

One-loop perturbative coupling of A and A_\star through the chiral overlap operator

Okuto Morikawa

Collaborators: Hiroki Makino, Hiroshi Suzuki

Kyushu University

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- H. Makino, O. M., and H. Suzuki, to appear in PTEP [arXiv:1704.04862 [hep-lat]].

Chiral gauge theory on the lattice

- No definition of chiral gauge theories on the lattice.
- Proposal of Grabowska–Kaplan [2015, 2016]
 - ▶ 5D domain-wall (DW) lattice formulation.
 - ▶ 4D lattice formulation based on **Chiral overlap operator**.
- Subtlety associated with topological charge [Okumura–Suzuki, Makino–Morikawa].
- We discuss gauge-breaking effects **even for anomaly-free cases**.

Chiral DW fermion [Grabowska–Kaplan 2015]

- 5D lattice space (x, s) with 4D DWs [Callan–Harvey, Kaplan]
 - ▶ Chiral zero modes are localized near two 4D DWs.
- **Gradient flow** [Narayanan–Neuberger, Lüscher]

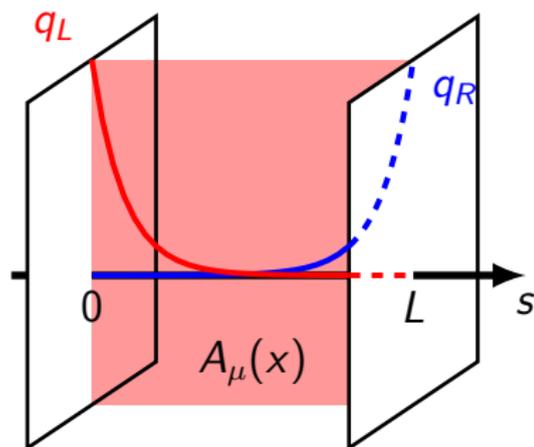
$$\partial_\tau B_\mu(x, s) = D_\nu G_{\nu\mu}(x, s)$$

- ▶ Flow time $\tau(s)$; Boundary condition $B_\mu(x, s=0) = A_\mu(x)$.

- Manifestly gauge invariant.
- B_μ goes to pure gauge.

$$\begin{aligned} A_{\star\mu}(x) &\equiv B_\mu(x, s=L) \\ &= g^{-1}(x)\partial_\mu g(x) \end{aligned}$$

- RH fermion **would couple** only to gauge mode.



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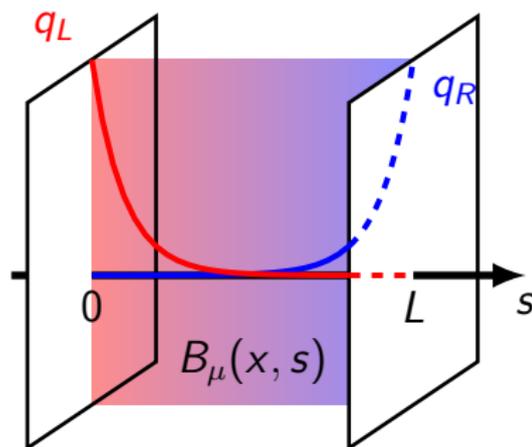
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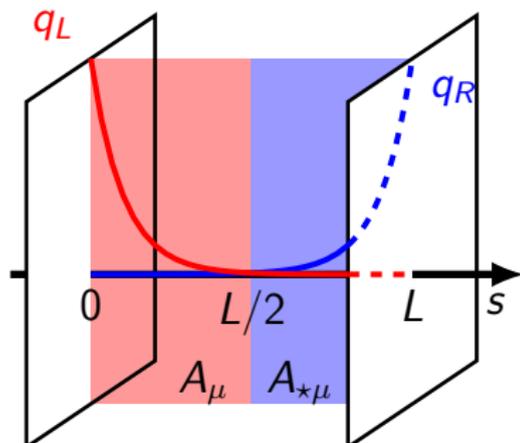
Chiral overlap operator [Grabowska–Kaplan 2016]

- DW formulation
→ 4D eff. theory [Neuberger, Vranas, Kikukawa–Noguchi]
- Assuming **abrupt transition (sudden flow)**,

$$a\hat{D}_\chi = 1 + \gamma_5 \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\epsilon_\star + 1} (1 - \epsilon) \right].$$

$$\epsilon = \frac{H_w[A]}{\sqrt{H_w[A]^2}}, \quad \epsilon_\star = \frac{H_w[A_\star]}{\sqrt{H_w[A_\star]^2}},$$

$$\begin{aligned} \gamma_5 H_w &= \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla_\mu^\star) \\ &\quad - \frac{1}{2} a \nabla_\mu \nabla_\mu^\star - m. \end{aligned}$$



Properties of $\hat{\mathcal{D}}_\chi$ [Grabowska–Kaplan 2016]

- 1 Ginsparg–Wilson relation

$$\gamma_5 \hat{\mathcal{D}}_\chi + \hat{\mathcal{D}}_\chi \gamma_5 = a \hat{\mathcal{D}}_\chi \gamma_5 \hat{\mathcal{D}}_\chi.$$

- 2 Classical continuum limit of $\hat{\mathcal{D}}_\chi$

$$am \hat{\mathcal{D}}_\chi = \gamma_\mu D_\mu(A) \frac{1 - \gamma_5}{2} + \gamma_\mu D_\mu(A_\star) \frac{1 + \gamma_5}{2} + \mathcal{O}(a),$$

where $D_\mu(A) = \partial_\mu + A_\mu$.

A couples only to LH fermion; A_\star couples only to RH fermion.

In the tree level, A and A_\star decouple from each other.

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In the tree level, A and A_\star decouple from each other.

- Investigate how the decoupling between A and A_\star is modified under radiative corrections.

One-loop effective action

- Fermion one-loop effective action

$$\ln \mathcal{Z}[A, A_\star] = \ln \int \prod_x [d\psi(x)d\bar{\psi}(x)] \exp \left[-a^4 \sum_x \bar{\psi}(x) \hat{\mathcal{D}}_x \psi(x) \right]$$

- Infinitesimal variations δ_A and δ_{A_\star}

$$\begin{aligned} \delta_A A_\mu &\neq 0, & \delta_A A_{\star\mu} &= 0, \\ \delta_{A_\star} A_{\star\mu} &\neq 0, & \delta_{A_\star} A_\mu &= 0. \end{aligned}$$

- Gauge transformation δ^ω

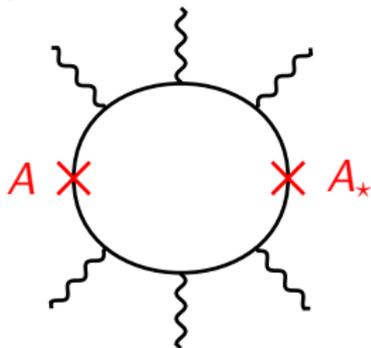
$$\begin{aligned} \delta_A^\omega A_\mu(x) &= \partial_\mu \omega(x) + [A_\mu(x), \omega], & \delta_A^\omega A_{\star\mu}(x) &= 0, \\ \delta_{A_\star}^\omega A_{\star\mu}(x) &= \partial_\mu \omega(x) + [A_{\star\mu}(x), \omega], & \delta_{A_\star}^\omega A_\mu(x) &= 0. \end{aligned}$$

- Gauge invariance $(\delta_A^\omega + \delta_{A_\star}^\omega) \ln \mathcal{Z}[A, A_\star] = 0$

One-loop perturbative coupling

- Let us focus on a variation of effective action:

$$\delta_A \delta_{A_*} \ln \mathcal{Z}[A, A_*]$$
$$\xrightarrow{a \rightarrow 0} - \int d^4x \mathcal{L}(A, A_*; \delta A, \delta_* A_*) \neq 0.$$



This implies breaking of BRS symmetry!

- Using gauge invariance, setting $A_* = 0$ ($\because A_*$: pure gauge),

$$\delta_A \delta_A^\omega \ln \mathcal{Z}[A, 0] \xrightarrow{a \rightarrow 0} \int d^4x \mathcal{L}(A, A_* = 0; \delta A, \delta_*^\omega A_* |_{A_* = 0}).$$

- $\delta_A \delta_A^\omega \ln \mathcal{Z}[A, 0]$ does not vanish **even for anomaly-free cases**.
- Thus, $\ln \mathcal{Z}[A, 0]$ is not invariant under gauge variation δ_A^ω .

Discussion with $A_{\star} = 0$ gauge

- Functional integral over A (A_{\star} is given from A)

$$\langle \mathcal{O}[A] \rangle = \int \mathcal{D}A \mathcal{Z}[A, A_{\star}] e^{-S_G[A]} \mathcal{O}[A].$$

- Faddeev–Popov determinant: $\Delta[A_{\star}] \int \mathcal{D}g \delta(A_{\star}^g) = 1$.
Inserting this into functional integral,

$$\begin{aligned} & \int \mathcal{D}g \int \mathcal{D}A \underline{\Delta[A_{\star}]} \delta(A_{\star}^g) \mathcal{Z}[A, A_{\star}] e^{-S_G[A]} \mathcal{O}[A] \\ & \propto \int \mathcal{D}A \underline{\Delta[A_{\star}]} \delta(A_{\star}) \mathcal{Z}[A, 0] e^{-S_G[A]} \mathcal{O}[A]. \end{aligned}$$

- Faddeev–Popov ghosts and Nakanishi–Lautrup field

$$S_{c\bar{c}} + S_{\text{gf}} = \int d^4x \left\{ \bar{c}_{\mu}^a \left[\partial_{\mu} c^a + f^{abc} A_{\star\mu}^b c^c \right] - i B_{\mu}^a A_{\star\mu}^a \right\}.$$

- $S_{c\bar{c}}$, S_{gf} , S_G and $\mathcal{O}[A]$ are BRS invariant; **In $\mathcal{Z}[A, 0]$ is not.**

Explicit forms of \mathcal{L} and $\ln \mathcal{Z}$

- Relation between \mathcal{L} and $\ln \mathcal{Z}$

$$-\delta_A \delta_{A_*} \ln \mathcal{Z}[A, A_*] \xrightarrow{a \rightarrow 0} \int d^4x \mathcal{L}(A, A_*; \delta A, \delta_* A_*).$$

- Expand around $a = 0$ and simplify in terms of

$$\bar{A}_\mu \equiv \frac{1}{2}(A_\mu + A_{*\mu}) \quad \begin{cases} \bar{D}_\mu &= \partial_\mu + [\bar{A}_\mu, \cdot] \\ \bar{F}_{\mu\nu} &= \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu + [\bar{A}_\mu, \bar{A}_\nu], \end{cases}$$

$$C_\mu \equiv A_{*\mu} - A_\mu.$$

C_μ transforms as **adjoint representation** under $\delta_A^\omega + \delta_{A_*}^\omega$.

Parity-odd part of $\mathcal{L} \sim \delta\delta \ln \mathcal{Z}$

$$-\frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{tr} \left[\left(\bar{F}_{\mu\nu} + \frac{1}{12} [C_\mu, C_\nu] \right) \{ \delta A_\rho, \delta_\star A_{\star\sigma} \} \right. \\ \left. - \frac{1}{3} C_\mu \left(\{ \delta A_\nu, \bar{D}_\rho \delta_\star A_{\star\sigma} \} + \{ \delta_\star A_{\star\nu}, \bar{D}_\rho \delta A_\sigma \} \right) \right]$$

- This is proportional to gauge anomaly coefficient $\text{tr } T^a \{ T^b, T^c \}$.

Parity-even, Lorentz-preserving part of $\mathcal{L} \sim \delta\delta \ln \mathcal{Z}$

$$\begin{aligned}
 & \frac{f_0}{a^2} \text{tr} \delta A_\mu \delta_\star A_{\star\mu} \\
 & + (-3f_1/2 + f_2/2 - f_3/2) \text{tr}[(\bar{D}_\mu \delta A_\mu) C_\nu \delta_\star A_{\star\nu} - C_\mu \delta A_\mu (\bar{D}_\nu \delta_\star A_{\star\nu})] \\
 & - (f_1/2 + f_2/2 - 3f_3/2) \text{tr}[C_\mu (\bar{D}_\nu \delta A_\mu) \delta_\star A_{\star\nu} - \delta A_\mu C_\nu (\bar{D}_\mu \delta_\star A_{\star\nu})] \\
 & - (f_1/2 + f_2/2 - 3f_3/2) \text{tr}[C_\nu \delta A_\mu (\bar{D}_\mu \delta_\star A_{\star\nu}) - (\bar{D}_\nu \delta A_\mu) C_\mu \delta_\star A_{\star\nu}] \\
 & + (-7f_1/2 + f_2/2 + f_3/2) \text{tr}[(\bar{D}_\mu C_\mu) \delta A_\nu \delta_\star A_{\star\nu} - \delta A_\nu (\bar{D}_\mu C_\mu) \delta_\star A_{\star\nu}] \\
 & - (3f_1/2 - f_2/2 + f_3/2) \text{tr}[\delta A_\mu C_\mu (\bar{D}_\nu \delta_\star A_{\star\nu}) - C_\nu (\bar{D}_\mu \delta A_\mu) \delta_\star A_{\star\nu}] \\
 & + (13f_1 - 3f_2 - 3f_3) \text{tr}(\bar{D}_\mu \delta A_\mu) (\bar{D}_\nu \delta_\star A_{\star\nu}) \\
 & + (9f_1 - 3f_2 - f_3) \text{tr}(\bar{D}_\mu \delta A_\nu) (\bar{D}_\mu \delta_\star A_{\star\nu}) \\
 & + (-19f_1 + 5f_2 + 5f_3) \text{tr}(\bar{D}_\nu \delta A_\mu) (\bar{D}_\mu \delta_\star A_{\star\nu}) \\
 & + (11f_1/6 - f_2/6 - 7f_3/6) \text{tr} C_\mu \delta A_\nu C_\mu \delta_\star A_{\star\nu} \\
 & + (-13f_1/6 + 11f_2/6 - 7f_3/6) \text{tr}(C_\mu \delta A_\mu C_\nu \delta_\star A_{\star\nu} + C_\nu \delta A_\mu C_\mu \delta_\star A_{\star\nu}) \\
 & + (-5f_1/12 + 19f_2/12 - 17f_3/12) \text{tr}(C_\nu C_\mu \delta A_\mu \delta_\star A_{\star\nu} + \delta A_\mu C_\mu C_\nu \delta_\star A_{\star\nu}) \\
 & + (19f_1/12 - 5f_2/12 - 5f_3/12) \text{tr}(C_\mu C_\nu \delta A_\mu \delta_\star A_{\star\nu} + \delta A_\mu C_\nu C_\mu \delta_\star A_{\star\nu}) \\
 & + (-17f_1/12 + 19f_2/12 - 11f_3/12) \text{tr}(C_\mu C_\mu \delta A_\nu \delta_\star A_{\star\nu} + \delta A_\nu C_\mu C_\mu \delta_\star A_{\star\nu})
 \end{aligned}$$

Parity-even, Lorentz-violating part of $\mathcal{L} \sim \delta\delta \ln \mathcal{Z}$

$$\begin{aligned}
 & \frac{3}{2} \left(9f_1 - f_2 - f_3 - \frac{f_4}{2} - \frac{f_5}{2} \right) \text{tr}[(\bar{D}_\nu C_\nu) \delta A_\nu \delta_\star A_{\star\nu} - \delta A_\nu (\bar{D}_\nu C_\nu) \delta_\star A_{\star\nu}] \\
 & - \left(9f_1 - f_2 - f_3 - \frac{f_4}{2} - \frac{f_5}{2} \right) \text{tr}(\bar{D}_\nu \delta A_\nu) (\bar{D}_\nu \delta_\star A_{\star\nu}) \\
 & + \left(\frac{47f_1}{2} - \frac{7f_2}{2} - \frac{7f_3}{2} + \frac{f_4}{4} - \frac{7f_5}{4} \right) \text{tr} C_\nu \delta A_\nu C_\nu \delta_\star A_{\star\nu} \\
 & + \left(\frac{67f_1}{4} - \frac{11f_2}{4} - \frac{11f_3}{4} + \frac{5f_4}{8} - \frac{11f_5}{8} \right) \\
 & \quad \times \text{tr}(C_\nu C_\nu \delta A_\nu \delta_\star A_{\star\nu} + \delta A_\nu C_\nu C_\nu \delta_\star A_{\star\nu})
 \end{aligned}$$

(coefficients $f_i = f_i(am)$)

Gauge anomaly

- Integrating $\int d^4x \mathcal{L} \sim \delta\delta \ln \mathcal{Z}$ with A , we can obtain
 - ① gauge anomaly (parity-odd)
 - ② effective action: $\ln \mathcal{Z}$ (parity-even).

$$\begin{aligned} & \delta_A^\omega \ln \mathcal{Z}[A, 0] \Big|_{\text{parity-odd}} \\ &= \frac{1}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} \int d^4x \operatorname{tr} \left[(\partial_\mu \omega) \left(A_\nu \partial_\rho A_\sigma + \frac{1}{2} A_\nu A_\rho A_\sigma \right) \right] \end{aligned}$$

Gauge *non-invariant* effective action

$$\begin{aligned} & \ln \mathcal{Z}[A, 0] \Big|_{\text{parity-even}} \\ &= \int d^4x \operatorname{tr} \left[\frac{f_0}{2a^2} A_\mu A_\mu \right. \\ & \quad + \frac{1}{2} (-13f_1 + 3f_2 + 3f_3) A_\mu \partial_\mu \partial_\nu A_\nu \\ & \quad + (5f_1 - f_2 - 2f_3) (A_\mu \partial_\nu \partial_\nu A_\mu - A_\mu A_\nu \partial_\mu A_\nu + A_\mu A_\nu \partial_\nu A_\mu) \\ & \quad + \frac{2}{3} (f_1 + f_2 - 2f_3) A_\mu A_\mu A_\nu A_\nu + \frac{1}{12} (-11f_1 + f_2 + 7f_3) A_\mu A_\nu A_\mu A_\nu \\ & \quad + \frac{1}{2} \left(9f_1 - f_2 - f_3 - \frac{f_4}{2} - \frac{f_5}{2} \right) A_{\mu} \partial_{\mu} \partial_{\mu} A_{\mu} \\ & \quad \left. + \frac{1}{12} \left(57f_1 - 9f_2 - 9f_3 + \frac{3f_4}{2} - \frac{9f_5}{2} \right) A_{\mu} A_{\mu} A_{\mu} A_{\mu} \right] \end{aligned}$$

- Even for **anomaly-free cases**, these terms do not generally vanish, and should be removed by **local counterterms**.

→ ~~Non-perturbative~~ formulation.

Summary

- Grabowska–Kaplan lattice formulation of chiral gauge theories.
 - ▶ Analytical model: abrupt transition.
 - ▶ **Tree-level decoupling** between A and A_* .
- We focused on **one-loop effective coupling** between A and A_* .
- Effective coupling $\neq 0$ even for anomaly-free cases.
- This means **BRS non-invariance** and **need of local counterterms!**
- *Improvement of $\hat{\mathcal{D}}_x$?*
- *Abrupt transition \rightarrow gradual flow scenario?*

Backup: Coefficients f_i

$$f_0(am) \equiv \int_p \left(-\frac{1}{4t} - \frac{s_\rho^2}{4t} - \frac{cc_\rho}{4t} \right),$$

$$f_1(am) \equiv \int_p \left(\frac{1}{64t^2} - \frac{c_\rho c_\sigma}{128t} + \frac{s_\rho^2 s_\sigma^2}{32t^2} \right),$$

$$f_2(am) \equiv \int_p \left(-\frac{c_\rho c_\sigma}{32t} + \frac{7s_\rho^2 s_\sigma^2}{64t^2} + \frac{cs_\rho^2 c_\sigma}{32t^2} + \frac{c^2 c_\rho c_\sigma}{64t^2} \right),$$

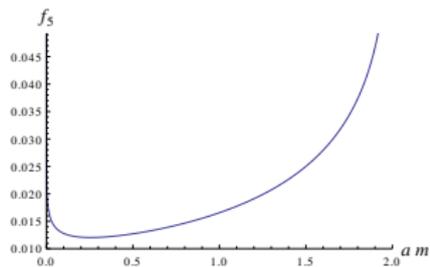
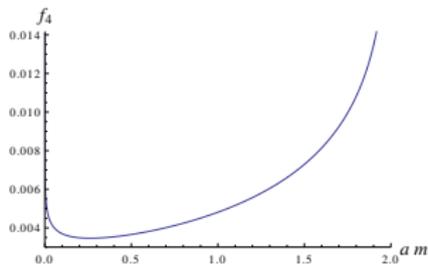
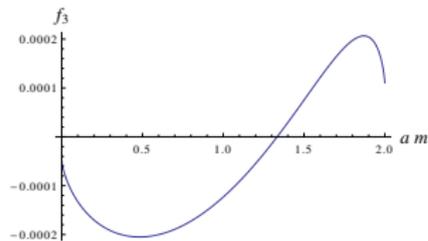
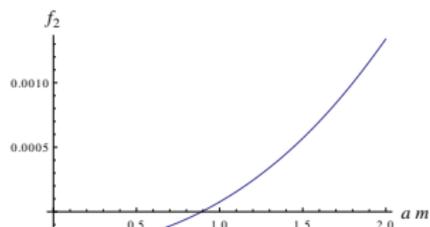
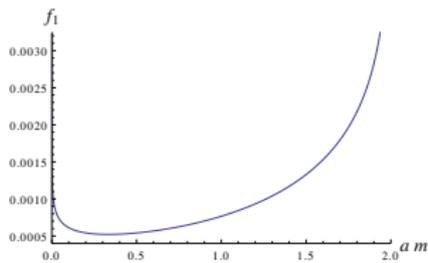
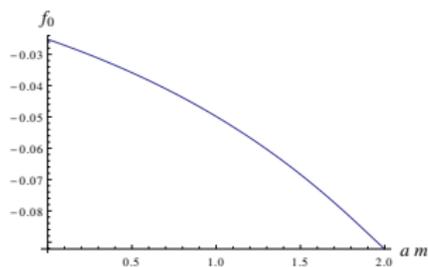
$$f_3(am) \equiv \int_p \left(-\frac{c_\rho c_\sigma}{32t} + \frac{3s_\rho^2 s_\sigma^2}{32t^2} - \frac{s_\rho^2}{32t^2} - \frac{cc_\rho}{32t^2} \right),$$

$$f_4(am) \equiv \int_p \left(\frac{1}{96t} + \frac{s_\rho^2}{96t} + \frac{cc_\rho}{96t} + \frac{1}{16t^2} \right),$$

$$f_5(am) \equiv \int_p \left(\frac{1}{16t} + \frac{c_\rho c_\sigma}{32t} + \frac{7}{32t^2} - \frac{c^2}{32t^2} + \frac{cc_\rho}{16t^2} + \frac{s_\rho^2}{32t^2} \right).$$

$$s_\mu = \sin p_\mu, \quad c_\mu = \cos p_\mu, \quad c = \sum_\mu (c_\mu - 1) + am, \quad t = \sum_\mu s_\mu^2 + c^2.$$

Backup: Coefficients f_i



Backup: Is A_\star pure gauge?

- Gradient flow

$$\partial_t B_\mu(x, t) = D_\nu G_{\nu\mu}(x, t), \quad B_\mu(x, t=0) = A_\mu(x).$$

- In abelian theory, solution to flow eq. is given by

$$\begin{aligned} B_\mu(x, t) &= \int d^4 y \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} \left[\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) e^{-tp^2} + \frac{p_\mu p_\nu}{p^2} \right] A_\nu(y) \\ &\xrightarrow{t \rightarrow \infty} g(x)^{-1} \partial_\mu g(x): \text{Pure gauge,} \end{aligned}$$

$$g(x) = \exp \left[- \int d^4 y \int \frac{d^4 p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2} \partial_\mu A_\mu(y) \right].$$

- *Assumption:* topologically trivial \rightarrow We can take $A_\star = 0$ gauge.

Backup: Chiral overlap operator (gradual flow)

- Dirac operator of Grabowska–Kaplan

$$a\mathcal{D}_\chi = \lim_{L \rightarrow \infty} \left[1 + \gamma_5 \frac{1 - \prod_{s=L}^1 T(s)}{1 + \prod_{s=L}^1 T(s)} \right].$$

- Transfer matrix

$$T(s) = \begin{pmatrix} B_s^{-1} & -B_s^{-1}C_s \\ -C_s^\dagger B_s^{-1} & B_s + C_s^\dagger B_s^{-1}C_s \end{pmatrix},$$

where

$$B_s = 1 + a_5 \left(-\frac{a}{2} \nabla_{\mu,s} \nabla_{\mu,s}^* - m \right), \quad C_s = a_5 \sigma_\mu \frac{1}{2} (\nabla_{\mu,s} + \nabla_{\mu,s}^*).$$

Backup: Lattice modifications

- Modified γ_5

$$\hat{\gamma}_5 \equiv \gamma_5(1 - a\hat{\mathcal{D}}_x),$$

which satisfies

$$(\hat{\gamma}_5)^2 = 1, \quad \hat{\mathcal{D}}_x \hat{\gamma}_5 = -\gamma_5 \hat{\mathcal{D}}_x.$$

- Using modified chiral projection operators

$$\hat{P}_\pm = \frac{1}{2}(1 \pm \hat{\gamma}_5),$$

the chiral components are defined as

$$\begin{aligned} \hat{P}_- \psi_L(x) &= \psi_L(x), & \bar{\psi}_L P_+ &= \bar{\psi}_L(x), \\ \hat{P}_+ \psi_R(x) &= \psi_R(x), & \bar{\psi}_R P_- &= \bar{\psi}_R(x). \end{aligned}$$