Proton Mass Decomposition

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Introduction

The Higgs boson is believed to be the source for the quark masses. But how it is related to the proton mass and the mass of the observable matters is another question. The masses of the valence quarks in the proton are just ∼3 MeV which are due to the coupling to the Higgs boson, while the total proton mass is 938MeV. How large the quark and gluon contributions are to the proton mass is a question that can only be answered by solving QCD nonperturbatively, and with information from experiments. With phenomenological inputs, the first decomposition has been carried out by Ji [1]. As in Ref. [1, 2], the Hamiltonian of QCD can be decomposed as

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle = \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \quad (1)$$

$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle, \quad (2)$$

where H_E, H_m, H_g denote the contributions from the quark energy, the quark mass, and the glue field energy, respectively:

$$H_E = \sum_{u,d,s...}\int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma}), \quad H_m = \sum_{u,d,s\cdots}\int d^3x\, m\,\overline{\psi}\psi, \quad H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2), \quad (3)$$

and the QCD anomaly term H^a is the joint contribution from the quantum anomaly of both quark and glue,

$$H_a = H_g^a + H_m^\gamma, \quad H_g^a = \int d^3x\, \frac{-\beta(g)}{4g}(E^2 + B^2), \quad H_m^\gamma = \sum_{u,d,s\cdots}\int d^3x\, \frac{1}{4}\gamma_m m\,\overline{\psi}\psi. \quad (4)$$

All the $\langle H\rangle$ are defined by $\langle P|H|P\rangle/\langle P|P\rangle$. Note that the sum of the quark and glue energy, and of the quark mass and anomaly terms are scale and renormalization scheme independent, but the separate quark and glue energy terms are scale dependent.

Theoretical Framework

Due to the breaking of the energy-momentum conservation and the continuum equation of motion with lowest dimensional operators on the lattice, all the 4 components $\langle H_E\rangle$, $\langle H_m\rangle$, $\langle H_g\rangle$ and $\langle H_a\rangle$ can mix with each other which results in a 4-by-4 mixing and renormalization matrix to deal with. But with the strategy below, we can avoid the direct calculation of this 4-by-4 mixing matrix.

Since the quark mass term has been calculated before [3] and the nucleon mass is calculated from the nucleon two-point function, we can obtain $\langle H_a\rangle$ from Eq. (2). For $\langle H_E\rangle$ and $\langle H_g\rangle$, it is known that they are related (in the continuum) to the quark and glue momentum fractions $\langle x\rangle_{q,g}$ as

$$\langle H_E\rangle = \frac{3}{4}\langle x\rangle_q M - \frac{3}{4}\langle H_m\rangle, \; \langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M, \quad (5)$$

and the momentum fractions mix with each other. So we can calculate the momentum fractions on the lattice, renormalize them with the mixing effects, convert them to that under $\overline{MS}$, push to the continuum limit, and then we can use the continuum equation Eq. (5) to obtain the renormalized values of $\langle H_E\rangle$ and $\langle H_g\rangle$.

Simulation Setup

We use valence overlap fermion on $(2+1)$ flavor RBC/UKQCD DWF gauge configurations on four ensembles [4]. The parameters of the ensembles used are:

Symbol	$L^3\times T$	a (fm)	$m_s^{(s)}$	m_π	N_{cfg}
32ID	$32^3\times 64$	0.01431(7)	89.4	171	200
24I	$24^3\times 64$	0.1105(3)	120	330	203
32I	$32^3\times 64$	0.0828(3)	110	300	309
48I	$48^3\times 96$	0.1141(2)	94.9	139	81

We use the grid source for the proton 2pt and 3pt functions to improve the signal. In the disconnected insertion cases, all the time slides are looped over for the nucleon propagator and 5 steps of HYP smearing are applied on the glue operator to improve the signal.

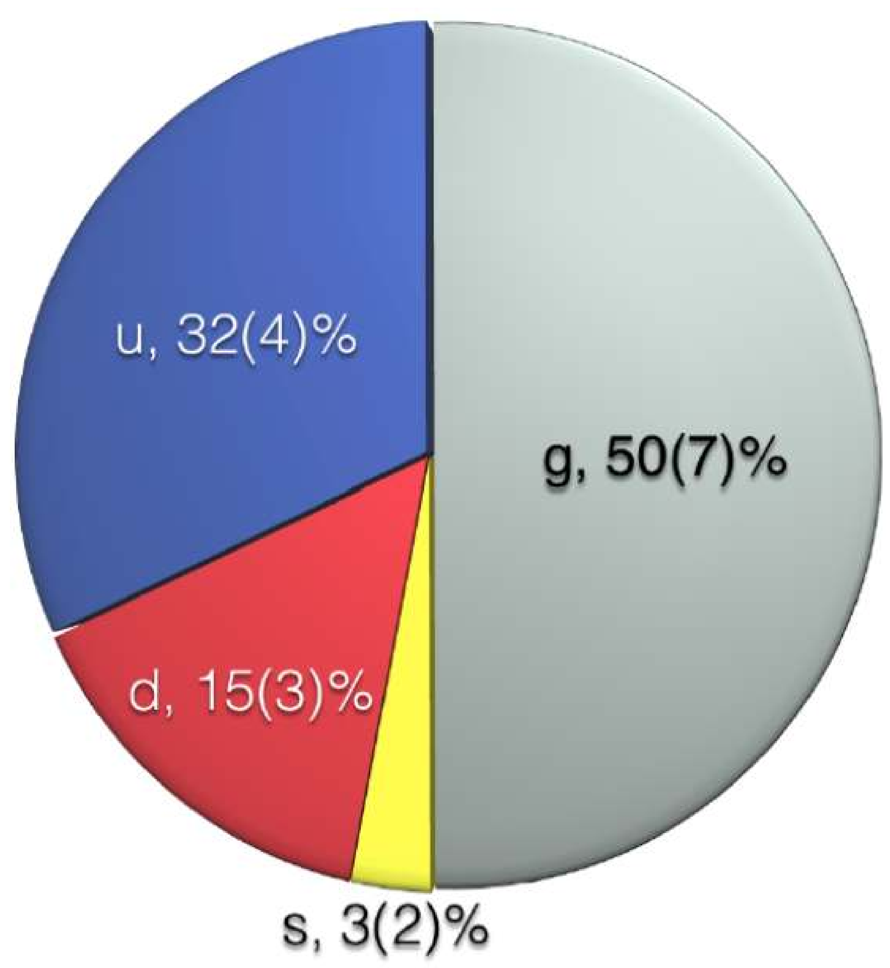
Results and Summary

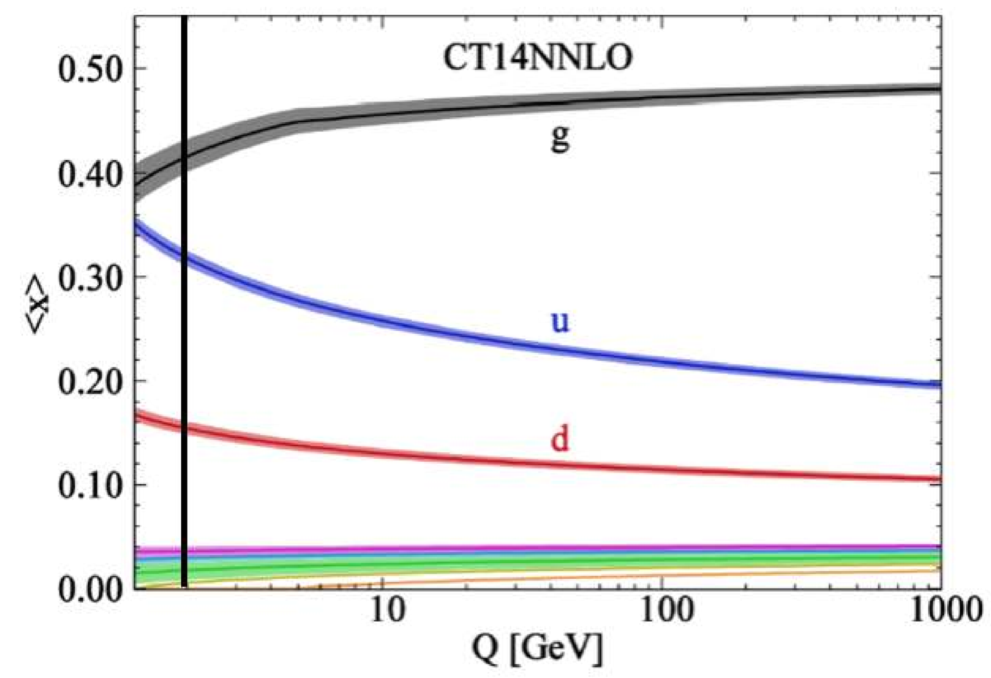
The numerical results of the mixing matrix that match the lattice bare quantities to those under $\overline{\rm MS}$ at $\mu = 1/a$ with $g^2 = 3$ (or β=2.0 equivalently) for the case of the chiral fermion D_c energy-momentum tensor (EMT) operators with 1-step HYP smeared Iwasaki gauge link and 5-steps HYP smeared gluon EMT operator are [5]

$$\begin{pmatrix} \overline{T}_Q^{\overline{\rm MS}} \\ \overline{T}_G^{\overline{\rm MS}} \end{pmatrix} = \begin{pmatrix} 1.0175 & -0.0207 \\ 0.1528 & Z_{gg} \end{pmatrix} \begin{pmatrix} \overline{T}_Q^{lat} \\ \overline{T}_G^{lat} \end{pmatrix} + O(g^4), \quad (6)$$

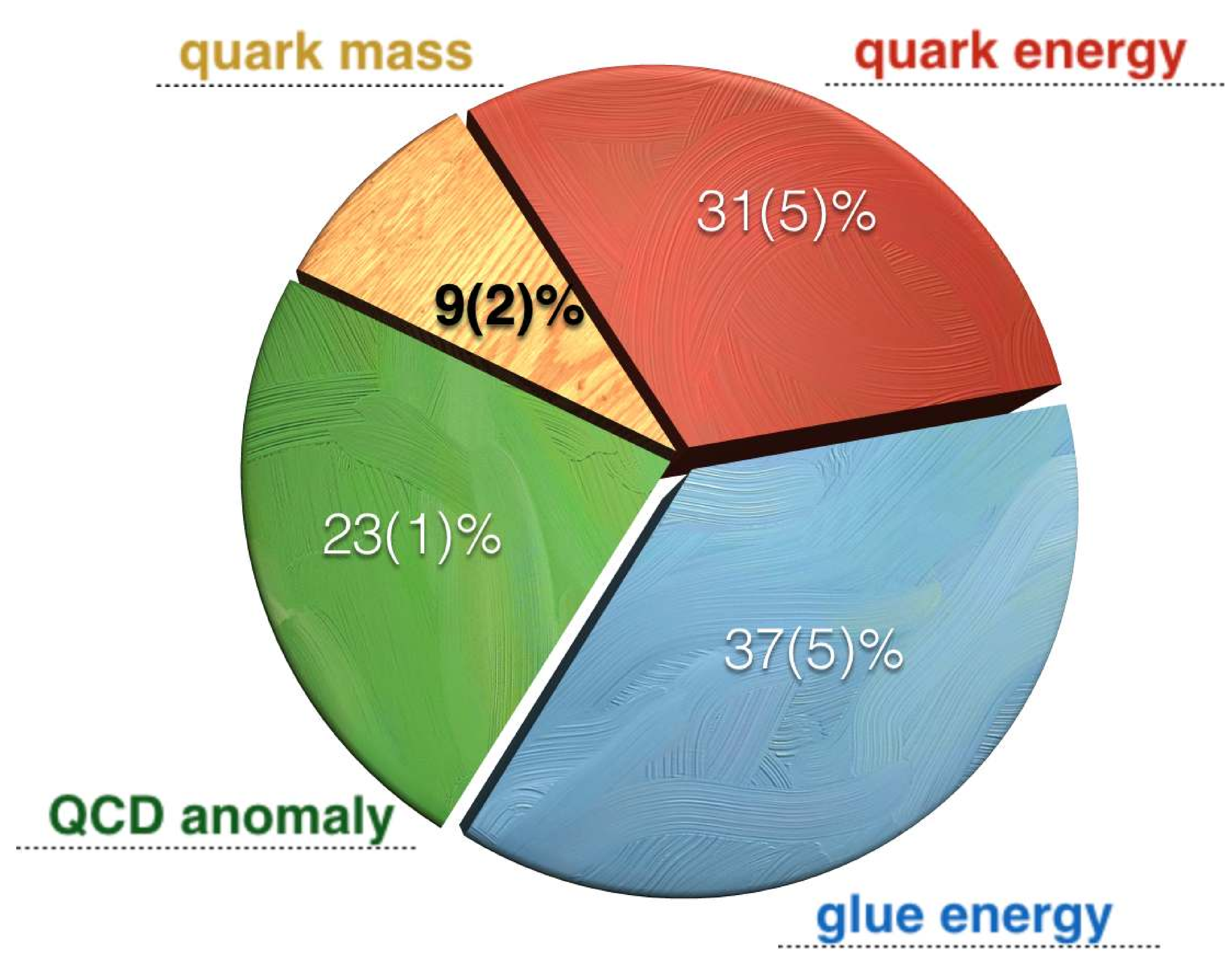
where Z_{gg} is to be determined from the sum rule of the momentum fractions. The other values in the mixing matrix are based on the 1-loop perturbative calculation [5].

Based on this mixing matrix, we get the renormalized quark and glue momentum fractions at $\overline{\rm MS}$ 2GeV and the results are illustrated in the upper panel of the following figure. We see that they are consistent with those obtained from the integration of PDF from the CT14NNLO global fitting [6] at $Q = 2$ GeV which are shown in the lower panel.





Combined with the quark mass term obtained in [3], the proton mass decomposition can be illustrated in the following chart:



Acknowledgements

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References

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