

The $B_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ Decay with Highly Improved Staggered Quarks & Improved NRQCD

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Lattice 2017, Granada

Outline

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② Calculation Setup

- Correlation Functions
- Continuum Current
- Form Factors

③ Preliminary Results

- $B_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ Form Factors close to q_{\max}^2
- Subleading Currents

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Motivation

- ▶ $B_{(s)} \rightarrow D_{(s)} l \bar{\nu}$ contains a $b \rightarrow c$ transition \implies can be used to determine $|V_{cb}|$.
- ▶ \exists many related tensions between prediction & experiment, including in

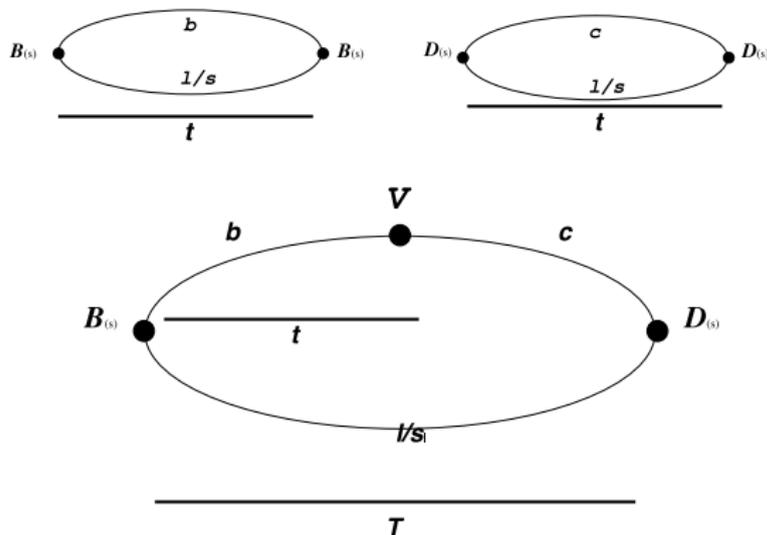
$$R(D) = \frac{\mathcal{B}(B \rightarrow D \tau \bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow D l \bar{\nu}_l)}$$

Calculation Setup

- ▶ Goal: Deduce $B_{(s)} \rightarrow D_{(s)}$ form factors $f_0(q^2), f_+(q^2)$ for $0 < q^2 < q_{\max}^2$. ($q^2 \equiv (p_{B_{(s)}} - p_{D_{(s)}})^2$)
- ▶ We use the Darwin cluster @ Cambridge, part of STFC's DiRAC II facility.
- ▶ 2nd Generation MILC Gluon Ensembles [1212.4768]:
 - ▶ Lüscher-Weisz Gauge action, improved up to $\mathcal{O}(N_f \alpha_s a^2)$.
 - ▶ 2+1+1 flavours in the sea, using HISQ action (Highly Improved Staggered Quark).
 - ▶ Many Lattice Spacings and quark masses, down to the physical point.

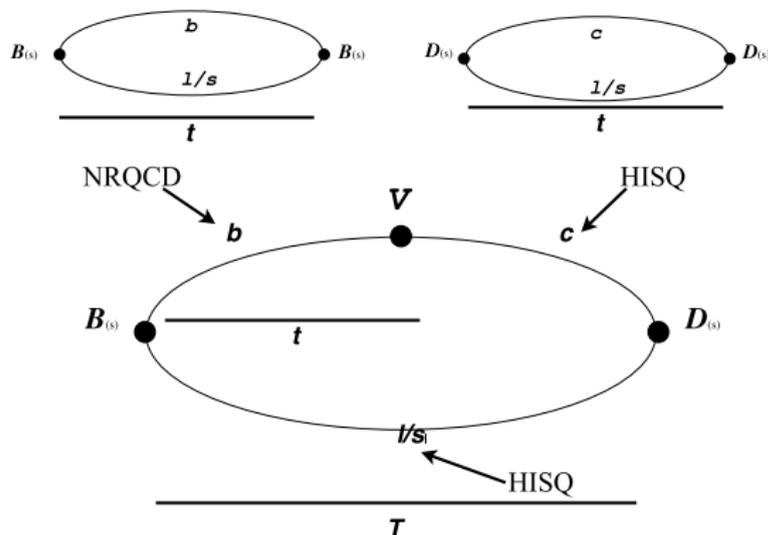
Correlation Functions

- ▶ Calculate 2 and 3-point correlation functions.
- ▶ Use exponential smearing on both $B_{(s)}$ and $D_{(s)}$ operators.
- ▶ Extract $\langle D_{(s)} | V_{\mu} | B_{(s)} \rangle$ from multiexponential fit using Bayesian constraints.



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Improved Non-Relativistic QCD (NRQCD)

- ▶ Lattice artifacts grow as $(am_b)^n, n > 0$, problem when $am_b > 1$.
Solution: non-relativistic formalism for b .
- ▶ NRQCD = action expanded in b velocity v , rest mass removed.

$$G(\underline{x}, t + 1) = e^{-a(H_0 + \delta H)} G(\underline{x}, t)$$

$$aH_0 = -\frac{\nabla^{(2)}}{2am_b},$$

$$a\delta H = -c_1 \frac{(\nabla^{(2)})^2}{8(am_b)^3} - c_4 \frac{1}{2am_b} \underline{\sigma} \cdot \underline{\tilde{B}} + \dots$$

- ▶ $\{c_i\}$ determined from perturbative matching to full continuum QCD. [1408.5768]
- ▶ Improved action: accurate through $\mathcal{O}(\alpha_s v^4)$.

Continuum Vector Current

- ▶ Continuum Vector current is expanded in terms of NRQCD-HISQ currents:

$$V_\mu = (1 + z_\mu \alpha_s) [V_\mu^{(0)} + V_\mu^{(1)}] + \dots$$
$$V_\mu^{(0)} = \bar{c} \gamma_\mu b \quad , \quad V_\mu^{(1)} = -\frac{1}{2m_b} \bar{c} \gamma_\mu \underline{\nabla} \cdot \underline{\nabla} b$$

- ▶ z_μ calculated by perturbative matching to full continuum QCD [1211.6966]
- ▶ Also \exists at $\mathcal{O}(\alpha_s, v)$:

$$V_\mu^{(2)} = -\frac{1}{2m_b} \bar{c} \gamma_\mu \underline{\nabla} \cdot \overleftarrow{\underline{\nabla}} b$$
$$V_k^{(3)} = -\frac{1}{2m_b} \bar{c} \underline{\nabla}_{-\mu} b \quad , \quad V_k^{(4)} = -\frac{1}{2m_b} \bar{c} \overleftarrow{\underline{\nabla}}_{-\mu} b$$

Form Factors

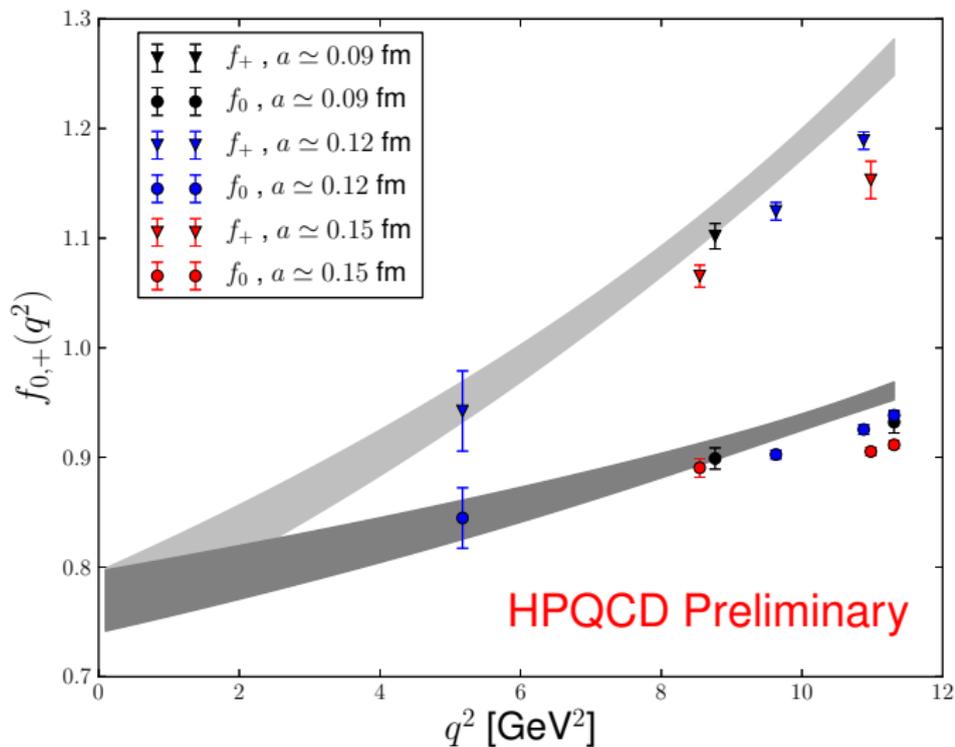
Transition element is parameterized in terms of form factors:

$$\begin{aligned} \langle D_{(s)}(p_{D_{(s)}}) | V^\mu | B_{(s)}(p_{B_{(s)}}) \rangle = & f_+(q^2) \left[p_1^\mu + p_2^\mu - \frac{M_{B_{(s)}}^2 - M_{D_{(s)}}^2}{q^2} q^\mu \right] \\ & + f_0(q^2) \frac{M_{B_{(s)}}^2 - M_{D_{(s)}}^2}{q^2} q^\mu \end{aligned}$$

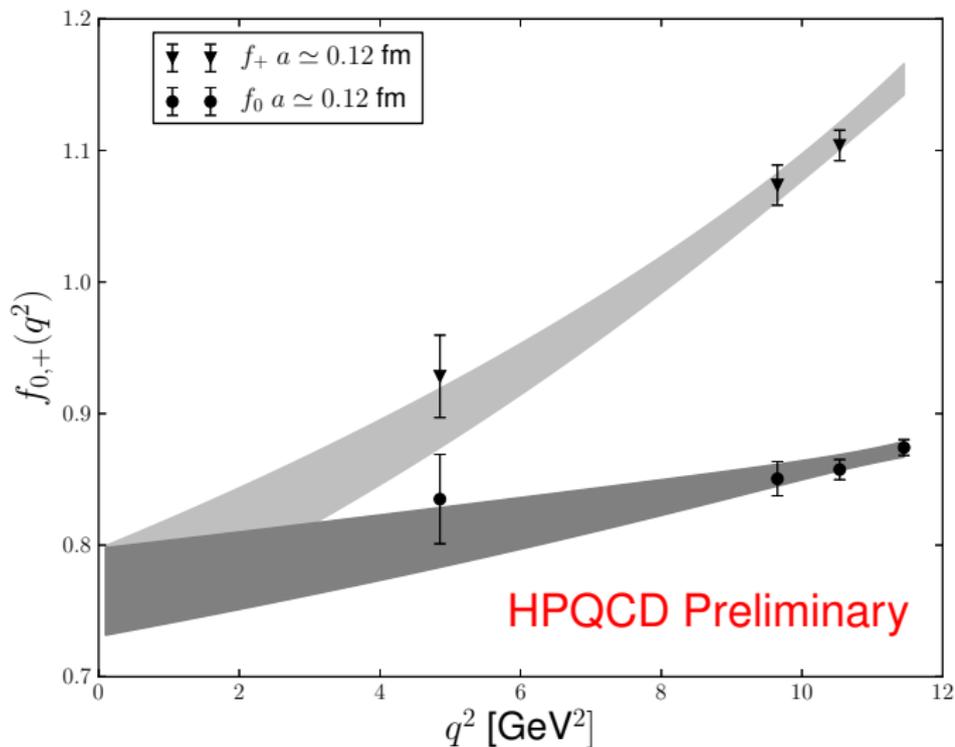
We computed $\langle D_{(s)} | V^\mu | B_{(s)} \rangle$ at varying $q^2 = (p_{B_{(s)}} - p_{D_{(s)}})^2$. We plan on fitting results to a z-fit.

$$f_{0,+}(z(q^2)) \propto \sum_k a_k^{(0,+)} z^k \quad , \quad z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

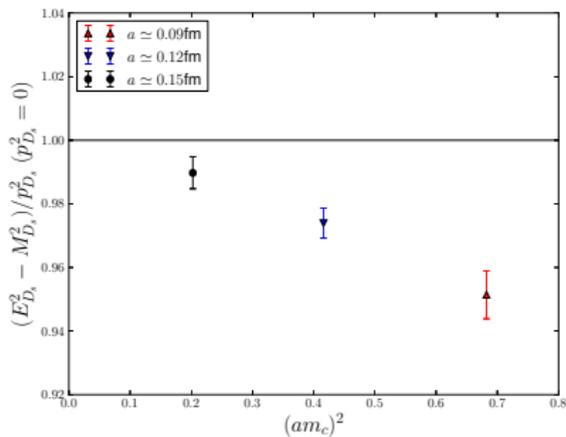
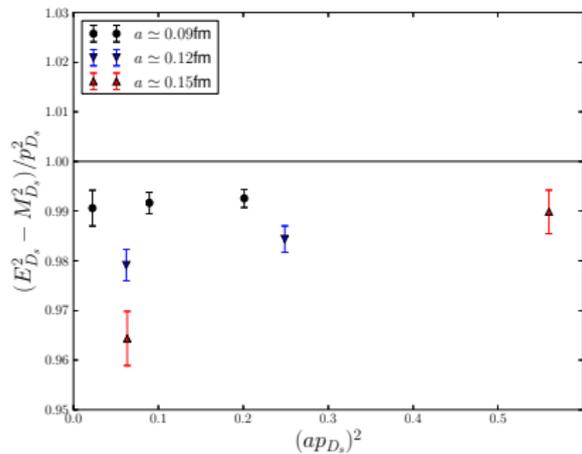
Preliminary Results: $B_s \rightarrow D_s$



Preliminary Results: $B \rightarrow D$

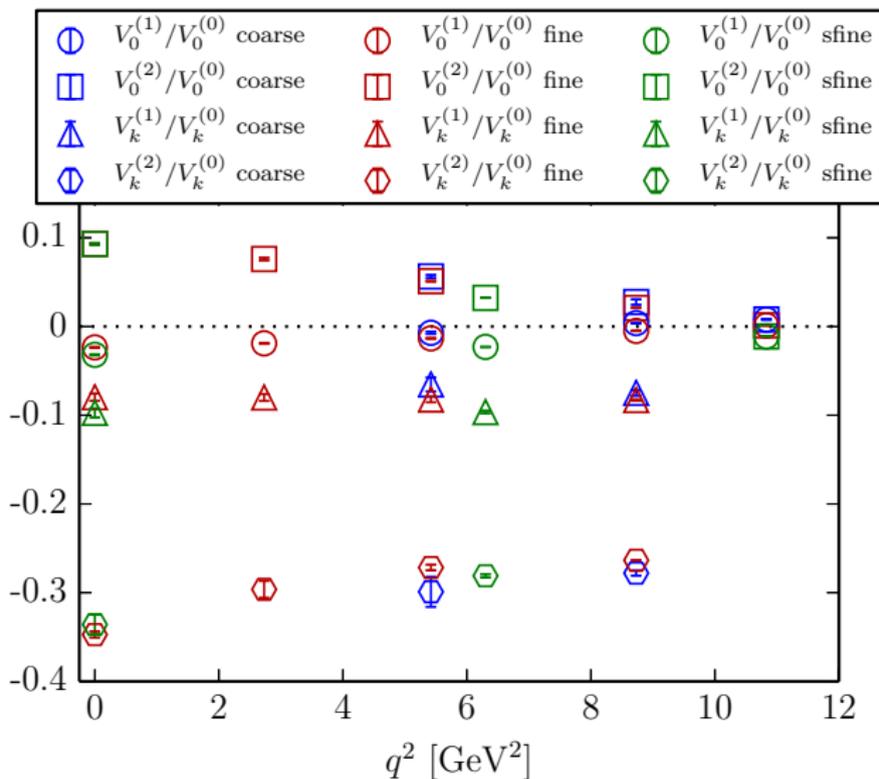


c^2 (speed of light) for D_s



See also 1208.2855.

Subleading Currents ($B_c \rightarrow \eta_c$)



Next Steps

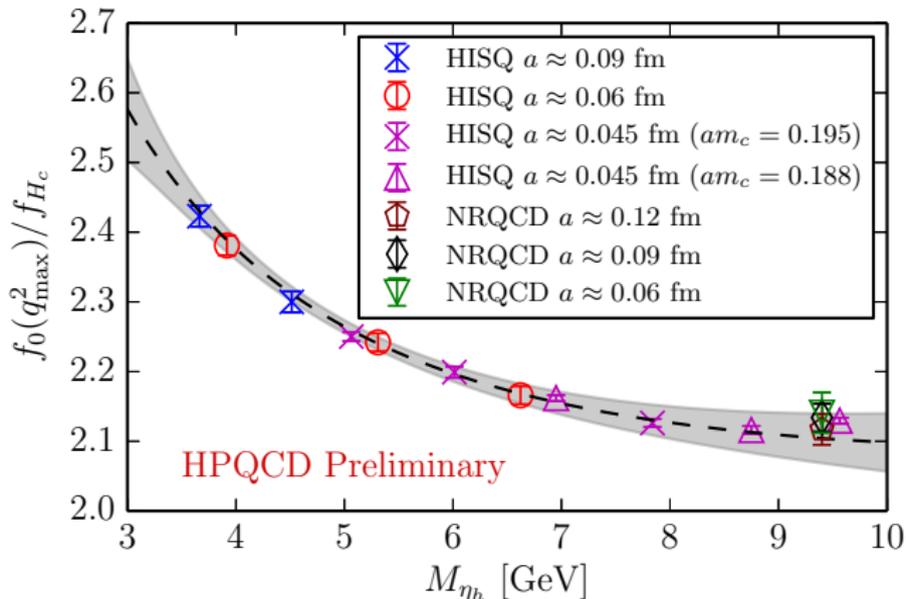
- ▶ Aim to extend data to minimum possible q^2 (requires overcoming signal/noise degradation).
- ▶ Fix renormalization of spacial currents non-perturbatively?
- ▶ New approach of extracting f_0 from scalar current:

$$S^{(0)} = \bar{c}b \quad , \quad m_b \left(1 - \frac{m_c}{m_b}\right) \langle D_s | S | B_s \rangle = (M_{B_s}^2 - M_{D_s}^2) f_0(q^2)$$

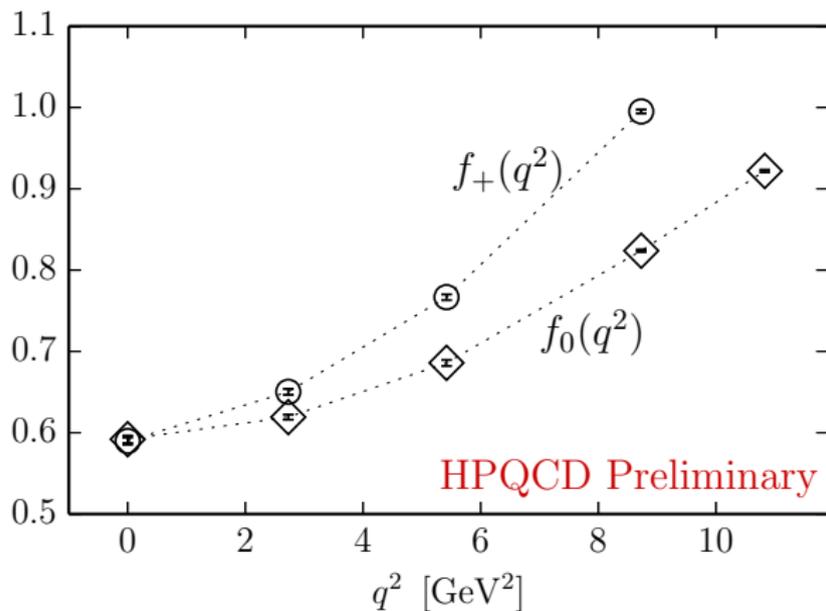
thanks for listening!

Backup Slides

NRQCD-HISQ Comparison



Proof of Principle: $B_c \rightarrow \eta_c$



[Colquhoun, Lytle et. al 1611.01987]

$$\mathcal{F}_\mu = \prod_{\rho \neq \mu} \left(1 + \frac{a^2 \delta_\rho^{(2)}}{4} \right)$$

$$X_\mu(x) \equiv \mathcal{U} \mathcal{F}_\mu U_\mu(x)$$

$$W_\mu(x) \equiv \left(\mathcal{F}_\mu - \sum_{\rho \neq \mu} \frac{a^2 (\delta_\rho)^2}{2} \right) \mathcal{U} \mathcal{F}_\mu U_\mu(x)$$

$$S_{\text{HISQ}} = \sum_x \bar{\psi}(x) \left(\gamma \cdot \left(\nabla_\mu(W) - \frac{a^2}{6} (1 + \epsilon) \nabla_\mu^3(X) \right) + m \right) \psi(x)$$

ϵ tuned to satisfy

$$\lim_{\underline{p} \rightarrow 0} \frac{E^2(\underline{p}) - m^2}{\underline{p}^2} = 1$$

$$G(\underline{x}, t + 1) = e^{-a(H_0 + \delta H)} G(\underline{x}, t)$$

$$aH_0 = -\frac{\nabla^{(2)}}{2am_b}, \tag{1}$$

$$a\delta H = -c_1 \frac{(\nabla^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2} (\underline{\nabla} \cdot \underline{\tilde{E}} - \underline{\tilde{E}} \cdot \underline{\nabla})$$

$$- c_3 \frac{1}{8(am_b)^2} \sigma \cdot (\underline{\nabla} \times \underline{\tilde{E}} - \underline{\tilde{E}} \times \underline{\nabla})$$

$$- c_4 \frac{1}{2am_b} \sigma \cdot \underline{\tilde{B}} + c_5 \frac{\nabla^{(4)}}{24am_b}$$

$$- c_6 \frac{(\nabla^{(2)})^2}{16n(am_b)^2} \tag{2}$$