$\pi\pi \to \pi\gamma^*$ radiative transition from lattice QCD

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radiative transition: $\pi\pi \to \pi\gamma$ or photoproduction: $\pi\gamma \to \pi\pi$

- experimentally clean (in experiment: photoproduction of a nuclei)
- but! mesons are easier and cleaner on the lattice
- great channel to learn the methodology
- (future benchmark for such studies)

- involves the QED current
- we can choose the quantum numbers of the $\pi\pi$ system ($J^{PC} = 1^{--}$)
- there is a resonance present: ρ almost fully elastic!

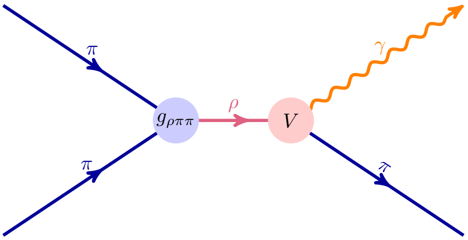
also: plenty of interesting phenomenological applications

previous studies: [R.M. Woloshyn Z.Phys. C33; Crisafulli & Lubicz PLB278; Owen et al. 1505.02876; HadSpec 1507.06622, 1604.03530]

Introduction
Outline

- about the $\pi\pi \to \pi\gamma^*$ process
- how to - on the lattice
- our lattice setup
- from 3-pt correlators to the infinite volume amplitude
- comparing to the Standard Approach (i.e. ρ assumed stable)

About $\pi\pi \to \pi\gamma^*$



$$\mathcal{A}(\pi\pi \to \pi\gamma) = \mathcal{A}(\pi\pi \to \rho) \ \times \ \mathcal{A}(\rho \to \pi\gamma)$$

$$\mathcal{A}(\rho \to \pi\pi) = \sqrt{\frac{8\pi s \Gamma(s)}{k}} \frac{1}{s - m_\rho^2 + i\sqrt{s}\Gamma(s)}$$

$$\mathcal{A}(\rho \to \pi\gamma)^{\nu,\mu} = \frac{iV(q^2, s)}{m_\rho} \varepsilon^{\mu\nu\alpha\beta} \epsilon^{*\nu} p_{\pi\pi}^{\alpha} p_{\pi}^{\beta}$$

On the $2 \to 1$ transition matrix elements in lattice

outline of the Procedure

- spectroscopy (the Lüscher method):
  - finite volume effects (L^n) lead to shifted energies in spectrum
  - quantization condition defines mapping between finite volume spectrum and infinite volume amplitude (review: [1706.06223])

$$\det\left(1 + i t_\ell(s)(1 + i\mathcal{M}^{\vec{p}_{\pi\pi}})\right) = 0$$

- transition matrix elements (the Briceño-Hansen-Walker-Loud formalism):
  - calculate the 3-pt functions in the and multi-hadron approach
  - use 2-pt GEVP info to construct the optimal 3-pt correlators
  - use δ_1 to determine the Lellouch-Lüscher factor

$$\sqrt{\frac{32\pi E_\pi \sqrt{s}}{k}\left[\frac{\partial \delta_1(\sqrt{s})}{\partial E_{\pi\pi}} - \frac{\partial \phi_1^{\vec{p}_{\pi\pi},\Lambda}(k)}{\partial E_{\pi\pi}}\right]}$$

  - map from finite-volume matrix elements to infinite-volume amplitudes [Lellouch & Lüscher hep-lat/0003023, Briceño et al. 1406.5965, Lin et al. hep-lat/0104006, Christ et al. hep-lat/0507009,Agadjanov et al. 1405.3476, ...]

Lattice setup

the C13 ensemble

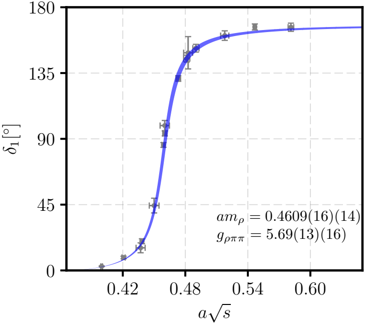
- (very!) preliminary results
- $N_f = 2 + 1$ Clover fermions
- isotropic lattice by Orginos et al.
- non-polynomial FV effects very small: $e^{-m_\pi L} \approx 0.3\%$

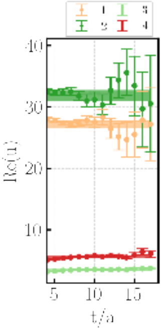
Label	$N_s^3 \times N_t$	a (fm)	L (fm)	m_π (MeV)	N_{config}	$N_{src}/config$
C13	$32^3 \times 96$	0.11403	3.65	317	1041	8

- Lüscher method: $I = 1$ P-wave scattering
- resonant Breit-Wigner:

$$\delta = \tan^{-1}\left(\frac{\sqrt{s}\Gamma(s)}{m_\rho^2 - s}\right)$$

$$\Gamma(s) = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k^3}{s}$$





Srijit P. @Monday: 1704.05439

3-pt correlators
operators

initial state: $I(J^P) = 1(1^-)$ multihadron operators

$$O_{\pi\pi}^{p,\Lambda\vec{p}_{\pi\pi}} \propto \sum_{R\in LG(\vec{p}_{\pi\pi})} \chi_\Lambda(R) \left(\pi^+(\frac{\vec{p}_{\pi\pi}}{2} + R\vec{p})\pi^0(\frac{\vec{p}_{\pi\pi}}{2} - R\vec{p}) - \pi^+ \leftrightarrow \pi^0\right),$$

$$O_{qq}^{p,\Lambda\vec{p}_{\pi\pi}} \propto \sum_{R\in LG(\vec{p}_{\pi\pi})} \chi_\Lambda(R) R O_{qq}(\vec{p}_{\pi\pi}). \quad \text{Where } \vec{p} = \frac{\vec{p}_{\pi\pi}}{2} + \frac{2\pi}{L}\vec{n}$$

final state: π:

$$O_\pi = \bar{u}\gamma_5 d(\vec{p}_\pi)$$

local current: QED

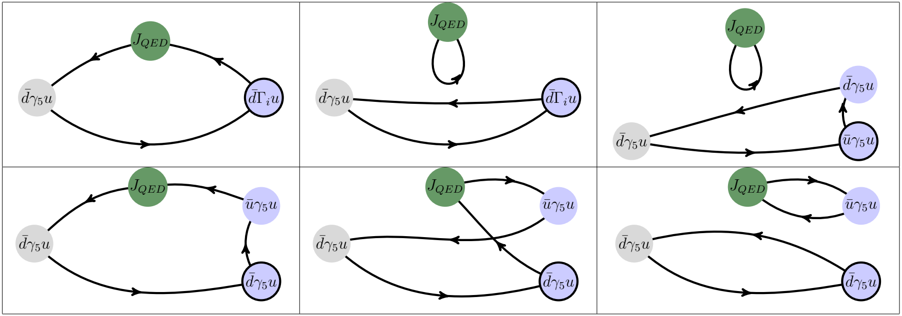
$$J^\mu = Z_V(\frac{2}{3}\bar{u}\gamma_\mu u(\vec{q}) - \frac{1}{3}\bar{d}\gamma_\mu d(\vec{q}))$$

$Z_V = 0.79700(24)$ [LHPC 1703.06703]

3-pt correlators
wick contractions

the 3-pt correlator:

$$C_3^{p,\mu}(t_{\pi\pi}, t_J, t_\pi; \Lambda\vec{p}_{\pi\pi}) = \langle O_\pi(t_\pi, \vec{p}_\pi) J^\mu(t_J, \vec{Q}) O_{\pi\pi}^{p,\Lambda\vec{p}_{\pi\pi}}(t_{\pi\pi})) \rangle$$



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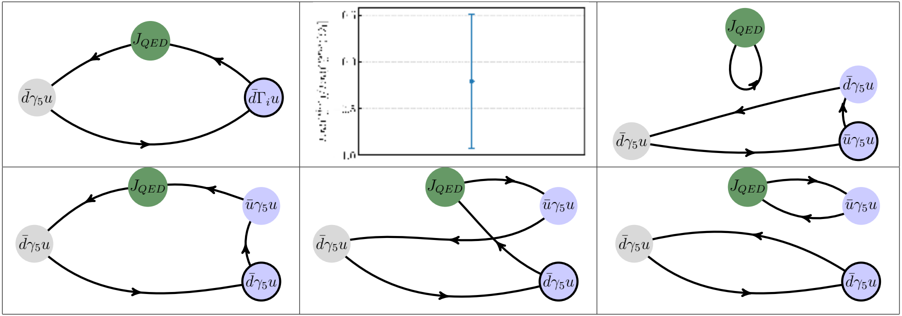


3-pt correlators
optimized three point function

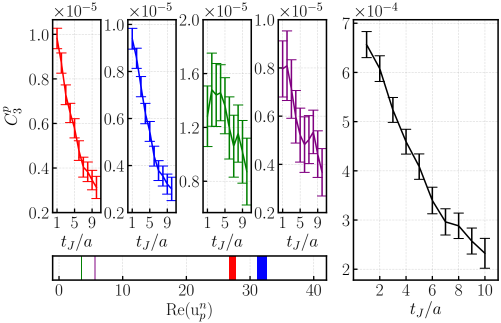
example: $\vec{p}_{\pi\pi} = 0$, T_1, n=1

- GEVP:

  $$C_{ij}(t)u_j^n = \lambda_n(t, t_0)C_{ij}(t_0)u_j^n$$

- optimized 3-pt correlator:

  $$C_3^{\mu}(t_{\pi\pi}, t_J, t_{\pi}; \Lambda\vec{p}_{\pi\pi}; n) = u_p^n C_3^{p,\mu}(t_{\pi\pi}, t_J, t_{\pi}; \Lambda\vec{p}_{\pi\pi})$$



  - allows us to project to the n-th state of the spectrum
  - reduces excited state contamination

[HadSpec 0902.2241, Becirevic 1411.6426, HadSpec 1501.07457+]

Matrix elements
the Ratio

3-pt function decomposition ($\Omega^n_{\pi\pi} = e^{-E_n t_0/2} u^n_p O^{p,\Lambda\vec{p}_{\pi\pi}}_{\pi\pi|qq}$):

$$C_3^{\mu}(t_{\pi\pi}, t_J, t_\pi; \Lambda\vec{p}_{\pi\pi}; n)) =$$
$$\langle 0|\Omega^n_{\pi\pi}|n, \Lambda\vec{p}_{\pi\pi}\rangle\langle n, \Lambda\vec{p}_{\pi\pi}|J^\mu(\vec{q})|\pi, \vec{p}_\pi\rangle\langle\pi|O_\pi|0\rangle \frac{e^{-E_n(t_n-t_J)}e^{-E_\pi(t_J-t_\pi)}}{2E_\pi E_n}$$

matrix elements obtained from ratio:

$$R_\mu(t_J; \Lambda\vec{p}_{\pi\pi}; n)) = \frac{C_3(t_J, \Delta t)C_3^*(\Delta t - t_J, \Delta t)}{C_2^{(n)}(\Delta t)C_2^{(\pi)}(\Delta t)} \rightarrow$$
$$\rightarrow \; |\langle\Lambda\vec{p}_{\pi\pi}; n|J^\mu, \vec{q}|\pi, \vec{p}_\pi\rangle|^2$$

- $|\langle\Lambda\vec{p}_{\pi\pi}; n|J^\mu, \vec{q}|\pi, \vec{p}_\pi\rangle|$ affected by finite volume effects!
- Briceno-Hansen-Walker-Loud formalism to map from FV to IV

Matrix elements
mapping from FV to IV

- LL factor maps the amplitude at different $\sqrt{s}$ differently
- maps from FV matrix element to IV amplitude (for full process)
- this correction is different for each $\pi\pi$ system momentum (and polarization)
- form factor is the residue:

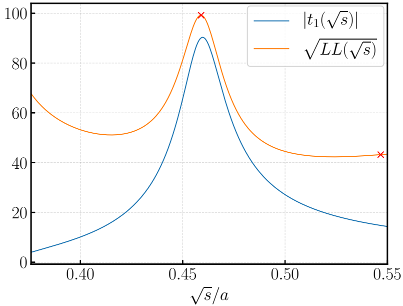
$$\frac{1}{Res^2} = \frac{\partial}{\partial s}\frac{1}{A(s,q^2)}\Big|_{s=m_\rho^2}$$

(not a nice definition)

  - use product defintion instead :

$$\mathcal{A}(\pi\pi\to\pi\gamma) = \mathcal{A}(\pi\pi\to\rho)\ \times\ \mathcal{A}(\rho\to\pi\gamma)$$

$\vec{p}_{\pi\pi} = 0,\ \vec{p}_{\pi} = \frac{2\pi}{L}(0,0,-1)$

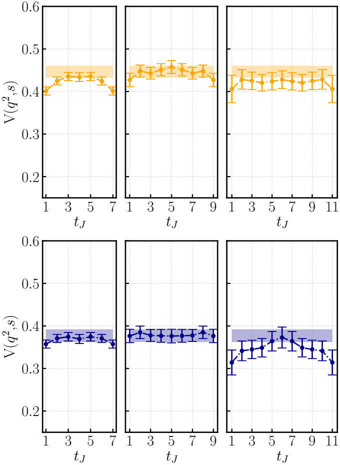


e.g. experiment [Belle 1306.2781]

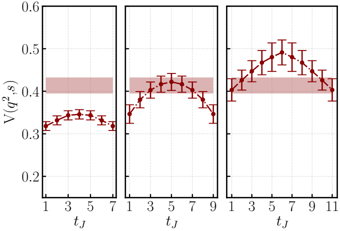
Matrix elements

how do they look?

ground state :)



excited state :/



- could be better?
- vary u_p^n fit ranges

determining the resonant form factor

what do we fit?

z-expansion for q^2 dependance [Bourley et al. 0807.2722]:

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_0 = 0 \quad t_+ = 3m_\pi^2$$

then q^2 of amplitude:

$$\bar{V}(q^2) = \frac{\sum_{n=0}^{N} a_n z^n}{1 - q^2/m_P^2}$$

(for now) limit $N = 1$ and $m_P = m_\omega \approx m_\rho$

Breit-Wigner amplitude in s dependance:

$$t_1(s) = \sqrt{\frac{8\pi s \Gamma(s)}{k}} \frac{1}{m_\rho^2 - s - i\sqrt{s}\Gamma(s)}$$

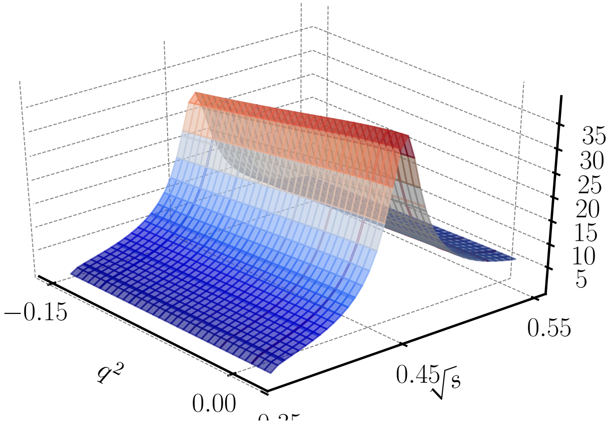
(for now) only P-wave $\Gamma(s)$:

$$\Gamma(s) = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k^3}{s}$$

join into:

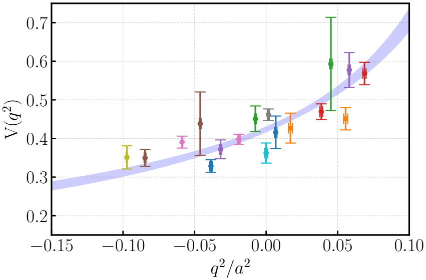
$$V(q^2, s) = \bar{V}(q^2) \times t_1(s) + O(?)$$

the infinite volume amplitude

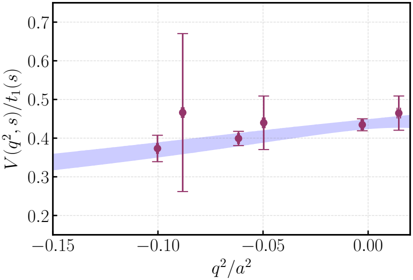


comparing to the Standard Approach
(when ρ is assumed to be stable)

ρ assumed stable



multi-hadron



- multi hadron has seems visually more pleasing, but more points need to be added for realistic comparison

Conclusion

- still plenty to do
- intimate understanding of the data
- investigate systematics:
  - excited state contamination
  - FV $\rightarrow$ IV mapping functional form dependance
- phenomenology
- once confident apply to heavy mesons

Thank you :)