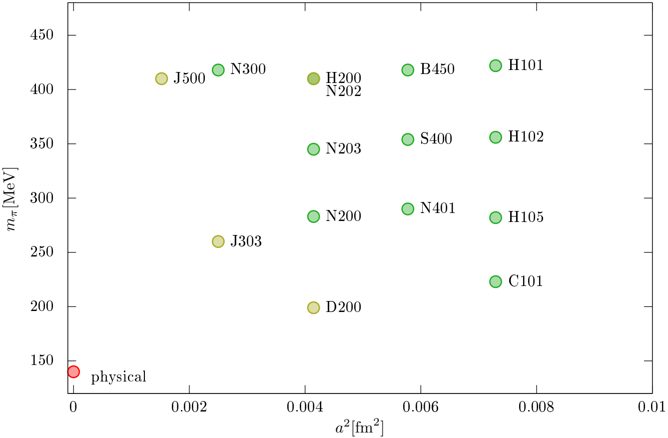
Estimation of $\kappa_{\mathbf{charm}}$



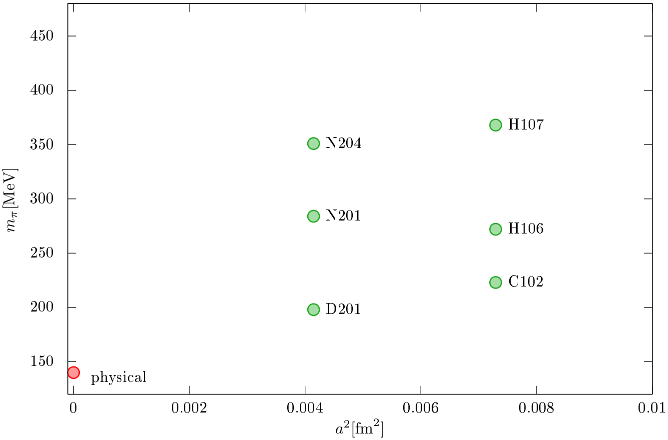
Ensembles for the $\overline{m} = m_{\mathbf{sym}}$ line



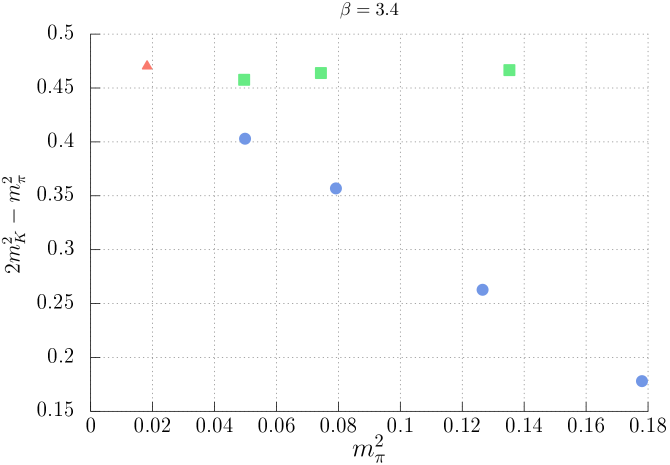
Ensembles analysed so far for the $\overline{m} = m_{\mathbf{sym}}$ line



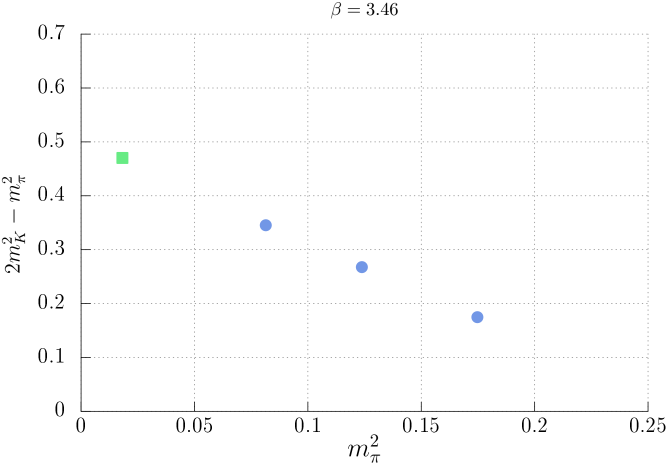
Ensembles analysed so far for the $\hat{m}_s = \hat{m}_s^{\mathbf{phys}}$ line



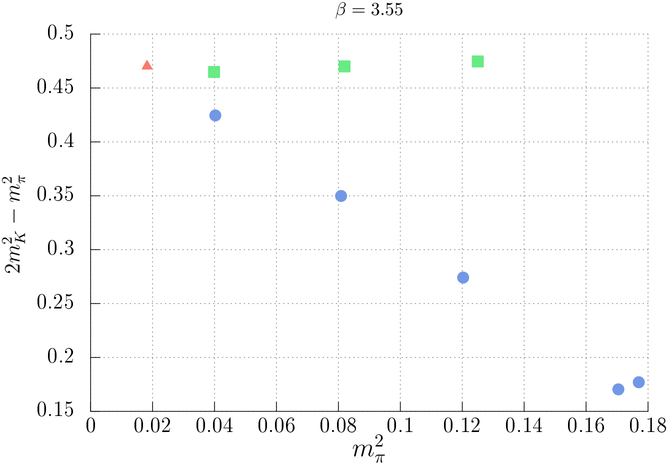
Ensembles analysed so far at $\beta = 3.4$



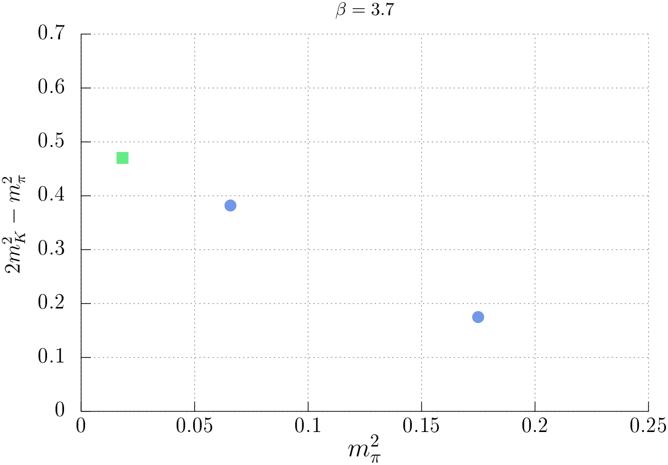
Ensembles analysed so far at $\beta = 3.46$



Ensembles analysed so far at $\beta = 3.55$



Ensembles analysed so far at $\beta = 3.7$



Ensembles analysed so far at $\beta = 3.85$