A precise determination of the HVP contribution to the muon anomalous magnetic moment from lattice QCD

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BNL and RBRC

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RBC/UKQCD $g-2$ effort

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Antonin Portelli (Edinburgh)

The magnetic moment

- The magnetic moment $\vec{\mu}$ determines the shift of a particle's energy in the presence of a magnetic field $\vec{B}$

$$V = -\vec{\mu} \cdot \vec{B}$$

- The intrinsic spin $\vec{S}$ of a particle contributes

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{S}$$

  with electric charge e, particle mass m, and Landé factor g.

- Anomalous magnetic moment $a = (g-2)/2$ accounts for radiative corrections to Dirac's result $g = 2$.

- In general $a_\ell^{\text{New Physics}} \propto m_\ell^2$ for a lepton $\ell = e, \mu, \tau \Rightarrow a_\mu$ good target to find new physics

Theory status for a_μ – summary

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
FNAL E989/J-PARC E34 goal		$\approx$ **1.6**

We currently observe a $\sim 3\sigma$ tension.

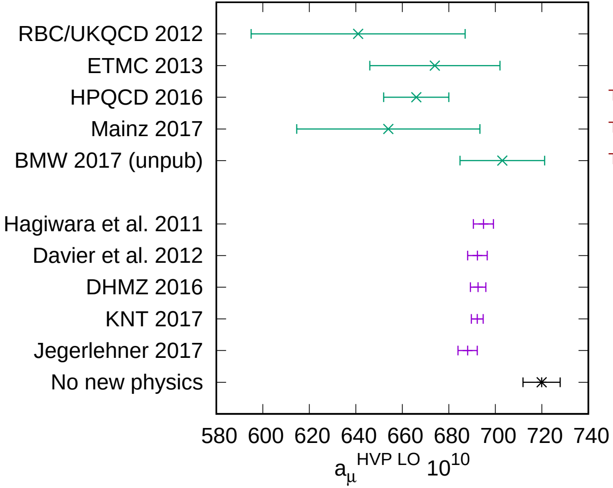
New experiment: Fermilab E989

Aims at a 4× reduction in experimental uncertainty. Need better theory!

(Higher muon intensity at FNAL, same storage ring)

Alternative proposal using ultra-cold muons allowing for a 66cm storage device proposed at J-PARC (E34)

Theory status for a_μ – HVP LO



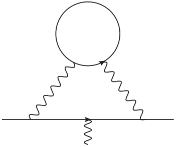
Talk (update) by Van de Water

Talk by Wittig

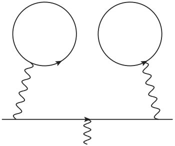
Talk by Miura

Further updates and related talks at Lattice 2017 by Bijnens, Bussone, D. Giusti, Golterman, Guelpers, Harrison, Shintani, and Silvano

First-principles approach to HVP LO

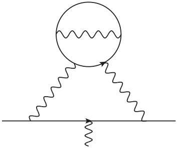


Quark-connected piece with by far dominant part from up and down quark loops, $\mathcal{O}(700 \times 10^{-10})$



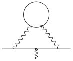
Quark-disconnected piece, $-9.6(4.0) \times 10^{-10}$

Phys.Rev.Lett. 116 (2016) 232002



QED corrections, $\mathcal{O}(10 \times 10^{-10})$

All results below are obtained using domain-wall fermions at physical pion mass with lattice cutoffs $a^{-1} = 1.73$ GeV and $a^{-1} = 2.36$ GeV.



HVP quark-connected contribution

Starting from the vector current

$$J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x) \gamma_\mu \Psi_f(x)$$

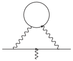
we may write

$$a_\mu^{\mathrm{HVP}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the diagram (Bernecker-Meyer 2011).



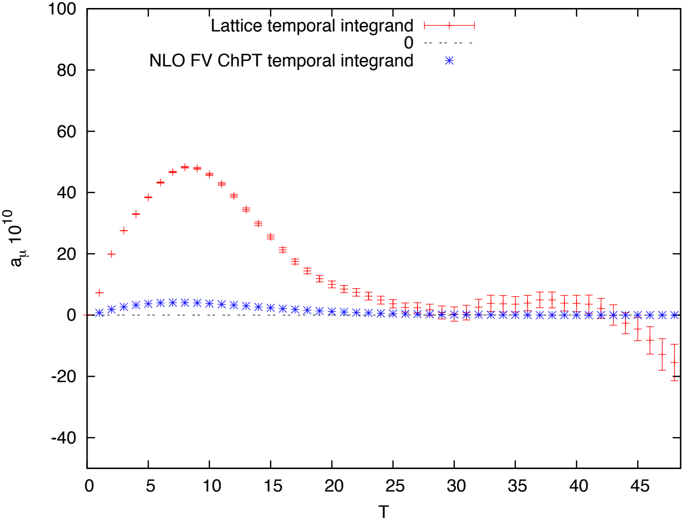
HVP quark-connected contribution

Biggest challenge to direct calculation at physical pion masses is to control statistics and potentially large finite-volume errors.

Statistics: for strange and charm solved issue, for up and down quarks existing methodology less effective

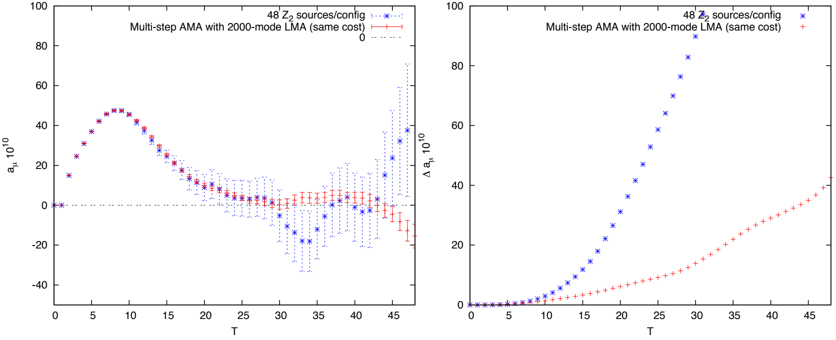
Finite-volume errors are exponentially suppressed in the simulation volume but may be sizeable

Integrand $w_T C(T)$ for the light-quark connected contribution:



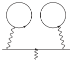
$m_\pi = 140$ MeV, $a = 0.11$ fm (RBC/UKQCD 48^3 ensemble)

Statistical noise from long-distance region



Significant error reduction using full-volume low-mode average (DeGrand & Schäfer 2004) in addition to a multi-level all-mode average.

New method: Multi-Grid Lanczos utilizing local coherence of eigenvectors yields 10× reduction in memory cost (Poster by C.L. at Lattice 2017)



HVP quark-disconnected contribution

First results at physical pion mass with a statistical signal
Phys.Rev.Lett. 116 (2016) 232002

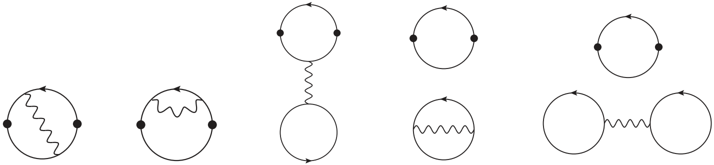
Statistics is clearly the bottleneck; calculation was a potential road-block of a first-principles calculation for a long time; due to very large pion-mass dependence calculation at physical pion mass is crucial.

New stochastic estimator allowed us to get result

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}$$

from a modest computational investment ($\approx$ 1M core hours).

HVP QED contribution

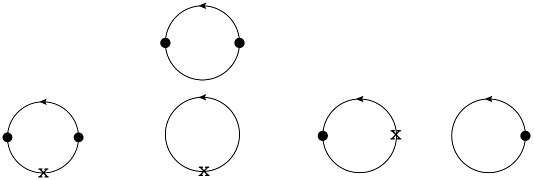


(a) V (b) S (c) T (d) D1 (e) D2

(f) F (g) D3

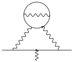
New method: use importance sampling in position space and local vector currents

HVP strong IB contribution



(a) M (b) R (c) O

Calculate strong IB effects via insertions of mass corrections in an expansion around isospin symmetric point



HVP QED+strong IB contributions

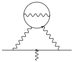
Strategy

1. Re-tune parameters for QCD+QED simulation
   (m_u, m_d, m_s, a) via (m_{π^+}, m_{π^0}, m_{K^+}, m_{K^0}, m_{Ω^-})

2. Verify simple observables ($m_{\pi^+} - m_{\pi^0}$, ...)

3. Calculate QED and strong IB corrections to HVP LO

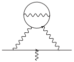
All results shown below are preliminary! For now focus on diagrams S, V, F; preliminary study below does not yet include re-tuning of a.



HVP QED+strong IB contributions

Result of parameter retuning (bare lattice masses):

$$\Delta m_u = -0.000678(83)\,,$$
$$\Delta m_d = 0.000519(83)\,,$$
$$\Delta m_s = -0.000431(32)\,,$$
$$\frac{m_{res} + m_l + \Delta m_u}{m_{res} + m_l + \Delta m_d} = 0.373(59)\,,$$
$$\frac{m_u^{\mathrm{PDG}}}{m_d^{\mathrm{PDG}}} = 0.48(11)\,.$$



HVP QED+strong IB contributions

Diagrams S, V for pion mass:

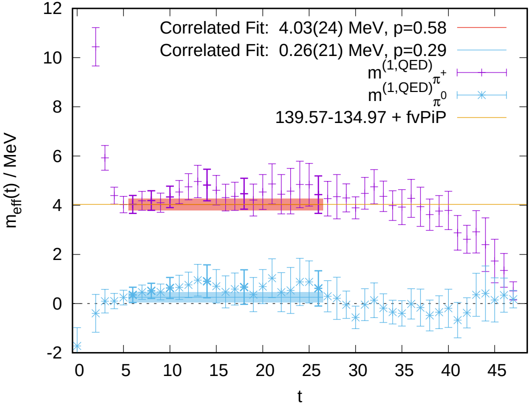
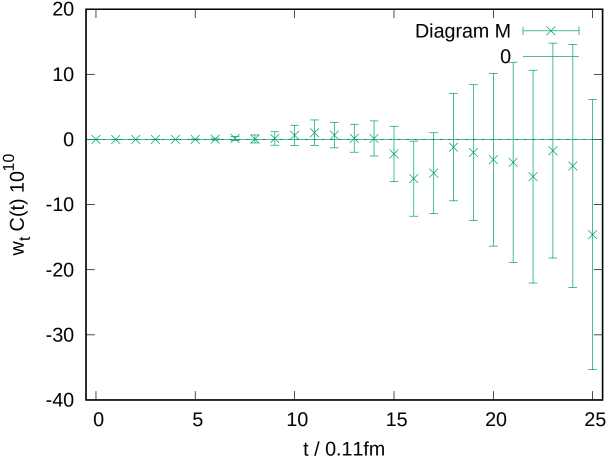


Diagram F for pion mass is measured and tiny.

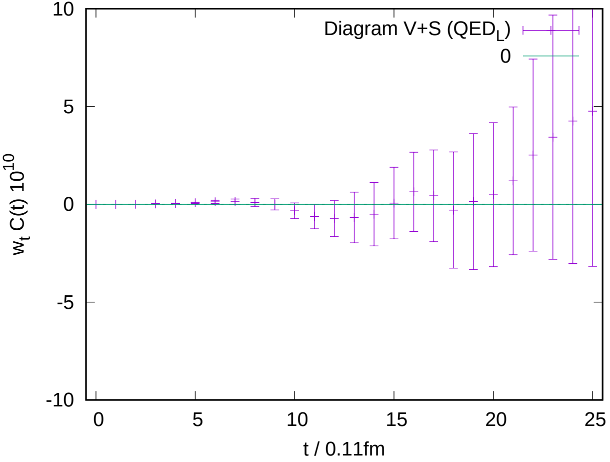
HVP QED+strong IB contributions

HVP strong IB effect



HVP QED+strong IB contributions

HVP QED example: diagram V+S



Statistics improvements available soon

A connection to the R-ratio data

We now have all ingredients to compare to the R-ratio data

We can connect $C(t)$ to the R-ratio data (Bernecker, Meyer 2011) as

$$\Pi(-Q^2) = \frac{1}{\pi}\int_0^\infty ds\,\frac{s}{s+Q^2}\sigma(s, e^+e^- \to \mathrm{had})$$

with

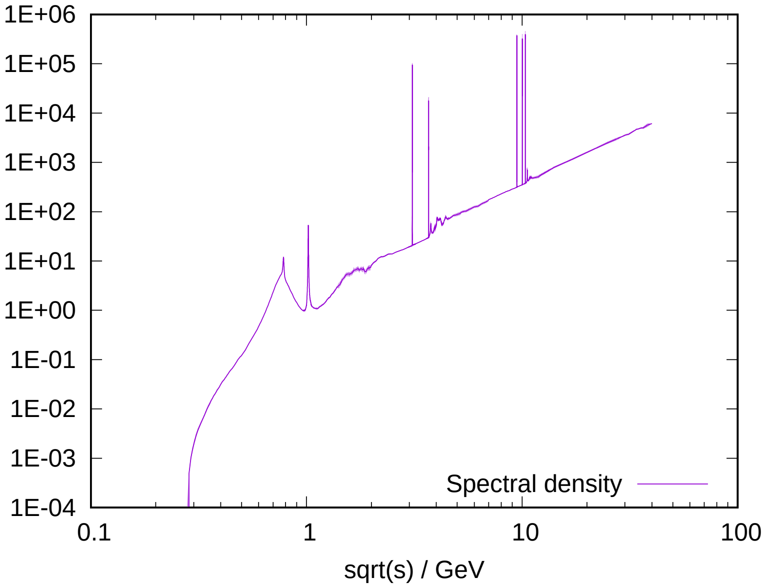
$$R(s) = \frac{\sigma(s, e^+e^- \to \mathrm{had})}{\sigma(s, e^+e^- \to \mu^+\mu^-, \mathrm{tree})} = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})\,.$$

A Fourier transform then gives

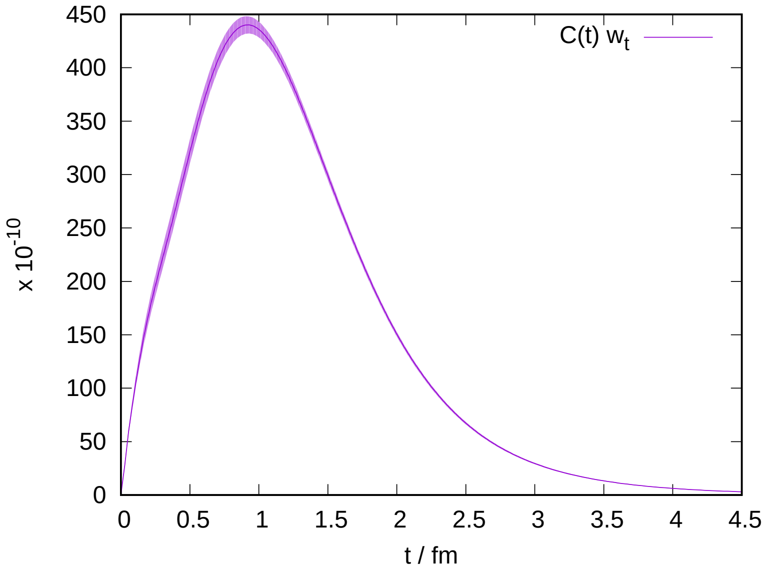
$$C(t) \propto \int_0^\infty d(\sqrt{s})R(s)se^{-\sqrt{s}t} \equiv \int_0^\infty d(\sqrt{s})\rho(\sqrt{s})e^{-\sqrt{s}t}$$

with spectral density $\rho(\sqrt{s})$.

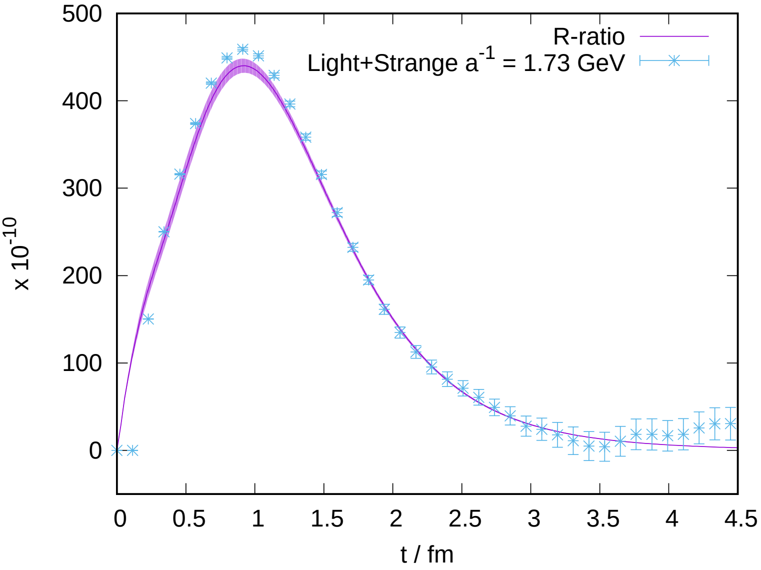
Below the $R(s)$ is taken from Jegerlehner 2016:



Contributions to a_μ as function of t:



Lattice data agrees quite well with the R-ratio data



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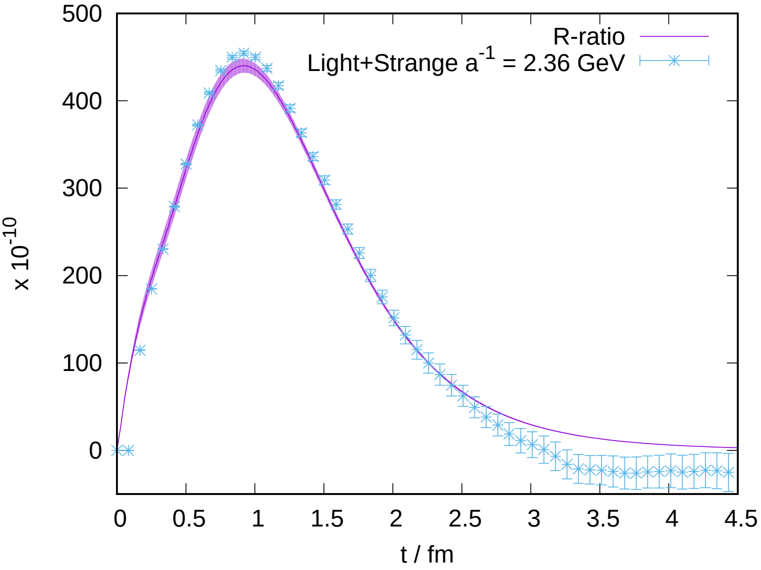
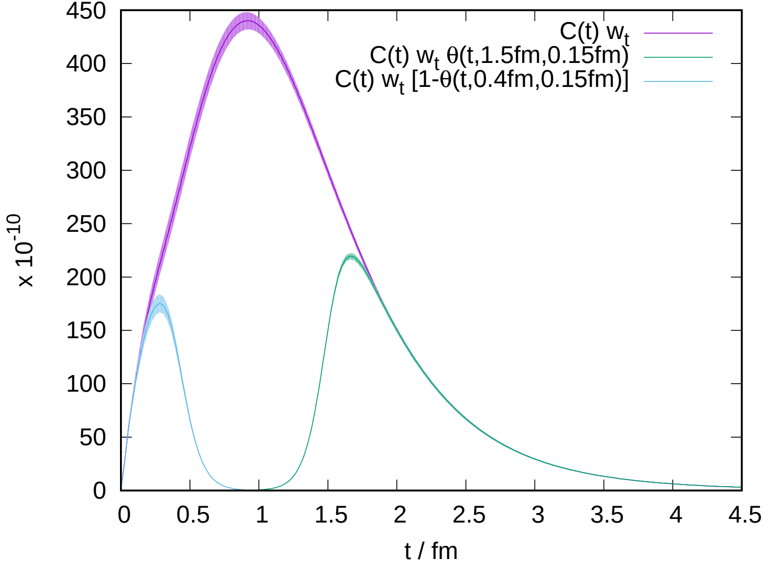
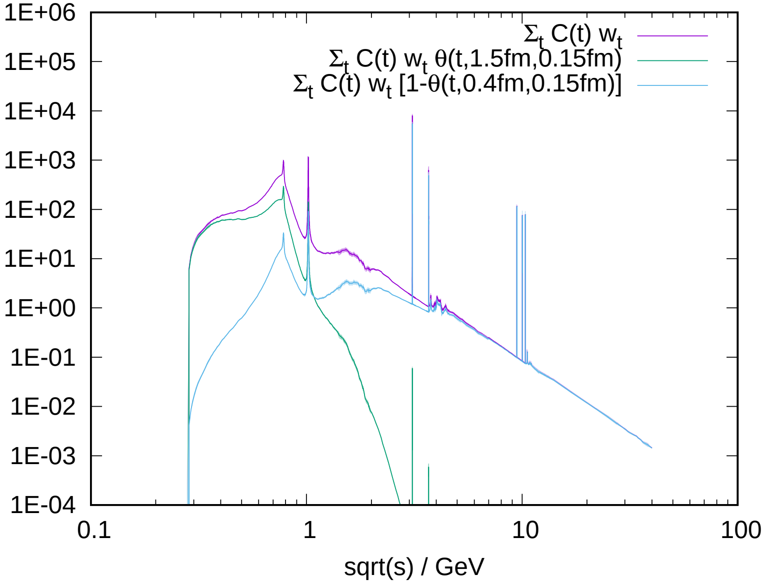


We can also select a window in t by defining a smeared Θ function:



$$\Theta(t, \mu, \sigma) \equiv [1 + \tanh[(t-\mu)/\sigma]]/2$$

Selecting a window in t can be translated to re-weighting contributions in $\sqrt{s}$. Here contributions to a_μ:



This allows us to devise a “Window method”:

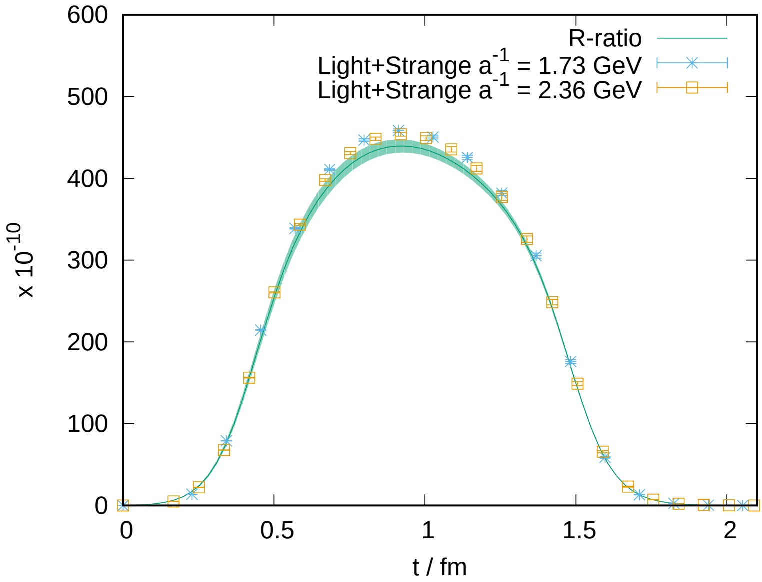
$$a_\mu = \sum_t w_t C(t) \equiv a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$$

with

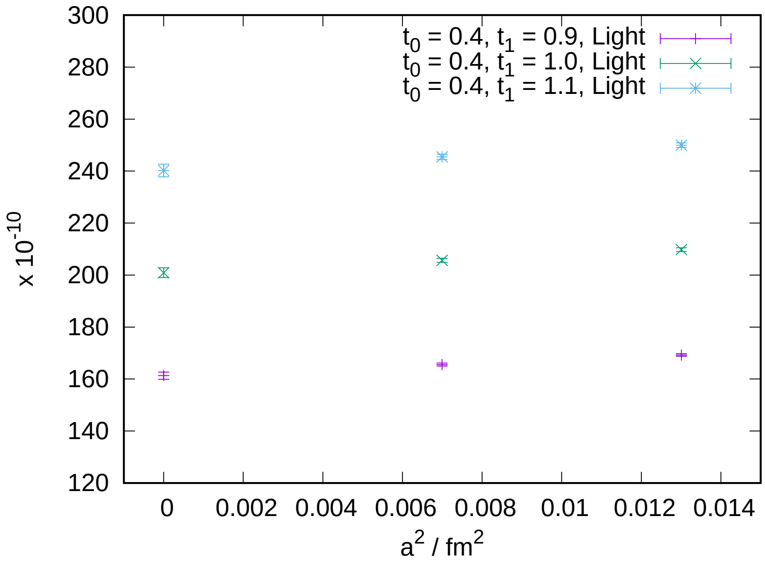
$$\begin{aligned} a_\mu^{\rm SD} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\ a_\mu^{\rm W} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\ a_\mu^{\rm LD} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta) \end{aligned}$$

and each contribution accessible from both lattice and R-ratio data.

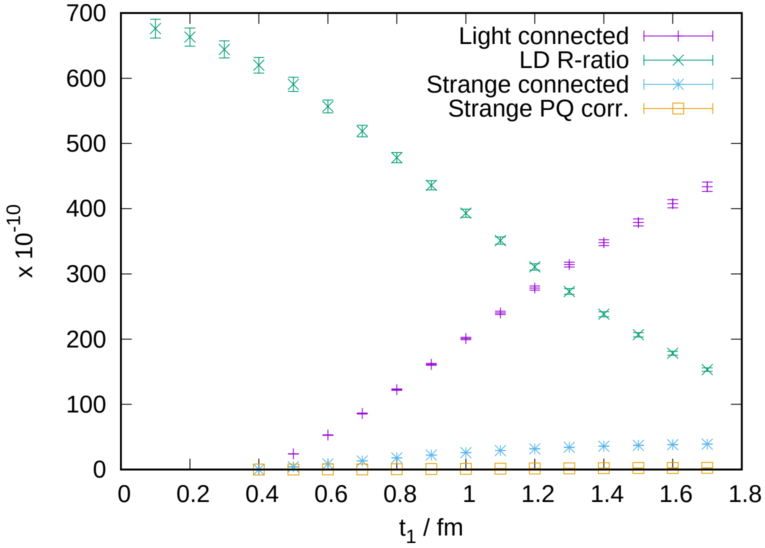
Example contribution to a_μ^{W} with $t_0 = 0.4$ fm, $t_1 = 1.5$ fm, $\Delta = 0.15$ fm:



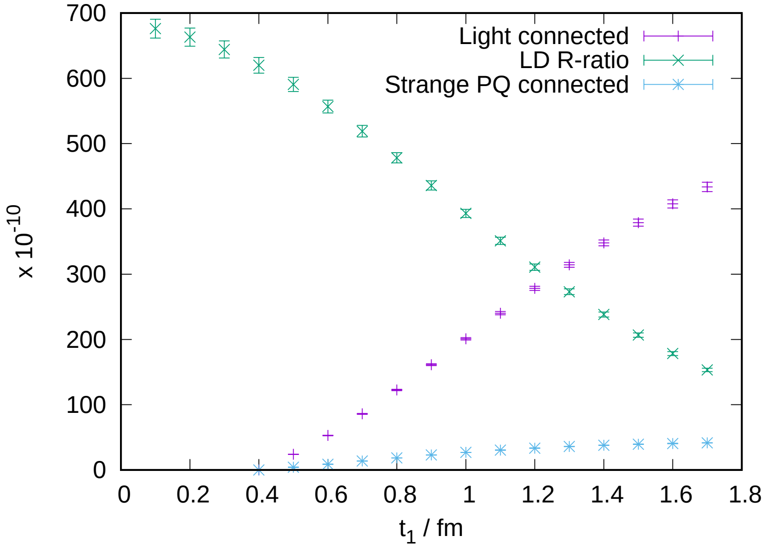
Continuum limit of a_μ^{W} from our lattice data; below $t_0 = 0.4$ fm and $\Delta = 0.15$ fm



Re-combine a_μ^{W} from lattice with a_μ^{LD} from R-ratio:



Re-combine a_μ^{W} from lattice with a_μ^{LD} from R-ratio:



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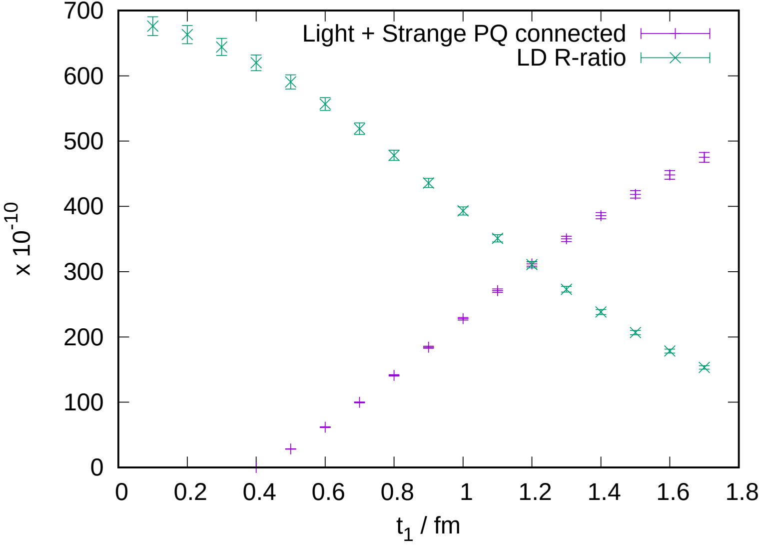

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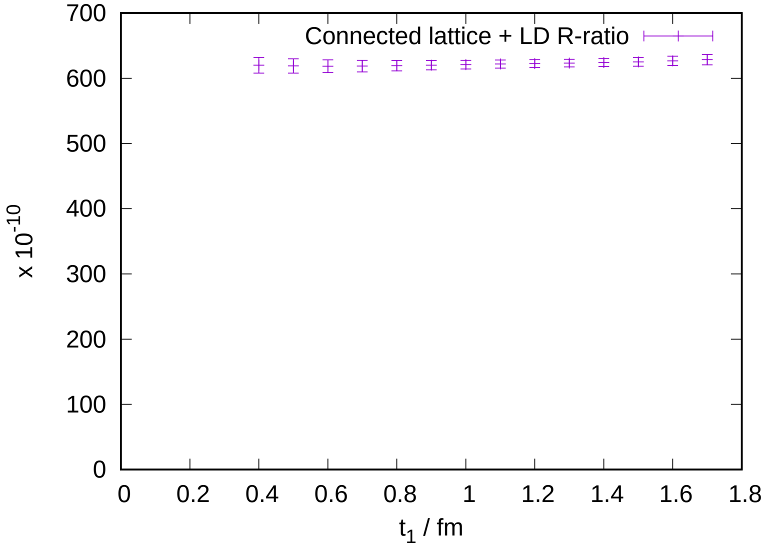
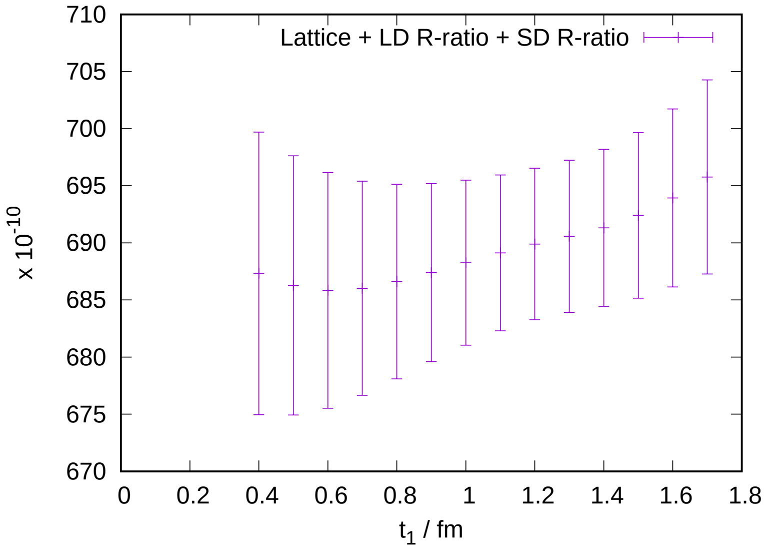
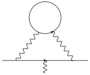
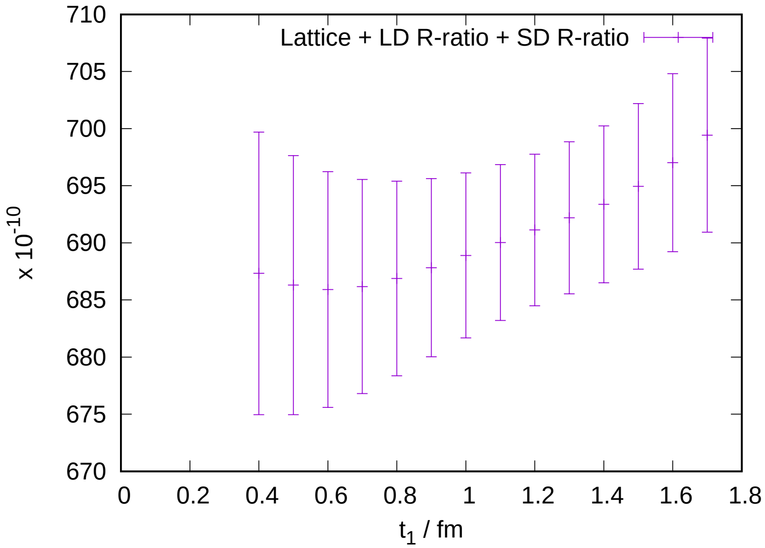
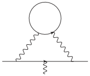


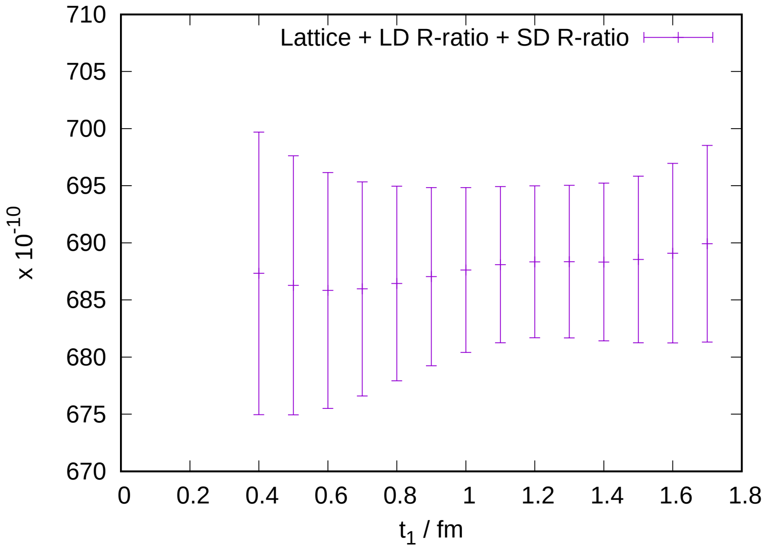

Lattice + LD R-ratio + SD R-ratio
710
705
700
695
690
685
680
675
670
x 10^-10
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
t1 / fm

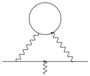
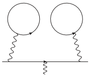
Lattice:

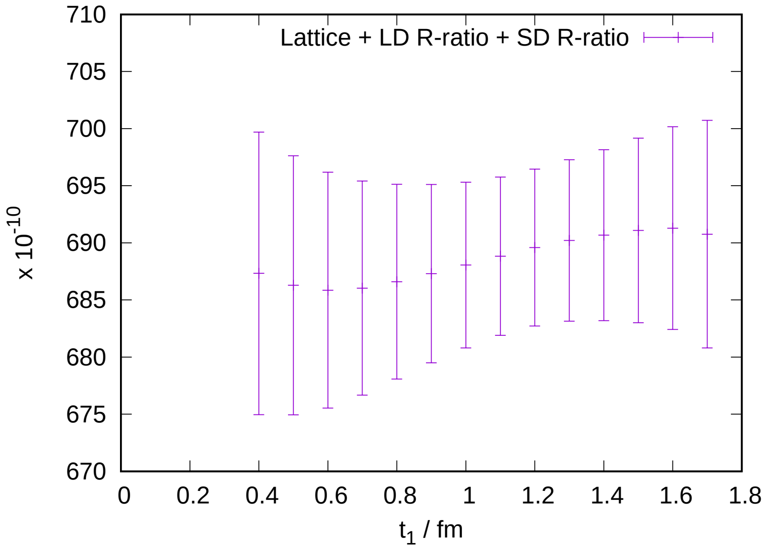


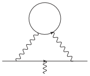
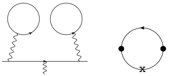


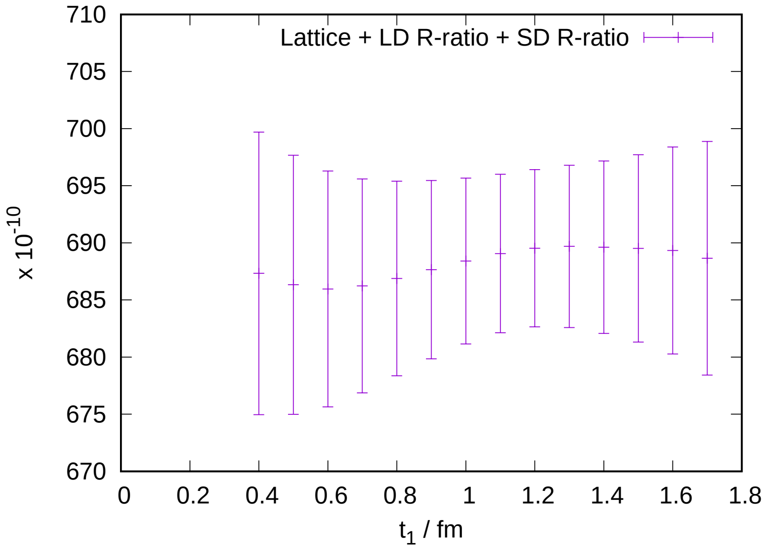
Lattice:  sQED FV correction



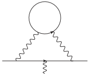
Lattice:  sQED FV correction 



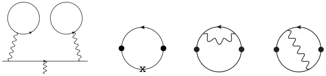
Lattice:  sQED FV correction 

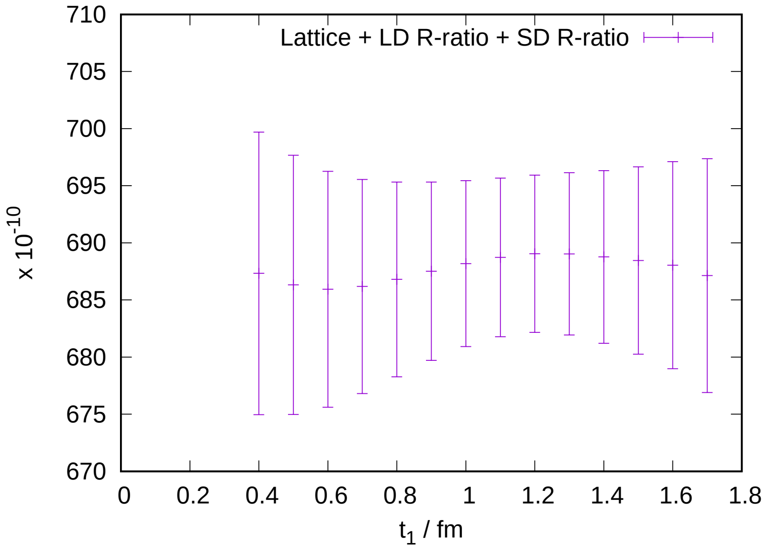


Lattice:

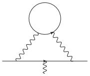


sQED FV correction

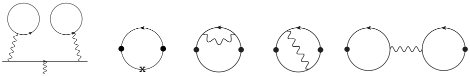


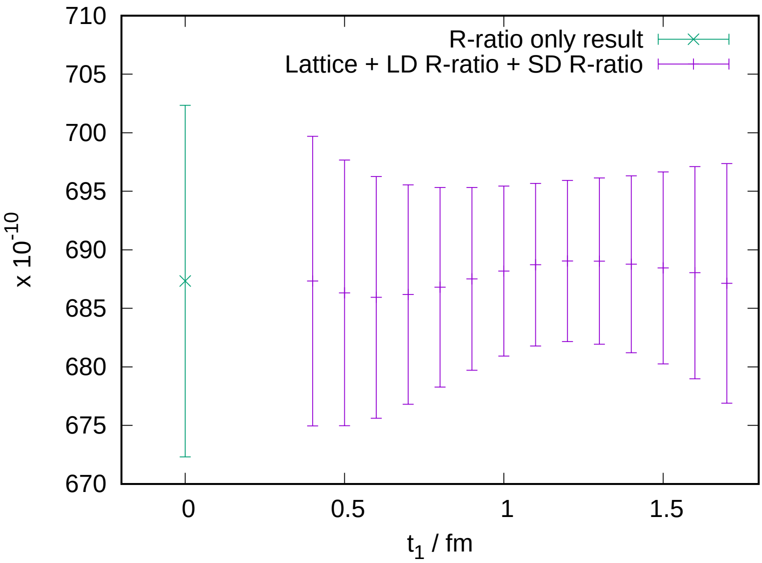


Lattice:

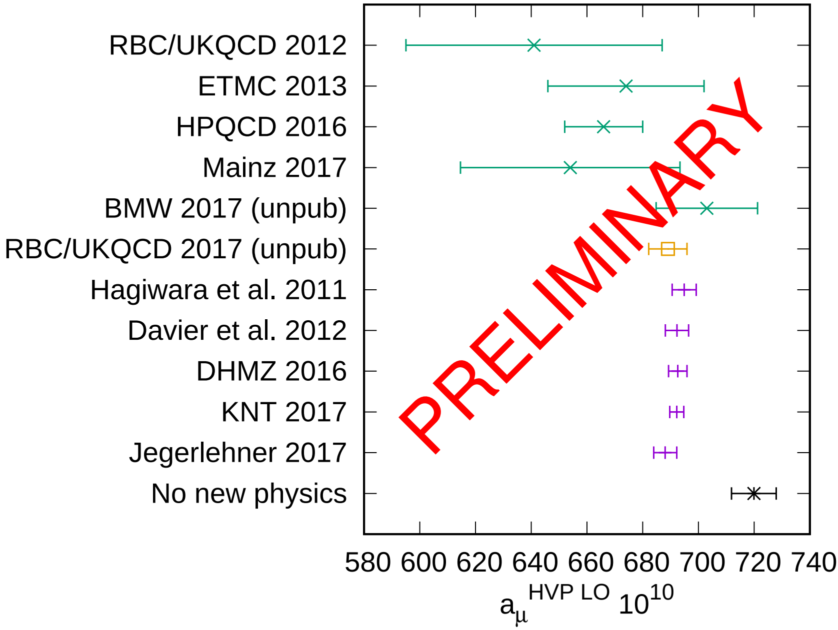


sQED FV correction





Note: combined lattice and R-ratio is more precise than R-ratio alone!
Error minimal for $t_1 = 1.2$ fm.


RBC/UKQCD 2012
ETMC 2013
HPQCD 2016
Mainz 2017
BMW 2017 (unpub)
RBC/UKQCD 2017 (unpub)
Hagiwara et al. 2011
Davier et al. 2012
DHMZ 2016
KNT 2017
Jegerlehner 2017
No new physics
PRELIMINARY
580 600 620 640 660 680 700 720 740
$a_\mu^{\text{HVP LO}}\ 10^{10}$

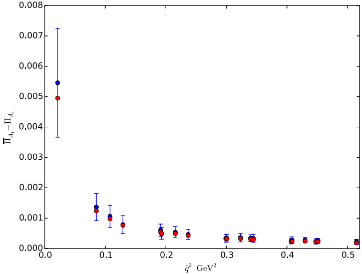
Summary

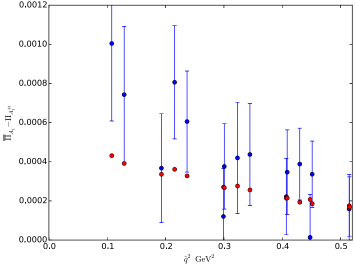
- Improved statistical estimators significantly reduced our statistical uncertainty and new Multi-Grid Lanczos further improves its efficiency.
- For the disconnected contributions the new method allowed for a precise calculation at physical pion mass.
- For the QED contributions we use a stochastic position space sampling technique; we still need to re-tune lattice spacing in presence of QED.
- We devise a window method to combine and cross-check lattice and R-ratio data. This method allows for further reduction in uncertainty over the already very precise R-ratio results.
- Here we used the results of Jegerlehner 2016 for a combined analysis and obtained a result with $\delta a_\mu^{\mathrm{HVP\ LO}} = 6.8 \times 10^{-10}$; further improvements require full knowledge of correlation of R-ratio data.
- Eventually the window can be widened to obtain a pure lattice result.

Thank you

Addressing the finite-volume problem

From Aubin et al. 2015 (arXiv:1512.07555v2)



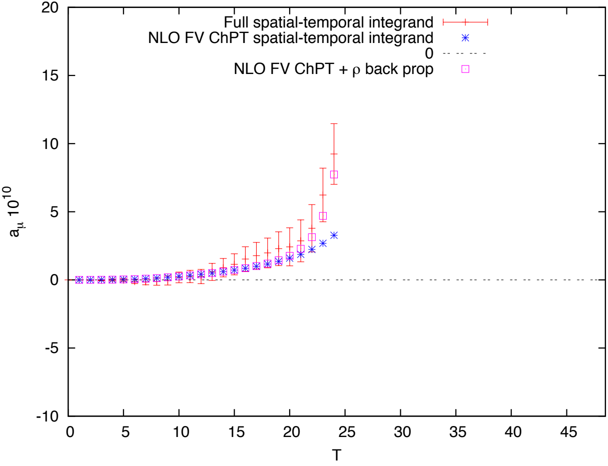


MILC lattice data with $m_\pi L = 4.2$, $m_\pi \approx 220$ MeV; Plot difference of $\Pi(q^2)$ from different irreps of 90-degree rotation symmetry of spatial components versus NLO FV ChPT prediction (red dots)

While the absolute value of a_μ is poorly described by the two-pion contribution, the volume dependence may be described sufficiently well to use ChPT to control FV errors at the 1% level; this needs further scrutiny

Aubin et al. find an $O(10\%)$ finite-volume error for $m_\pi L = 4.2$ based on the $A_1 - A_1^{44}$ difference (right-hand plot)

Compare difference of integrand of $48 \times 48 \times 96 \times 48$ (spatial) and $48 \times 48 \times 48 \times 96$ (temporal) geometries with NLO FV ChPT $(A_1 - A_1^{44})$:



$m_\pi = 140$ MeV, $p^2 = m_\pi^2/(4\pi f_\pi)^2 \approx 0.7\%$

Our efforts to control the finite-volume error:

- We have generated three additional lattices with physical pion mass and $L = 4.8$fm, 6.4fm, and 9.6fm; we have started first measurements on these lattices.

- We are currently tuning our new Multi-Grid Lanczos method on the largest volumes to continue to use our noise-reduction techniques for these studies. For these ensembles the improved Multi-Grid Lanczos is critical.

Addressing the long-distance noise problem

There are two general classes of solutions to the long-distance noise problem

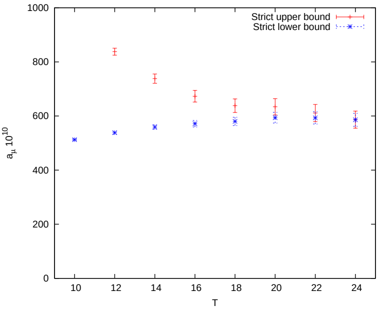
- Statistics $\rightarrow$ Systematics: One can reduce statistical uncertainty at the cost of introducing an additional systematic uncertainty that then needs to be controlled; This requires additional care in estimating a potential systematic bias but may be overall beneficial.
- Statistics $\uparrow$: One can devise improved statistical estimators without additional systematic uncertainties

Concrete recent proposals:

- Replace $C(t)$ for large t with model, say multi-exponentials for $t \geq t^*$ HPQCD arXiv:1601.03071 (Statistics $\rightarrow$ Systematics)

- Define stochastic estimator for strict upper and lower bounds of a_μ which have reduced statistical fluctuations RBC/UKQCD 2015, BMWc arXiv:1612.02364 (Statistics $\uparrow$)

More details, e.g., talk C.L. at Rutgers 2015



Bound $C_l(t) \leq C(t) \leq C_u(t)$ with

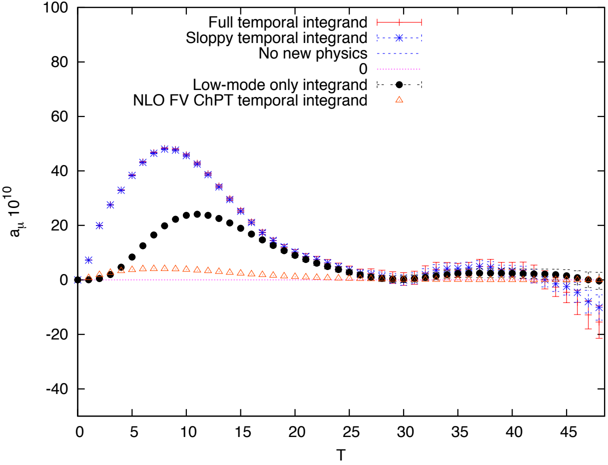
$$C_{l/u}(t) = \begin{cases} C(t) & t < T\,, \\ C(T)e^{-(t-T)\bar{E}_{l/u}} & t \geq T \end{cases}$$

with $\bar{E}_u$ being the ground state of the VV correlator and

$$\bar{E}_l = \log(C(T)/C(T+1))\,.$$

Concrete recent proposals (continued):

- RBC/UKQCD 2015 Improved stochastic estimator; hierarchical approximations including exact treatment of low-mode space DeGrand & Schäfer 2004: (Statistics ↑):



Concrete recent proposals (continued):

- Phase reweighting (Savage et al.) (Statistics $\rightarrow$ Systematics)

$$C(t) \rightarrow C(t)\,\mathrm{Sign}[C(t-\Delta)]$$

  extrapolate to $\Delta \rightarrow \infty$

- Multi-level gauge field generation (Ce/Giusti/Schafer) (Statistics $\uparrow$)
  - Action is local $\Rightarrow$ independent evolution of gauge fields in sub-domains possible
  - Recombination of independent samples over all subdomains may lead to exponential reduction of noise
  - We are currently investigating this method for the HVP (M. Bruno for RBC/UKQCD)

The setup:

$$C(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \langle \mathcal{V}_j(t+t')\mathcal{V}_j(t')\rangle_{\mathrm{SU(3)}} \tag{1}$$

where V stands for the four-dimensional lattice volume, $\mathcal{V}_\mu = (1/3)(\mathcal{V}_\mu^{u/d} - \mathcal{V}_\mu^s)$, and

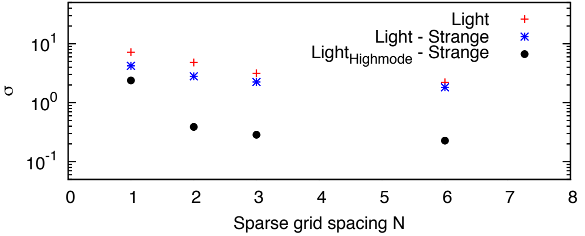
$$\mathcal{V}_\mu^f(t) = \sum_{\vec{x}} \mathrm{Im}\,\mathrm{Tr}[D^{-1}_{\vec{x},t;\vec{x},t}(m_f)\gamma_\mu]\,. \tag{2}$$

We separate 2000 low modes (up to around m_s) from light quark propagator as $D^{-1} = \sum_n v^n (w^n)^\dagger + D^{-1}_{\mathrm{high}}$ and estimate the high mode stochastically and the low modes as a full volume average Foley 2005.

We use a sparse grid for the high modes similar to Li 2010 which has support only for points x_μ with $(x_\mu - x_\mu^{(0)}) \bmod N = 0$; here we additionally use a random grid offset $x_\mu^{(0)}$ per sample allowing us to stochastically project to momenta.

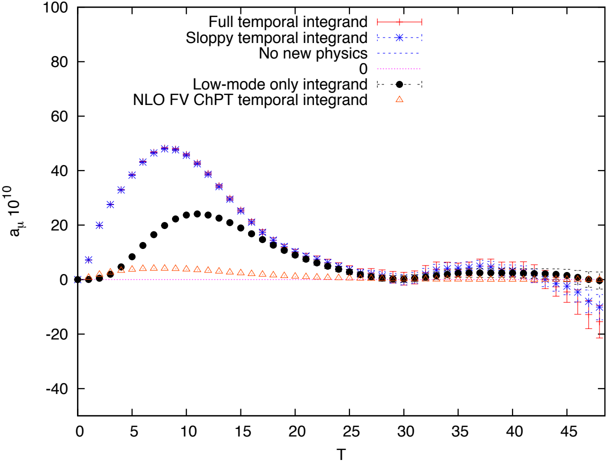
Combination of both ideas is crucial for noise reduction at physical pion mass!

Fluctuation of $\mathcal{V}_\mu$ (σ):

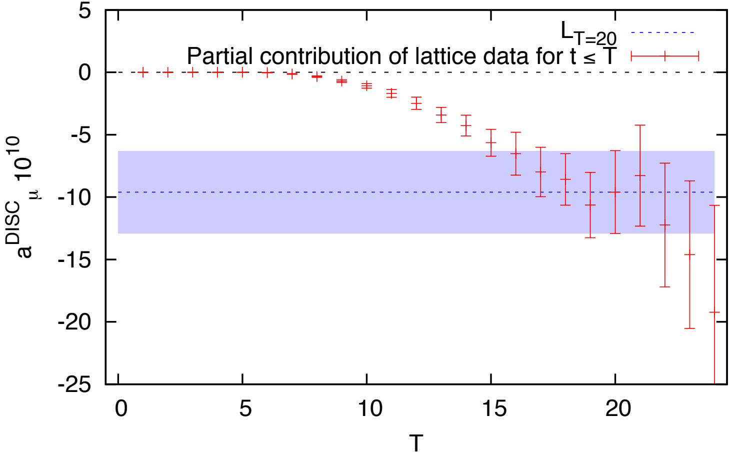


Since $C(t)$ is the autocorrelator of $\mathcal{V}_\mu$, we can create a stochastic estimator whose noise is potentially reduced linearly in the number of random samples, hence the normalization in the lower panel

Low-mode saturation for physical pion mass (here 2000 modes):

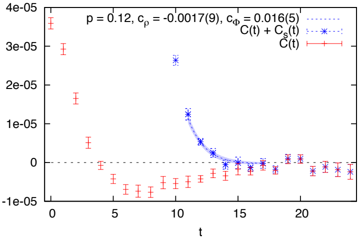


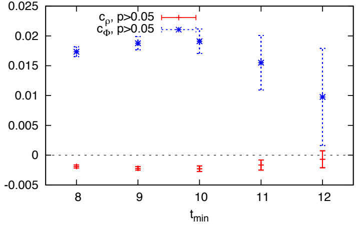
Result for partial sum $L_T = \sum_{t=0}^{T} w_t C(t)$:



For $t \geq 15$ $C(t)$ is consistent with zero but the stochastic noise is t-independent and $w_t \propto t^4$ such that it is difficult to identify a plateau region based only on this plot

Resulting correlators and fit of $C(t) + C_s(t)$ to $c_\rho e^{-E_\rho t} + c_\phi e^{-E_\phi t}$ in the region $t \in [t_{\min}, \ldots, 17]$ with fixed energies $E_\rho = 770$ MeV and $E_\phi = 1020$. $C_s(t)$ is the strange connected correlator.

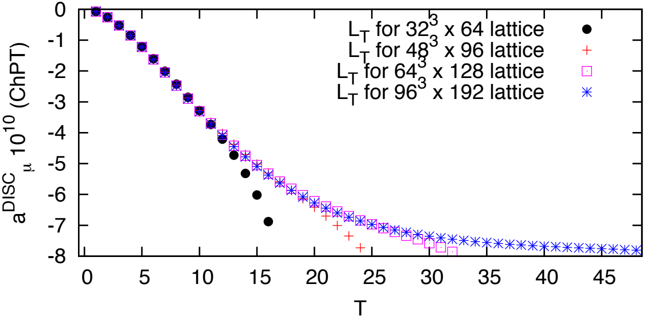




We fit to $C(t) + C_s(t)$ instead of $C(t)$ since the former has a spectral representation.

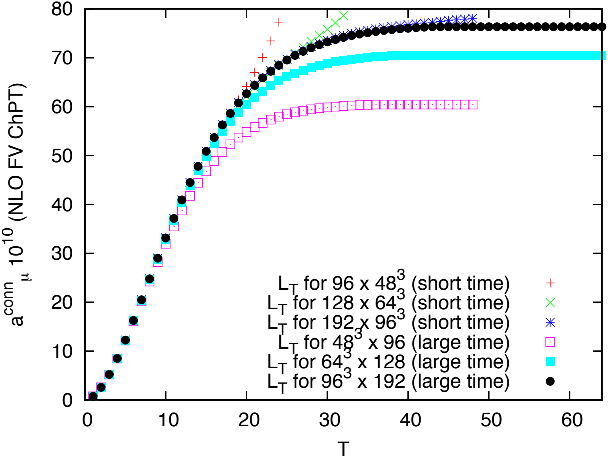
We could use this model alone for the long-distance tail to help identify a plateau but it would miss the two-pion tail

We therefore additionally calculate the two-pion tail for the disconnected diagram in ChPT:



A closer look at the NLO FV ChPT prediction (1-loop sQED):

We show the partial sum $\sum_{t=0}^{T} w_t C(t)$ for different geometries and volumes:



The dispersive approach to HVP LO

The dispersion relation

$$\Pi_{\mu\nu}(q) = i\left(q_\mu q_\nu - g_{\mu\nu} q^2\right)\Pi(q^2)$$
$$\Pi(q^2) = -\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}\frac{\mathrm{Im}\Pi(s)}{q^2-s} \, .$$

allows for the determination of a_μ^{HVP} from experimental data via

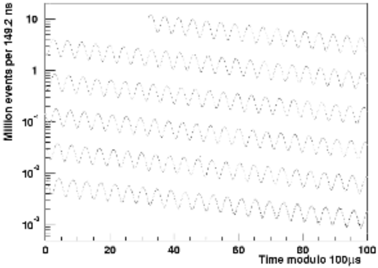
$$a_\mu^{\mathrm{HVP\ LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left[\int_{4m_\pi^2}^{E_0^2} ds \frac{R_\gamma^{\mathrm{exp}}(s)\hat{K}(s)}{s^2} + \int_{E_0^2}^{\infty} ds \frac{R_\gamma^{\mathrm{pQCD}}(s)\hat{K}(s)}{s^2}\right] ,$$
$$R_\gamma(s) = \sigma^{(0)}(e^+e^- \to \gamma^* \to \mathrm{hadrons})/\frac{4\pi\alpha^2}{3s}$$

Experimentally with or without additional hard photon (ISR:
$e^+e^- \to \gamma^*(\to \mathrm{hadrons})\gamma$)

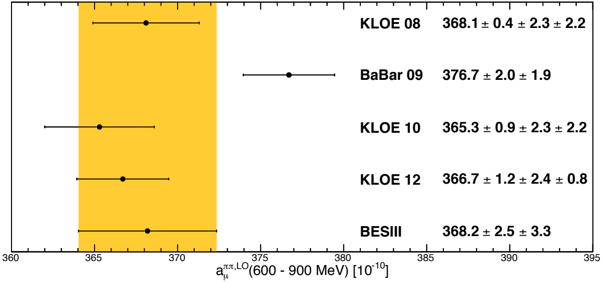
Experimental setup: muon storage ring with tuned momentum of muons to cancel leading coupling to electric field

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Because of parity violation in weak decay of muon, a correlation between muon spin and decay electron direction exists, which can be used to measure the anomalous precession frequency ω_a:



BESIII 2015 update:



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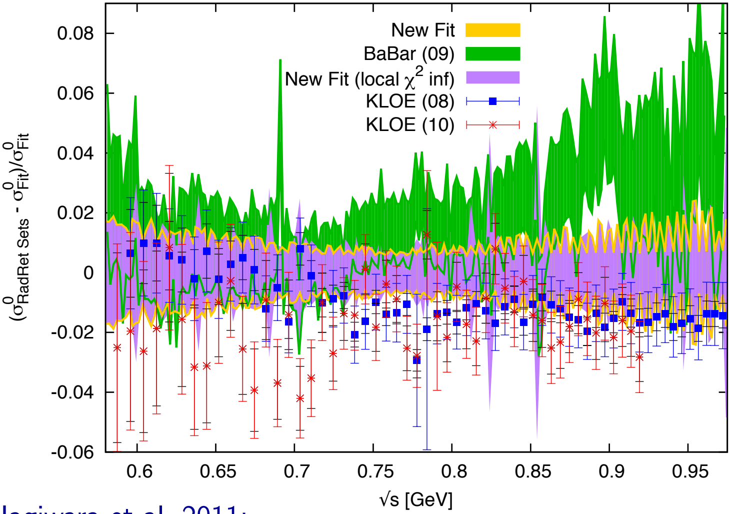


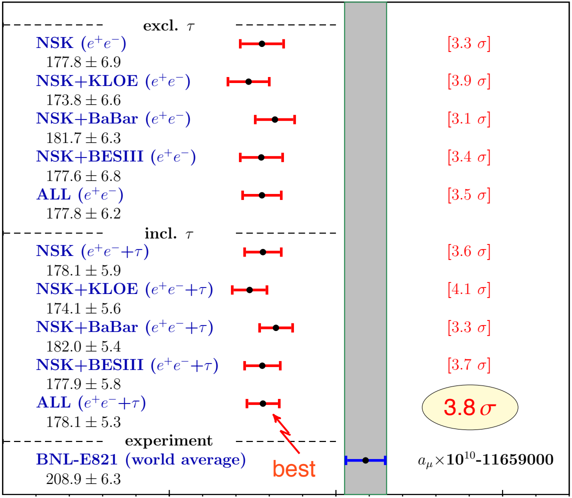
Hagiwara et al. 2011:

Jegerlehner FCCP2015 summary:

final state	range (GeV)	$a_\mu^{\mathrm{had}(1)} \times 10^{10}$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	507.55 (0.39) (2.68)[2.71]	0.5%	39.9%
ω	(0.42, 0.81)	35.23 (0.42) (0.95)[1.04]	3.0%	5.9%
ϕ	(1.00, 1.04)	34.31 (0.48) (0.79)[0.92]	2.7%	4.7%
J/ψ		8.94 (0.42) (0.41)[0.59]	6.6%	1.9%
Υ		0.11 (0.00) (0.01)[0.01]	6.8%	0.0%
had	(1.05, 2.00)	60.45 (0.21) (2.80)[2.80]	4.6%	42.9%
had	(2.00, 3.10)	21.63 (0.12) (0.92)[0.93]	4.3%	4.7%
had	(3.10, 3.60)	3.77 (0.03) (0.10)[0.10]	2.8%	0.1%
had	(3.60, 9.46)	13.77 (0.04) (0.01)[0.04]	0.3%	0.0%
had	(9.46,13.00)	1.28 (0.01) (0.07)[0.07]	5.4%	0.0%
pQCD	(13.0,∞)	1.53 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	687.06 (0.89) (4.19)[4.28]	0.6%	0.0%
total		688.59 (0.89) (4.19)[4.28]	0.6%	100.0%

Results for $a_\mu^{\mathrm{had}(1)} \times 10^{10}$. Update August 2015, incl SCAN[NSK]+ISR[KLOE10,KLOE12,BaBar,BESIII]

Jegerlehner FCCP2015 summary ($\tau \leftrightarrow e^+e^-$):



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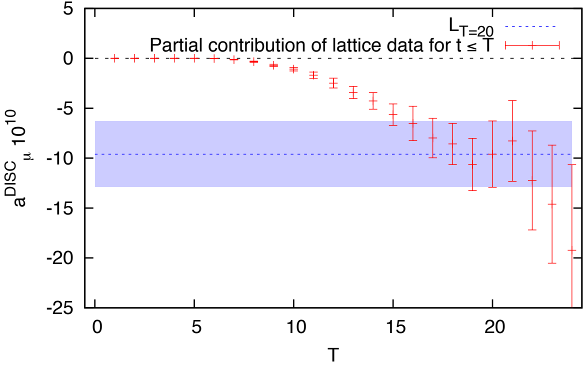
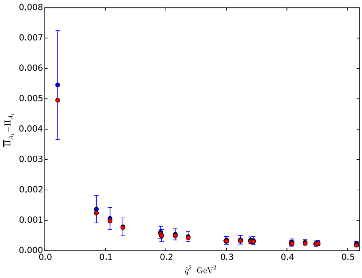
Study $L_T = \sum_{t=T+1}^{\infty} w_t C(t)$ and use value of T in plateau region (here $T = 20$) as central value. Use a combined estimate of a resonance model and the two-pion tail to estimate systematic uncertainty.

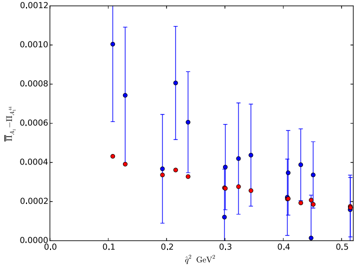


Combined with an estimate of discretization errors, we find

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10} \,. \qquad (5)$$

From Aubin et al. 2015 (arXiv:1512.07555v2)





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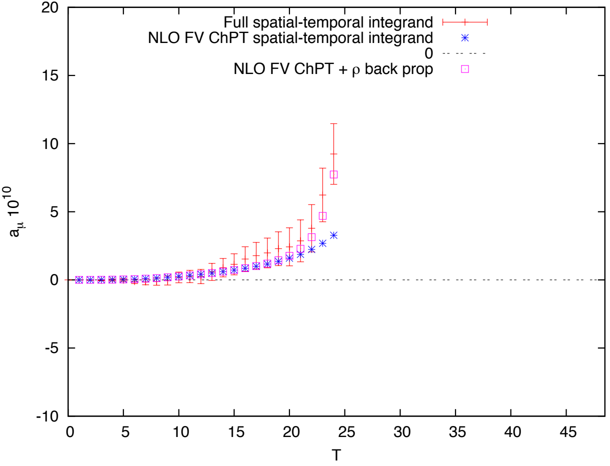
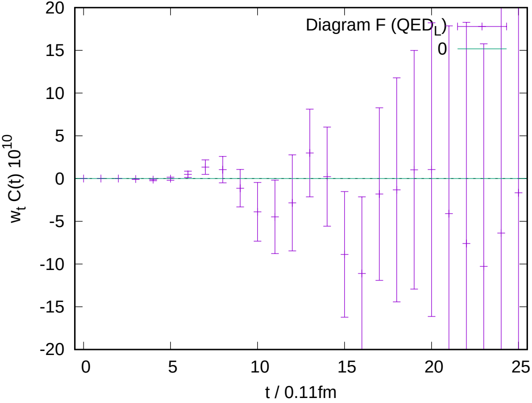
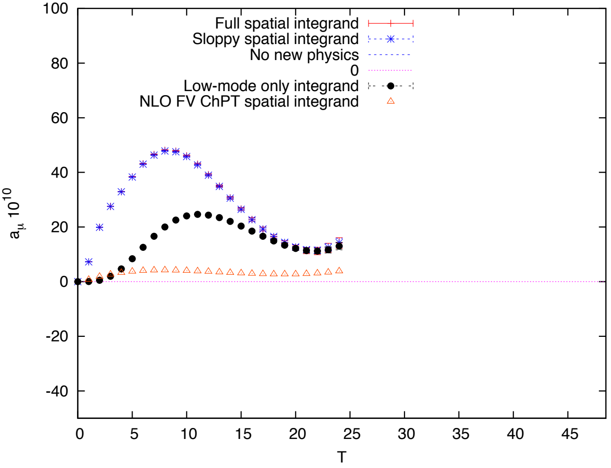


$m_\pi = 140$ MeV, $p^2 = m_\pi^2/(4\pi f_\pi)^2 \approx 0.7\%$

HVP QED+strong IB contributions

HVP QED diagram F





Full spatial integrand
Sloppy spatial integrand
No new physics
0
Low-mode only integrand
NLO FV ChPT spatial integrand
a_μ 10^10
T
100
80
60
40
20
0
-20
-40
0
5
10
15
20
25
30
35
40
45