

The η' meson at the physical point with $N_f = 2$ Wilson twisted mass fermions

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Part of the Sino-German CRC 110

European Twisted Mass Collaboration

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Motivation

- $U(1)$ axial anomaly thought to be responsible for large η' mass

$$\partial^\mu J_{\mu 5}^q = 2m_q(\bar{q}i\gamma_5 q) + \frac{\alpha_s}{4\pi} F\tilde{F}$$

- tightly connected to topology
 - results at physical pion mass missing so far
 - how do errors scale towards the physical point?
 - $N_f = 2$ technically easier: no mixing of η - η'
 - test bed for e.g. $\eta' \rightarrow \gamma^* \gamma^*$
- ⇒ relevant for HLbL contribution to $g - 2$ of the muon

Status η and η' Mesons

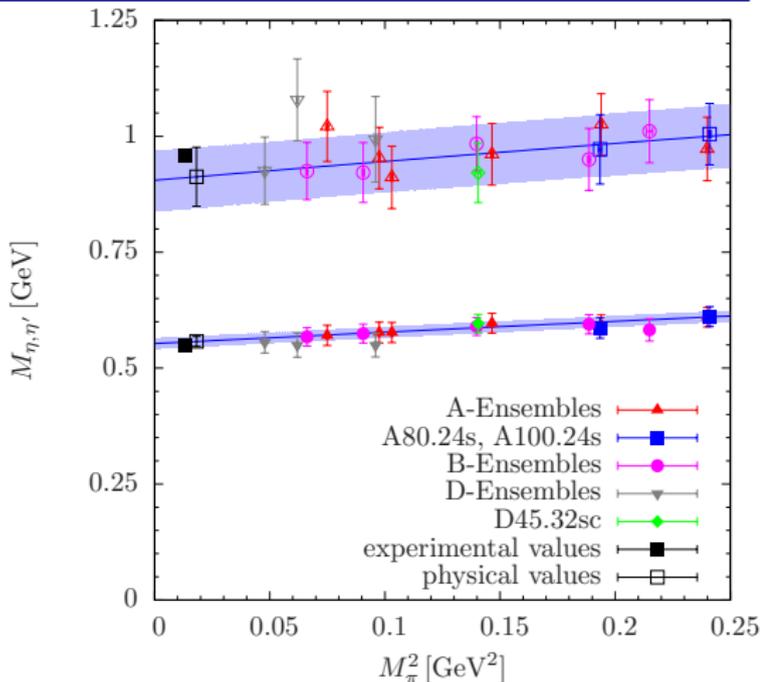
- $N_f = 2 + 1 + 1$ computation with three values of the lattice spacing

[Michael, Ottnad, CU, PRL (2013)]

- here: updated results

[Ottnad, Urbach, to be published]

- extrapolations including lattice artefacts and quark mass effects (ChPT reliable?)



- long extrapolation from $M_{\pi} \sim 250$ MeV
- decay constants available as well

[Ottnad, Michael, CU, (LATTICE2013); Ottnad, CU, to be published]

- Iwasaki gauge action at $\beta = 2.10$
- $N_f = 2$ Wilson twisted-mass clover ensembles
[ETMC, LAT15 (2015), ETMC PRD (2017)]
- maximal twist guarantees $\mathcal{O}(a)$ improvement
[Frezzotti, Rossi, (2004)]
- clover term introduced to reduce pion mass splitting
- single lattice spacing $a \sim 0.9$ fm, $r_0/a = 5.317$
- three pion mass values from 130 MeV to 350 MeV
- two volumes per pion mass value from $L/a = 24$ to $L/a = 64$
- stochastic Laplacian Heaviside for correlators
[HSC, Peardon et al., PRD (2009), Morningstar et al., PRD (2011)]
- blocked bootstrap statistics

Pion Mass Splitting

- previously: large pion mass splitting in tmLQCD
- simulations @ M_π^{phys} only at tiny lattice spacing possible

- added clover term to action
- clover coefficient set to its non-perturbative value

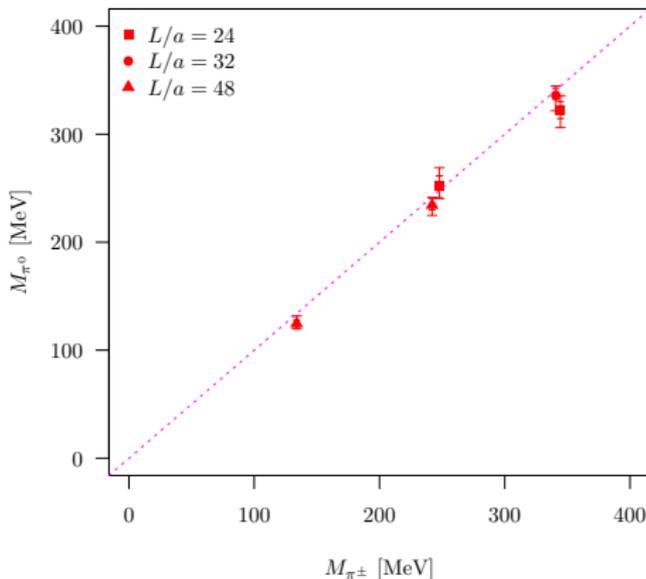
$$c_{\text{SW}} = 1.57551$$

[CP-PACS, JLQCD, (2006)]

⇒ mass splitting zero within errors

- c_{SW} -value also compatible with boosted PT

$$c_{\text{SW}} = 1 + \frac{g_0^2}{2\langle P \rangle}$$



The η' in $N_f = 2$ flavour QCD

- $N_f = 2$ light pseudo scalar mesons: a pion triplet and the η_2
- pions: (pseudo) GOLDSTONE bosons
- η_2 : related to the $U(1)$ axial anomaly, i.e. the η' for $N_f > 2$
- correlation function

$$C_{\eta_2}(t-t') = \langle \mathcal{O}^s(t) (\mathcal{O}^s(t'))^\dagger \rangle, \quad \mathcal{O}^s(t) = \frac{1}{\sqrt{2}} \sum_{\mathbf{x}} \bar{\psi} i\gamma_5 \mathbb{1}_f \psi(\mathbf{x}, t).$$

⇒ fermionic connected and disconnected contributions

- delicate cancellation and difficult statistical analysis

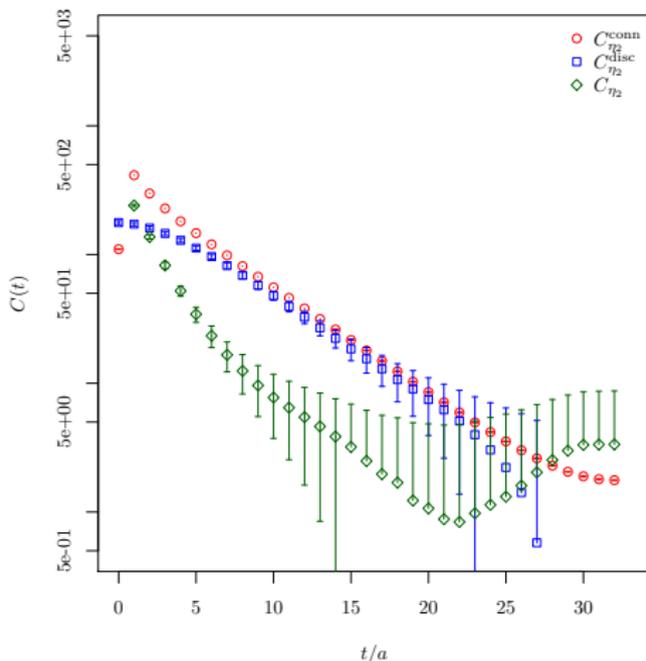
$$C_{\eta_2} = C_{\eta_2}^{\text{conn}} - C_{\eta_2}^{\text{disc}}$$

The η' in $N_f = 2$ flavour QCD

- delicate cancellation
- $C(t) \geq 0$ required by theory

$$\Rightarrow C_{\eta_2}^{\text{disc}} \leq C_{\eta_2}^{\text{conn}}$$

- signal lost early in time
- $M_\pi \approx 340$ MeV
- $L/a = 32$



Correlation Functions

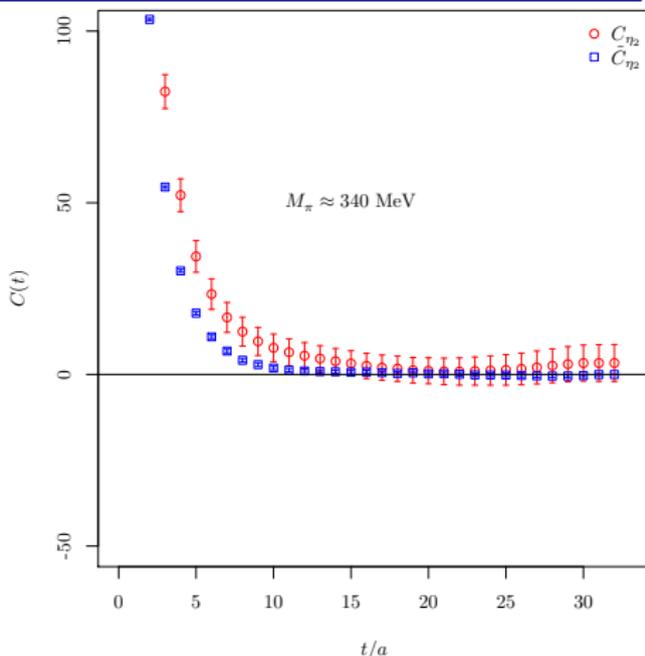
- disconnected contribution introduces large correlation

- increasing with $M_\pi \rightarrow 0$

- use instead

$$\tilde{C}_{\eta_2}(t) = C_{\eta_2}(t) - C_{\eta_2}(t+1)$$

- drastically reduces correlation



- also removes possible constant FSE [Aoki et al., PRD (2014), Bali et al., PRD (2014)]

$$\propto \frac{a^5}{T} \left(\chi t + \frac{Q^2}{V} \right)$$

Correlation Functions

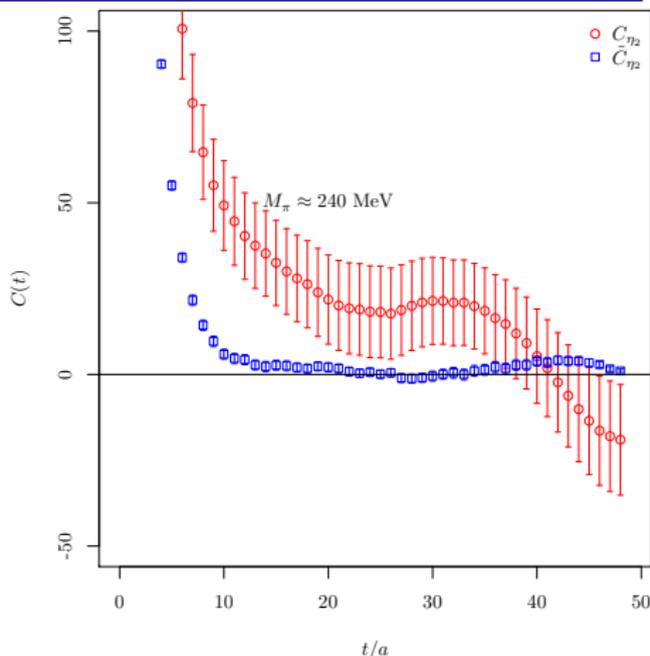
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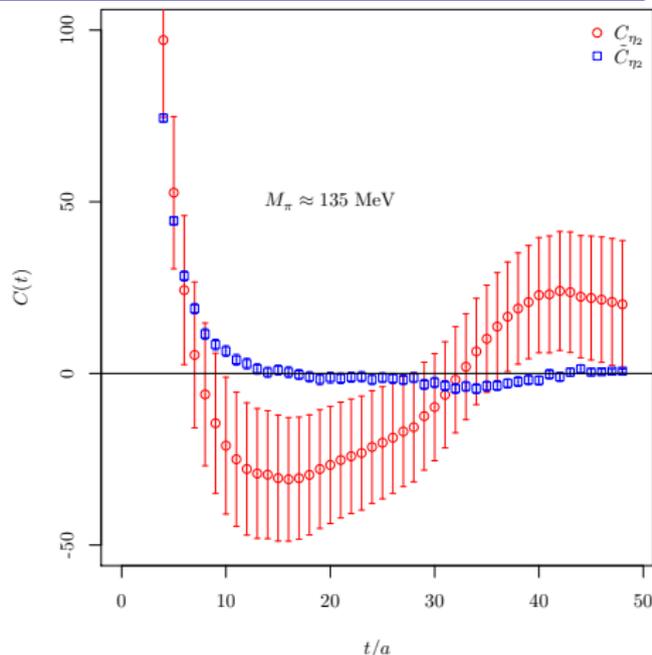
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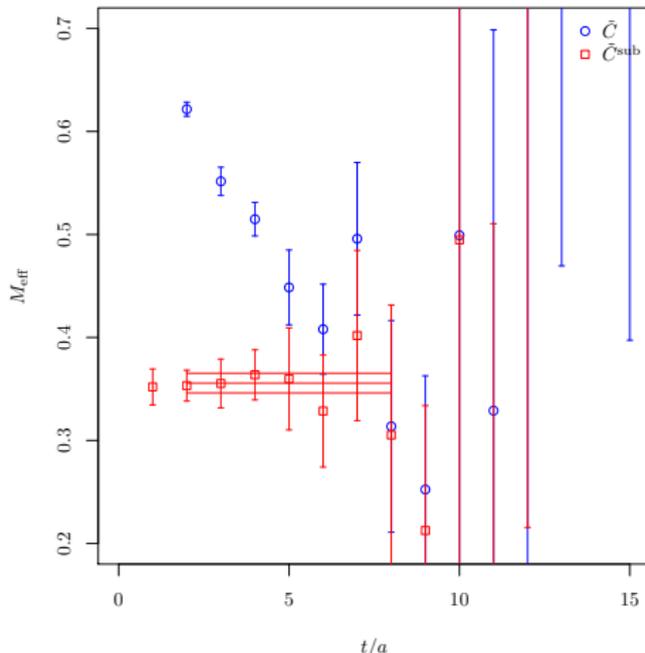
Excited State Subtraction

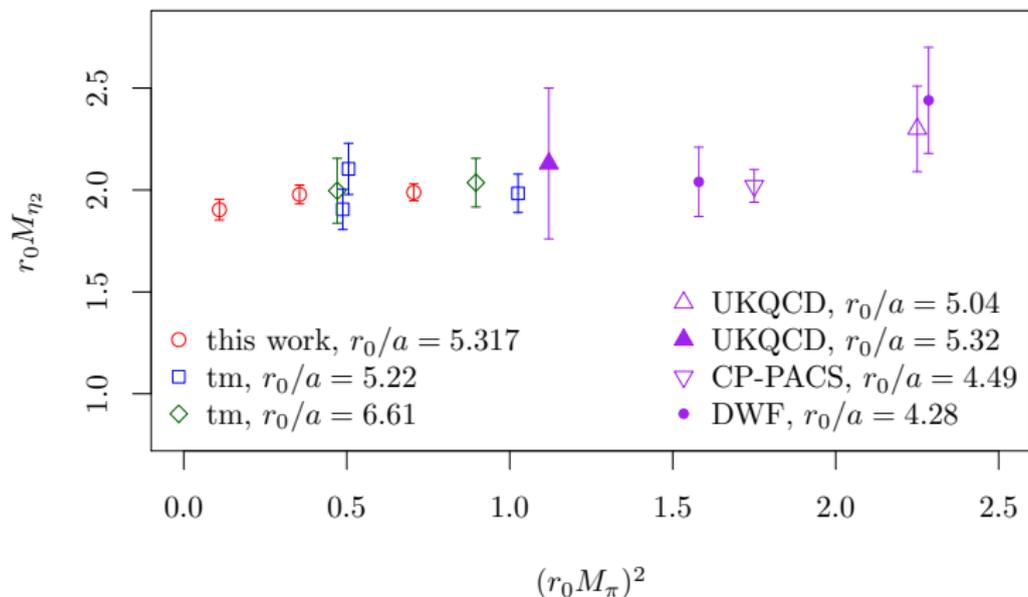
- signal lost early in time
- assume disconnected contribute only to ground state
- subtract excited states from connected part

[Neff et al. (2001), K.Jansen, C.Michael, C.U. (2008)]

- analyse subtracted connected plus disconnected correlation function \tilde{C}^{sub}

⇒ plateau very early in time!





- with $r_0 = 0.4907(5)$ fm we arrive at

$$M_{\eta_2}^{N_f=2} = 768(24) \text{ MeV}$$

Results Topological Susceptibility

- finite η_2 mass

⇒ χ_t should vanish in chiral limit

- but: lattice artefacts

$$t_1^2 \chi_t = c_2 t_1 M_\pi^2 + a^2 c_2 / t_1 + \dots$$

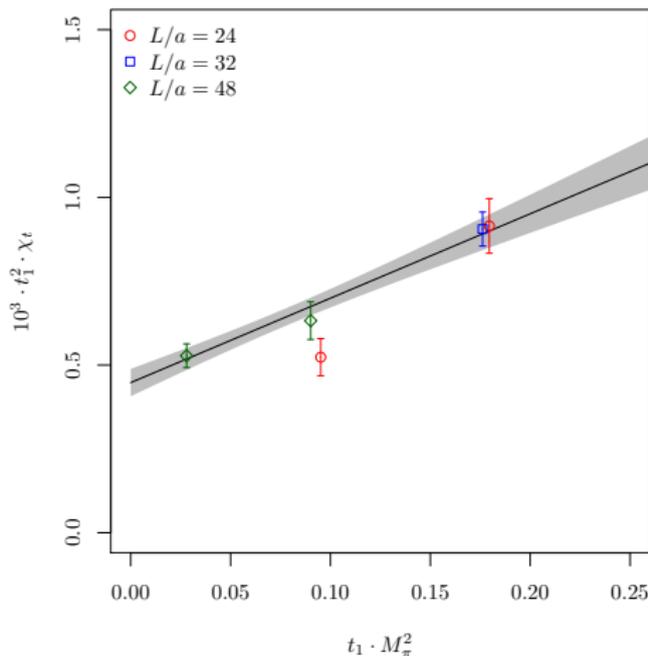
[ALPHA, JHEP (2014)]

- determine χ_t from gradient flow at flowtime t_1

[Lüscher, JHEP (2010)]

⇒ fits with expected LO ChPT prediction

- largish lattice artefact



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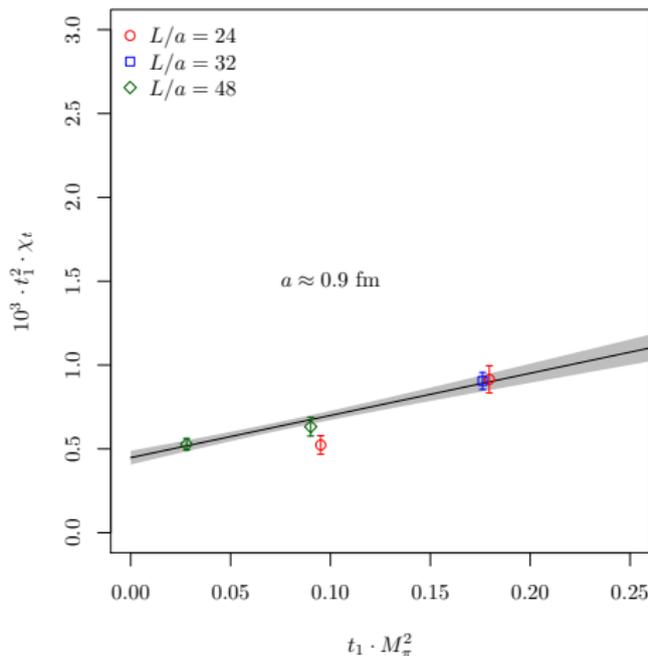
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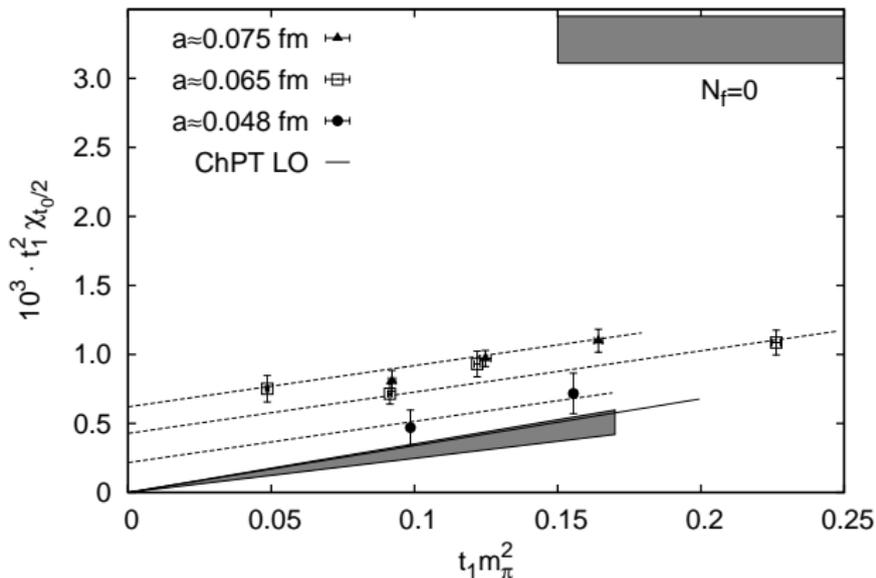
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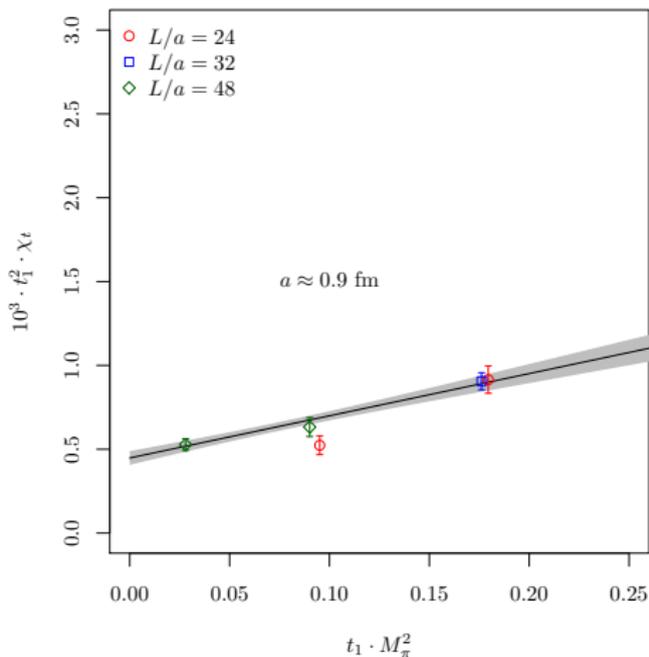
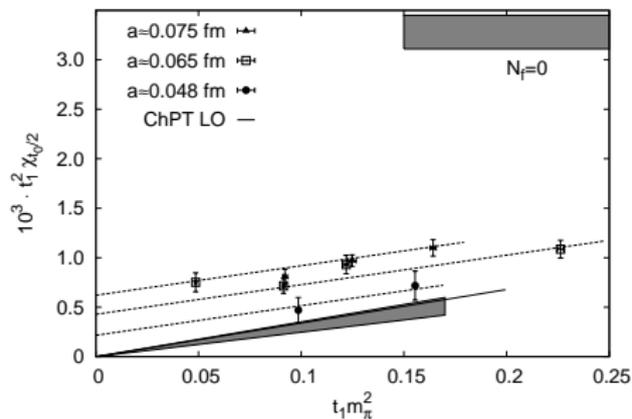


Comparison $N_f = 2$ Topological Susceptibility



- but not unseen large [ALPHA, JHEP (2014)]

Comparison $N_f = 2$ Topological Susceptibility



[ALPHA, JHEP (2014)]

- note the lattice spacing values!

Summary

- investigated $N_f = 2$ η' meson @ M_π^{phys}
- previous extrapolations are confirmed
- use of shifted correlation functions important
- determined topological susceptibility @ M_π^{phys}
- expected pion mass scaling plus $\mathcal{O}(a^2)$

Thanks to ...

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