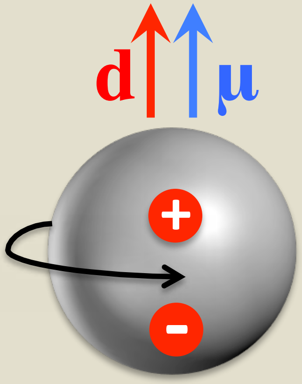
Neutron Electric Dipole Moment



Boram Yoon
Los Alamos National Lab

Lattice 2017
June 18-24, 2017, Granada

Contents

- Introduction
- nEDM induced by QCD θ-term
- nEDM induced by Quark EDM
- nEDM induced by Quark chromo EDM
- Other Related Results
- Summary

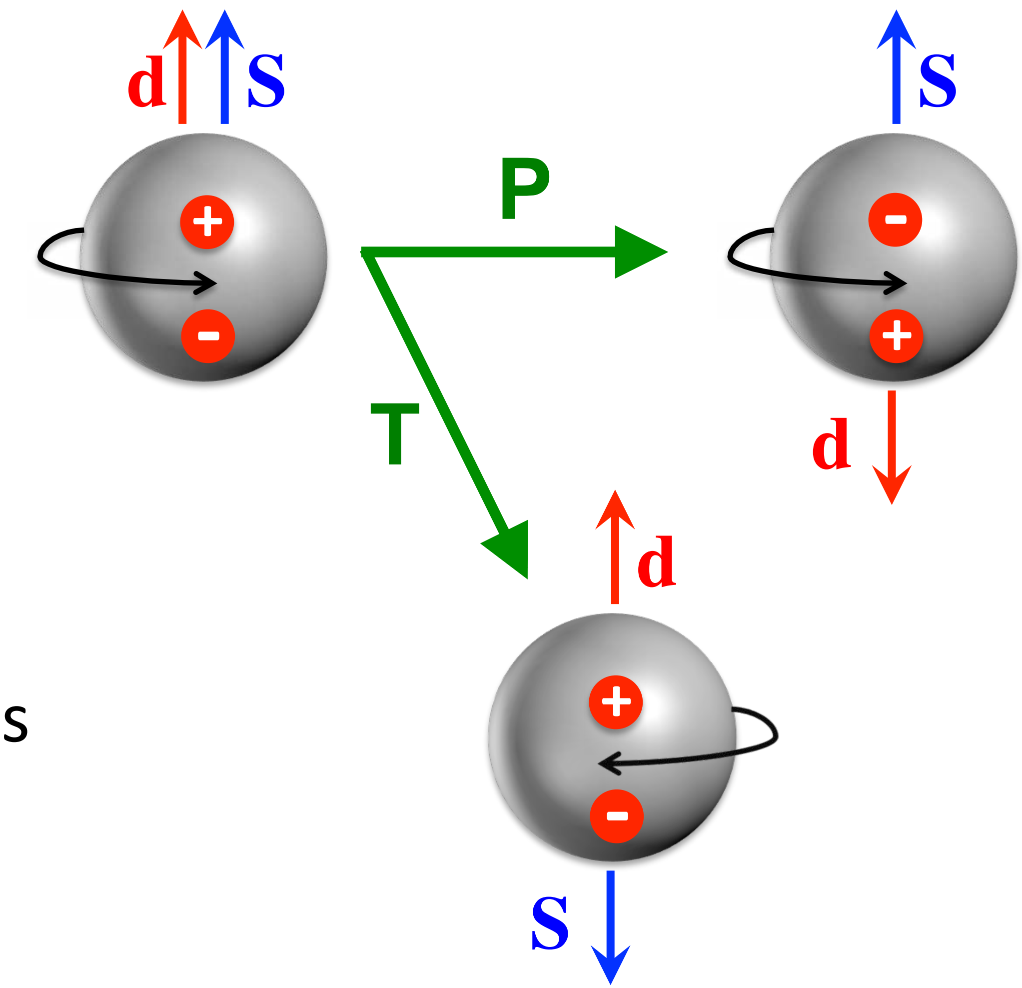
Neutron EDM and CP Violation

- Measures separation between centers of (+) and (-) charges

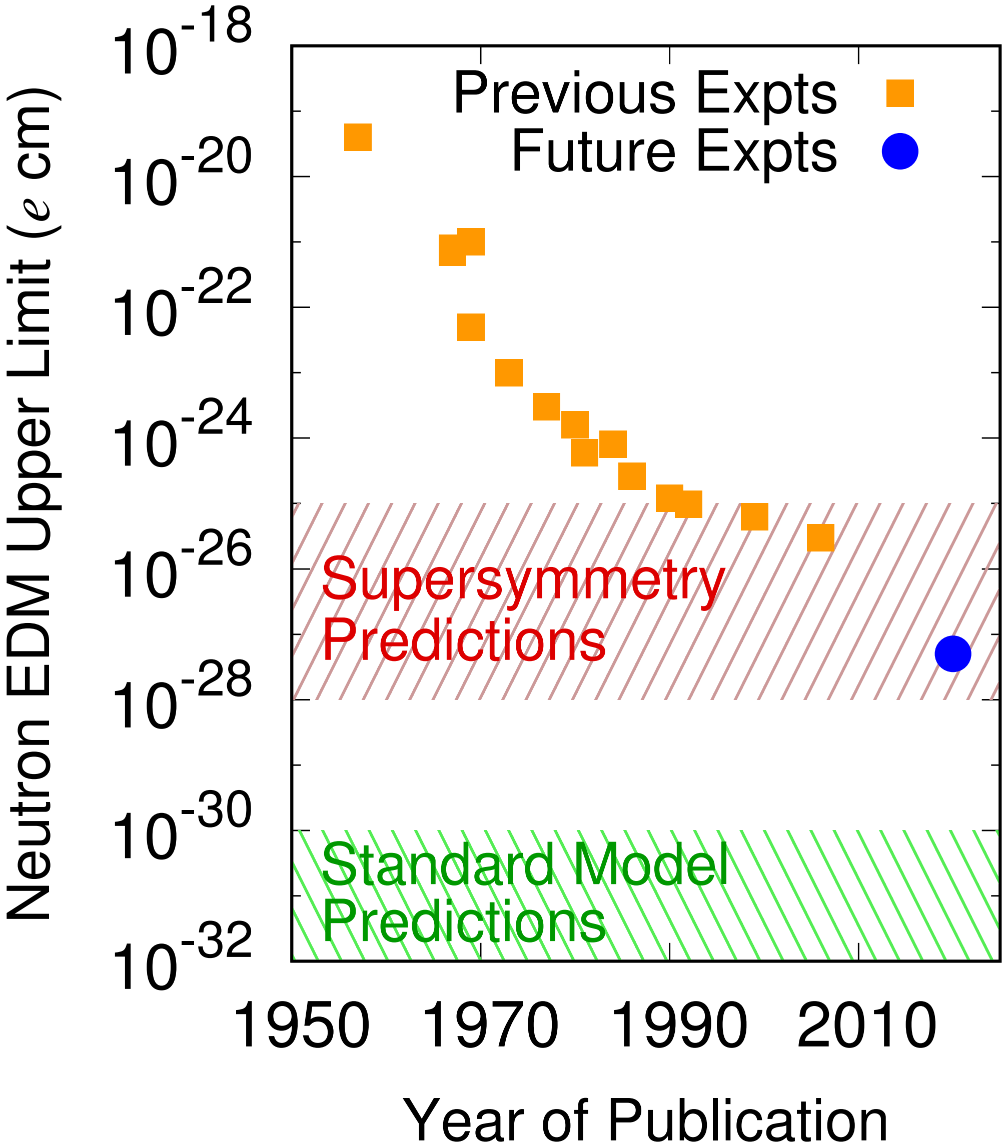
$$\delta H = d_N \hat{S} \cdot \vec{E}$$

- Current bound: $|d_n| < 2.9 \times 10^{-26}\ e{\cdot}\mathrm{cm}$

- Nonzero nEDM violates P and T, so CP



Neutron EDM Searches



- Predictions
  - Standard Model
    $|d_n| \sim 10^{-31}\ e{\cdot}\mathrm{cm}$
  - Supersymmetry
    $|d_n| \sim 10^{-25} - 10^{-28}\ e{\cdot}\mathrm{cm}$
- Experiments targeting $5 \times 10^{-28}\ e{\cdot}\mathrm{cm}$ precision
  - PSI EDM
  - Munich FRMII
  - RCNP/TRIUMF
  - SNS nEDM
  - JPARC

Tests of

- New source of CP violation
  - CPV in SM is not sufficient to explain observed baryon asymmetry
- Supersymmetry and other BSM models
  - nEDM predicted to be at 10^{-26}–10^{-28} e·cm

Effective Lagrangian at 1 GeV

$$\mathcal{L}_{\rm CPV}^{d\le 6} = -\frac{g_s^2}{32\pi^2}\bar{\theta} G\tilde{G}$$ dim=4 QCD θ-term

$$-\frac{i}{2}\sum_{q=u,d,s} d_q \bar{q}(\sigma\cdot F)\gamma_5 q$$ dim=5 Quark EDM (qEDM)

$$-\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \bar{q}(\sigma\cdot G)\gamma_5 q$$ dim=5 Quark Chromo EDM (CEDM)

$$+ d_w \frac{g_s}{6} G\tilde{G}G$$ dim=6 Weinberg 3g operator

$$+\sum_i C_i^{(4q)} O_i^{(4q)}$$ dim=6 Four-quark operators

- $\bar{\theta} \le O(10^{-9} - 10^{-11})$: Strong CP problem
- Dim=5 terms suppressed by $d_q \approx v/\Lambda_{\rm BSM}^2$; effectively dim=6
- Lattice QCD calculations play important role

QCD θ-term $-\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G}$

QCD θ-term

- Calculate d_N in presence of CP violating θ-term

$$S = S_{QCD} + S_\theta$$

$$S_\theta = -i\theta \int d^4x\, G\tilde{G} / 32\pi^2 = -i\theta\, Q_{\mathrm{top}}$$

- Lattice calculation strategies
  - Expansion in θ
  - External electric field method
  - Simulation with imaginary θ

Expansion in θ

$$\langle \mathcal{O}(x) \rangle_\theta = \frac{1}{Z_\theta} \int d[U,q,\bar{q}]\, \mathcal{O}(x)\, e^{-S_{QCD} + \boxed{i\theta Q_{\text{top}}}}$$

$$= \langle \mathcal{O}(x) \rangle_{\theta=0} + i\theta \langle \mathcal{O}(x) Q_{\text{top}} \rangle_{\theta=0} + O\left(\theta^2\right)$$

- Measurements performed on regular (θ=0) lattices
- The effect of the QCD θ-term is included by reweighting
- d_n extracted from form-factor F_3 extrapolated to q^2=0

$$\langle N | V_\mu | N \rangle_\theta = \bar{u}\left[F_1(q^2)\gamma_\mu + i\frac{F_2(q^2)}{2m_N}\sigma_{\mu\nu}q^\nu - \boxed{\frac{F_3(q^2)}{2m_N}\sigma_{\mu\nu}q^\nu\gamma_5} \right] u$$

$$d_N = \frac{F_3(q^2=0)}{2m_N}$$

Form Factors with Parity Mixing

Abramczyk, et al., arXiv:1701.07792

$$N = \varepsilon^{abc}\left(d^{Ta} C\gamma_5 u^b\right) d^c$$

	P, CP-even	P, CP-violating
Dirac Eq.	$\left(ip_\mu \gamma_\mu + m\right) u = 0$	$\left(ip_\mu \gamma_\mu + me^{-2i\alpha\gamma_5}\right)\tilde{u} = 0$
Parity Op.	γ_4 $u_{\vec{p}} \to \gamma_4 u_{-\vec{p}}$	$e^{2i\alpha\gamma_5}\gamma_4$ $\tilde{u}_{\vec{p}} \to e^{2i\alpha\gamma_5}\gamma_4 \tilde{u}_{-\vec{p}}$

- CPV interactions ➔ phase in neutron mass

  γ_4 no longer parity op of neutron state

- Introduce new parity op or
- Rotate neutron state so that γ_4 is the parity op:

$$\tilde{u} = e^{i\alpha\gamma_5} u, \qquad \bar{\tilde{u}} = \bar{u} e^{i\alpha\gamma_5}$$

[Syritsyn@ Wed 9:00]

Form Factors with Parity Mixing

Abramczyk, et al., arXiv:1701.07792

- FF analysis used in Lattice QCD calculations (since 2005)

$$\left\langle \tilde{N}\, V_\mu\, \bar{\tilde{N}} \right\rangle \;\Rightarrow\; \tilde{F}_1(q^2)\gamma_\mu + i\frac{\tilde{F}_2(q^2)}{2m_N}\sigma_{\mu\nu}q^\nu - \frac{\tilde{F}_3(q^2)}{2m_N}\sigma_{\mu\nu}q^\nu\gamma_5$$

γ_4 is not parity op ➔ $\tilde{F}_3$ **is not correct CP-odd FF of** $\sim \boldsymbol{E}\cdot\boldsymbol{S}$

- Correct analysis (Rotate: γ_4 is the parity operator)

$$e^{-i\alpha\gamma_5}\left\langle \tilde{N}\, V_\mu\, \bar{\tilde{N}} \right\rangle e^{-i\alpha\gamma_5} = \left\langle N\, V_\mu\, \bar{N} \right\rangle, \quad e^{-i\alpha\gamma_5}\left\langle \tilde{N}\, \bar{\tilde{N}} \right\rangle e^{-i\alpha\gamma_5} = \left\langle N\, \bar{N} \right\rangle$$

$$\Rightarrow\; F_1(q^2)\gamma_\mu + i\frac{F_2(q^2)}{2m_N}\sigma_{\mu\nu}q^\nu - \frac{F_3(q^2)}{2m_N}\sigma_{\mu\nu}q^\nu\gamma_5$$

Rotation applies only to neutron not quark states

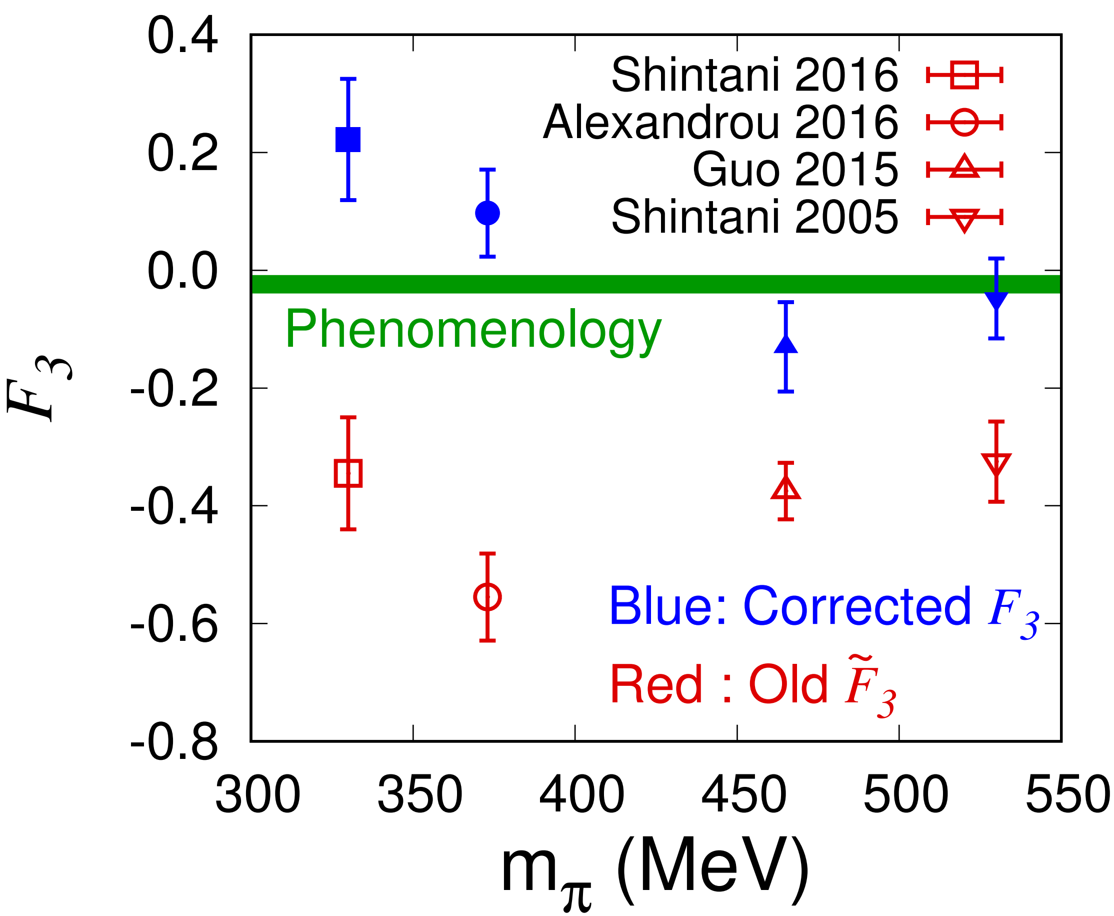
[Syritsyn@ Wed 9:00]

Form Factors with Parity Mixing

Abramczyk, et al., arXiv:1701.07792

- Without rotation, phase $e^{i\alpha\gamma_5}$ mixes F_2 and F_3

$$F_2 = \cos(2\alpha)\tilde{F}_2 - \sin(2\alpha)\tilde{F}_3$$
$$F_3 = \sin(2\alpha)\tilde{F}_2 + \cos(2\alpha)\tilde{F}_3$$



- Corrections calculated with assumptions & approximations
- Corrected lattice data consistent with zero
- May resolve tension between phenomenology and lattice results

Variance Reduction using Cluster Decomposition

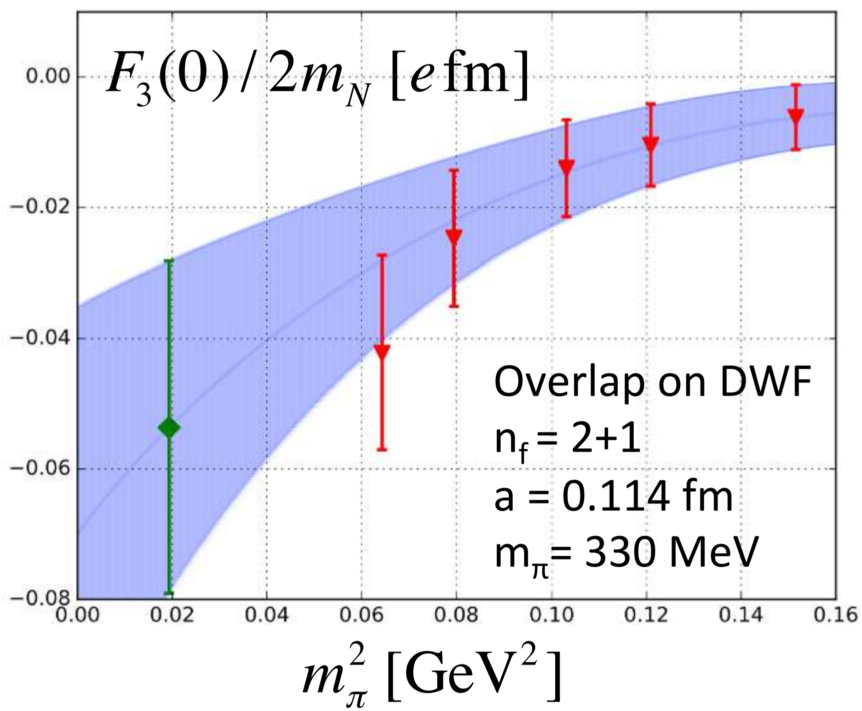
Liu, et al, arXiv:1705.06385

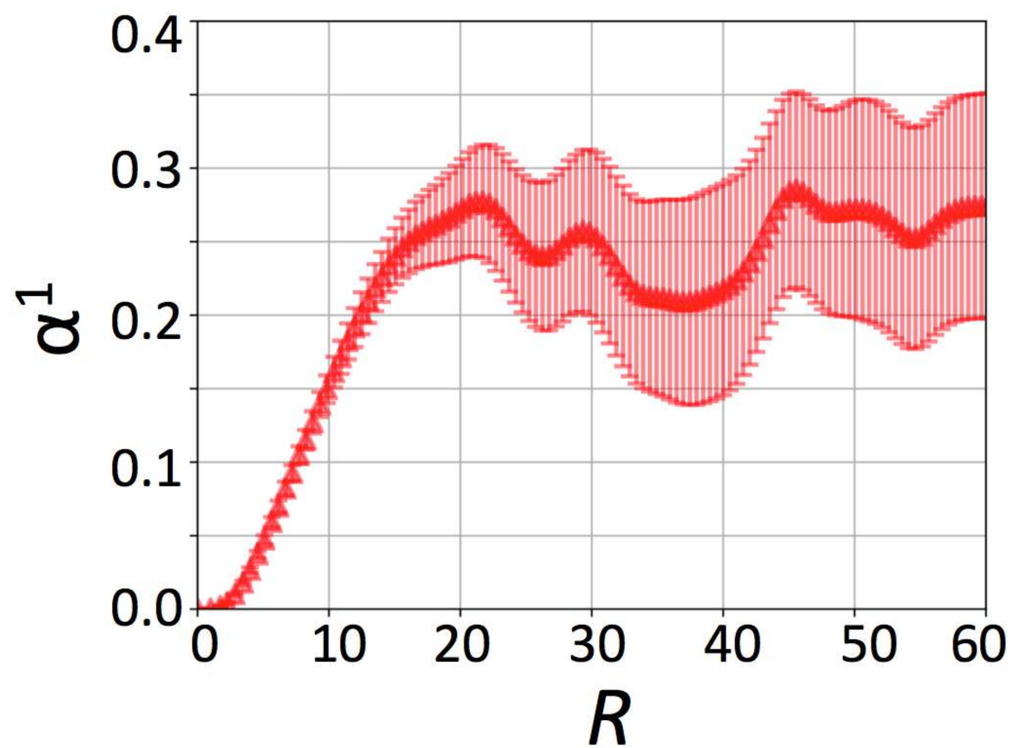
[Liu @ Wed 9:40]

$$C_R(t) = \left\langle \sum_{\vec{x}} N(\vec{x},t)\,\bar{N}(\text{[redacted]},0)\; Q_R(x) \right\rangle$$

$$Q_R(x) = \sum_{\|x-y\|<R} q(y)$$

Calculate topological charge only in the vicinity of the sink





External Electric Field Method

- In the presence of uniform electric field $\vec{\mathcal{E}}$, a change of energy for the nucleon state due to the θ-term is

$$E^{\theta}_{\vec{S}} - E^{\theta}_{-\vec{S}} \approx 2 d_N \theta\, \vec{S} \cdot \vec{\mathcal{E}}$$

- Neutron correlator with θ-term via reweighting

$$\left\langle N\bar{N} \right\rangle_{\theta} (\vec{\mathcal{E}}, t) = \left\langle N(t)\bar{N}(0)\, e^{i\theta Q_{\mathrm{top}}} \right\rangle_{\vec{\mathcal{E}}}$$

- Electric field applied only to valence quarks
- Does not need form-factor analysis and extrapolation to q^2=0

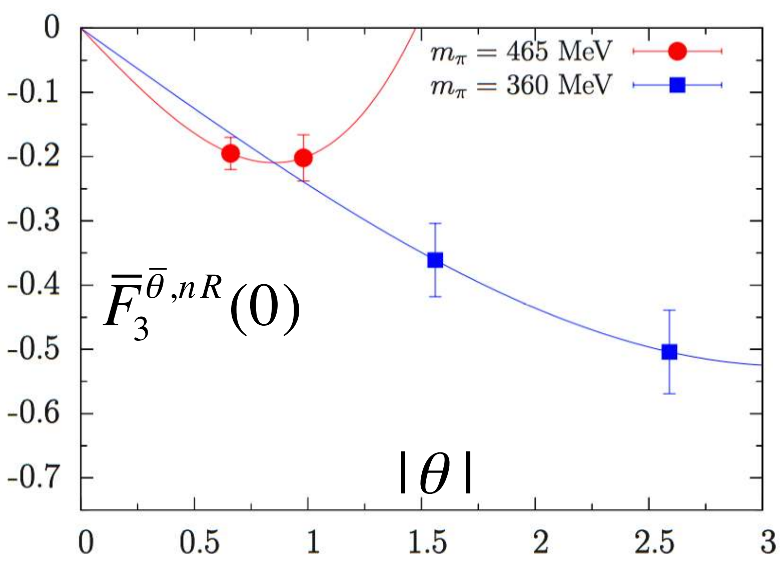
Simulation with Imaginary θ

- Avoid imaginary action (sign problem) by

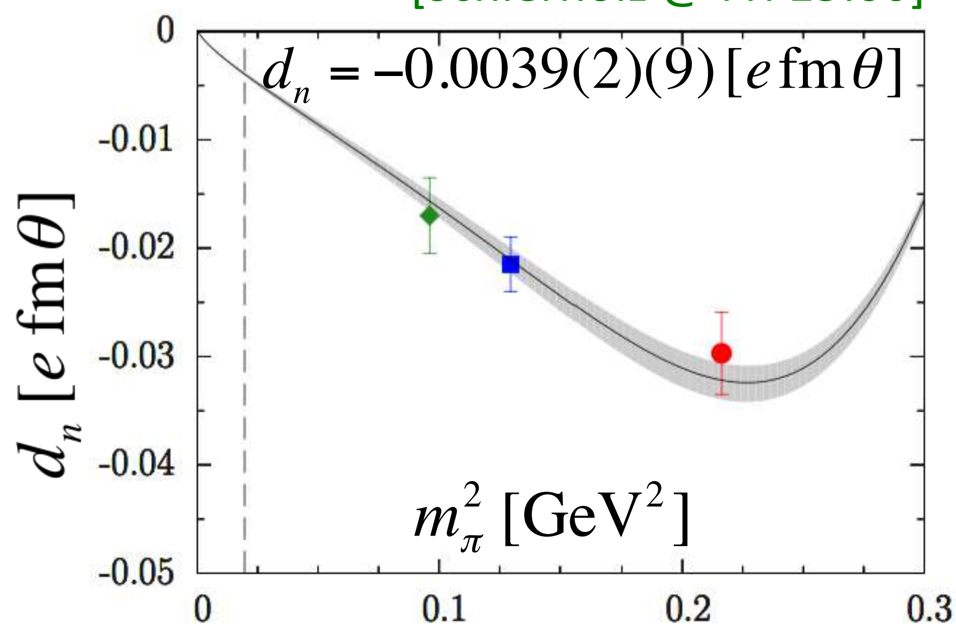
$$\boxed{\theta = i\tilde{\theta}} \qquad S_\theta^q = \tilde{\theta}\frac{m_l m_s}{2m_s + m_l}\sum_x \bar{q}\gamma_5 q$$

- Analytic continuation for small |θ|
- d_n is extracted from F_3

[Schierholz @ Fri 15:00]



Guo, et al, PRL 115 (2015) 062001



Stout Link Nonperturbative Clover, a = 0.074 fm

Quark EDM

$$-\frac{i}{2}\sum_{q=u,d,s} d_q \overline{q}(\sigma \cdot F)\gamma_5 q$$

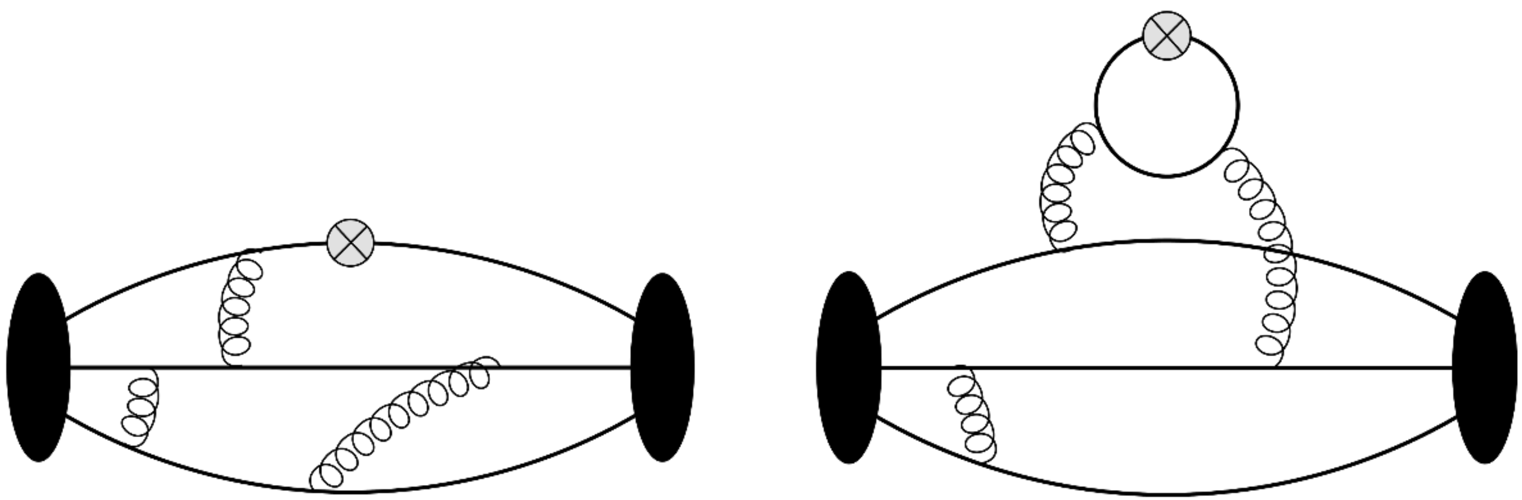
Quark EDM $d_q \overline{q}(\sigma \cdot F)\gamma_5 q$

- nEDM from qEDMs written in tensor charges g_T

$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s$$

$$\langle N | \overline{q} \sigma_{\mu\nu} q | N \rangle = g_T^q \, \overline{u}_N \sigma_{\mu\nu} u_N$$

- $d_q \propto m_q$ in many models; $m_u/m_d \approx 1/2$, $m_s/m_d \approx 20$
  Precise determination of g_T^s is important

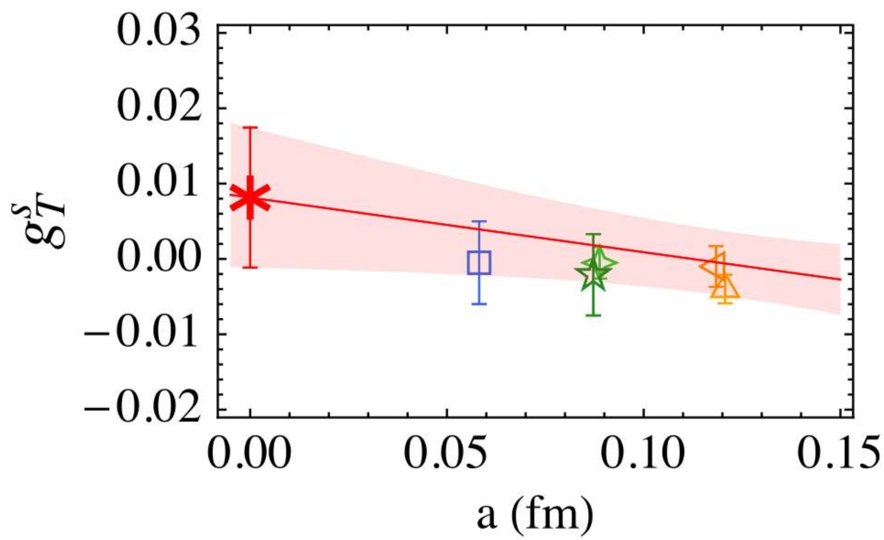


Quark EDM $d_q \bar{q}(\sigma \cdot F)\gamma_5 q$

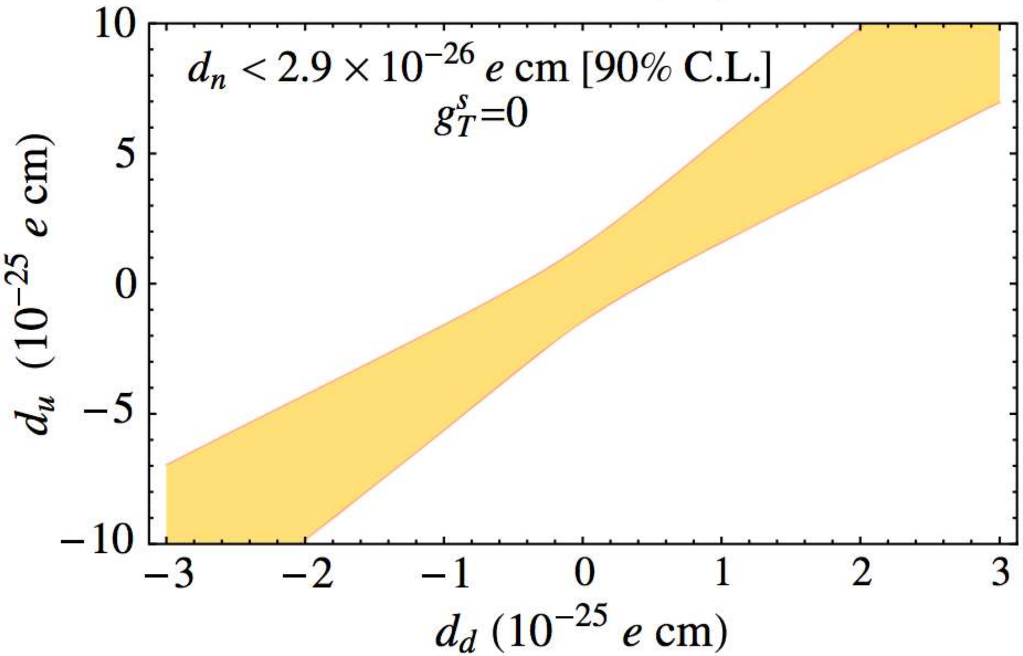
- Tensor charges

$$g_T^u = -0.23(3)$$
$$g_T^d = \ \ 0.79(7)$$
$$g_T^s = \ \ 0.008(9)$$

N_f=2+1+1 Clover-on-HISQ
a → 0, m_π → 135MeV



- Constraints on d_q using
$|d_n| < 2.9 \times 10^{-26}\, e{\cdot}\mathrm{cm}$
$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s$$



Bhattacharya, et al., PRL (2015)

Quark Chromo EDM (cEDM)

$$-\frac{i}{2}\sum_{q=u,d,s}\tilde{d}_q g_s \overline{q}(\sigma\cdot G)\gamma_5 q$$

Quark Chromo EDM

- Calculate d_N in presence of CP violating cEDM term

$$S = S_{QCD} + S_{cEDM}$$

$$S_{cEDM} = -\frac{i}{2}\int d^4x \;\; \tilde{d}_q g_s \overline{q}(\sigma \cdot G)\gamma_5 q$$

- Three methods explored
  - Expansion in $\tilde{d}_q$
  - External electric field method
  - Schwinger source method

Expansion in $\tilde{d}_q$

$$\left\langle N V_\mu \bar{N} \right\rangle_{CPV} = \left\langle N V_\mu \bar{N} \right\rangle + \tilde{d}_q \left\langle N V_\mu \bar{N} \cdot \sum_x O_{\text{cEDM}}(x) \right\rangle + O(\tilde{d}_q^2)$$

$$O_{\text{cEDM}} = \frac{i}{2} g_s \bar{q} (\sigma \cdot G) \gamma_5 q$$

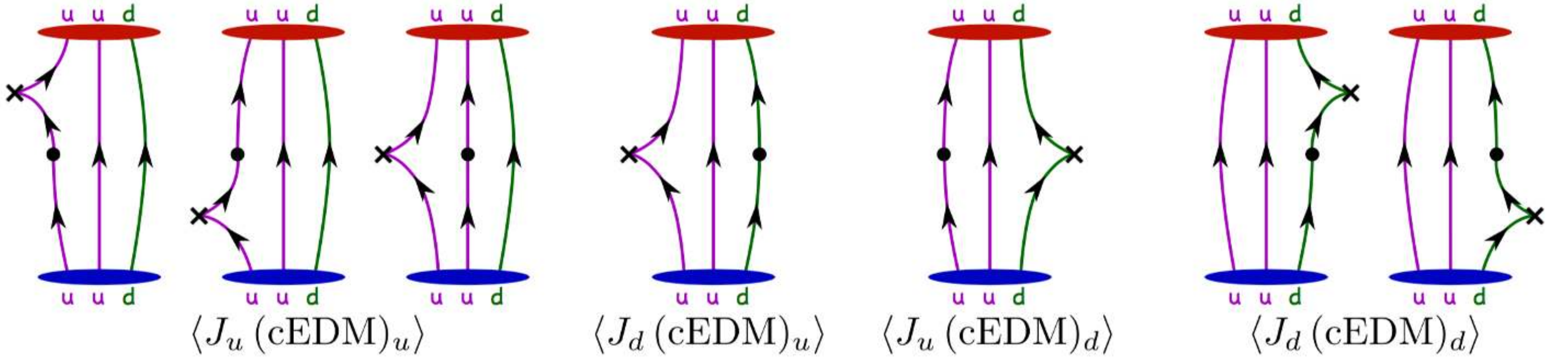
- d_n is extracted from the form-factor F_3

$$\left\langle N V_\mu \bar{N} \right\rangle_{\text{CPV}} = \bar{u} \left[F_1(q^2) \gamma_\mu + i \frac{F_2(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu - \frac{F_3(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu \gamma_5 \right] u$$

- Needs calculation of **four-point correlator**

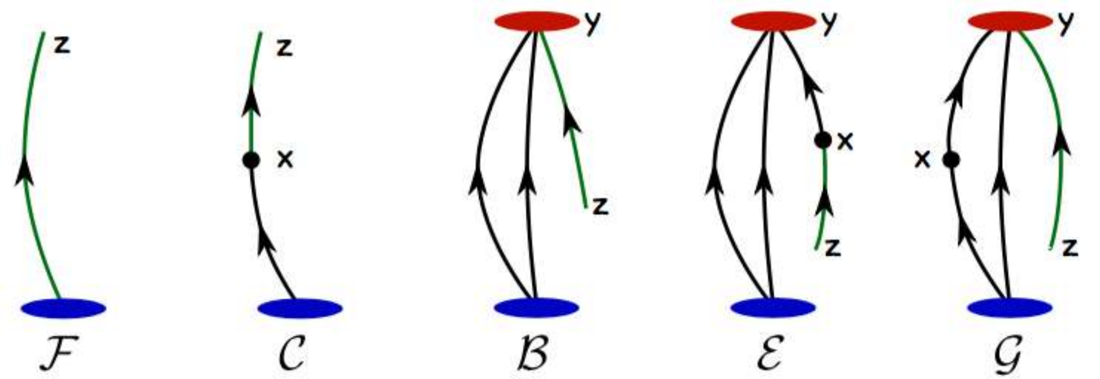
$$\boxed{\left\langle N V_\mu \bar{N} \sum_x O_{\text{cEDM}}(x) \right\rangle}$$

Expansion in $\tilde{d}_q$



Connected Diagrams

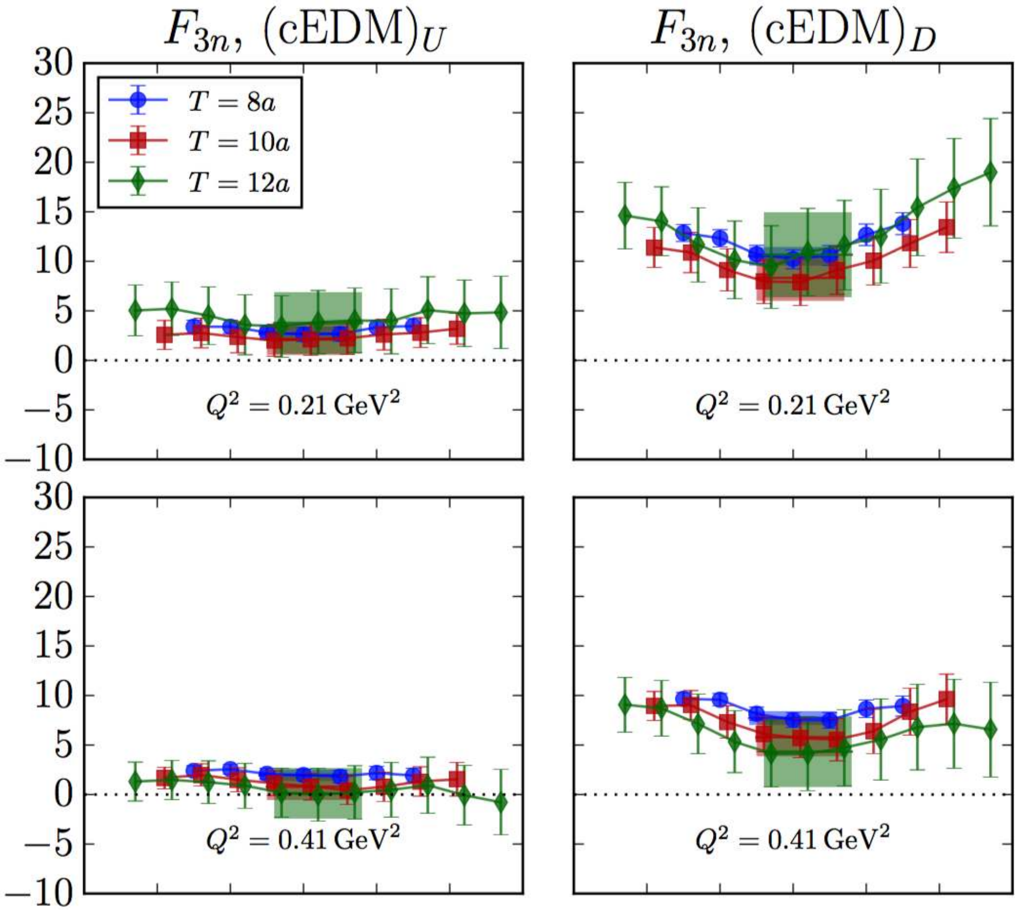
Propagators Needed

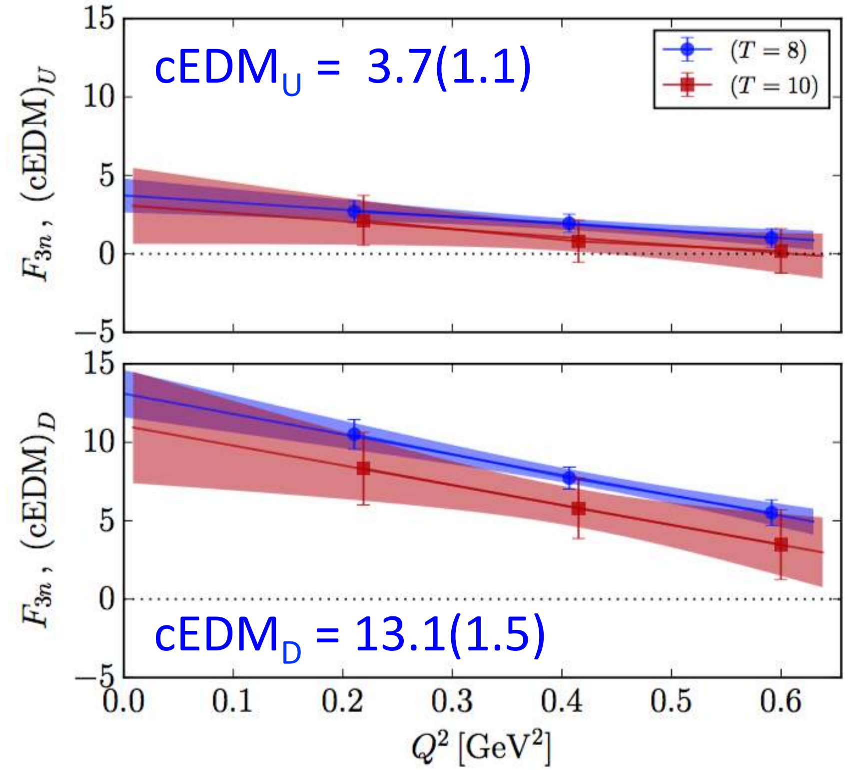


- Four-point correlator is evaluated using Regular and backward props (*F*, *B*), cEDM sequential prop (*C*) and doubly-sequential props ($\mathcal{E}$, $\mathcal{G}$)

Abramczyk, et al., arXiv:1701.07792

Expansion in $\tilde{d}_q$





- DWF
- a = 0.11fm
- m_π=340 MeV

[Syritsyn@ Wed 9:00]

Abramczyk, et al., arXiv:1701.07792

External Electric Field Method

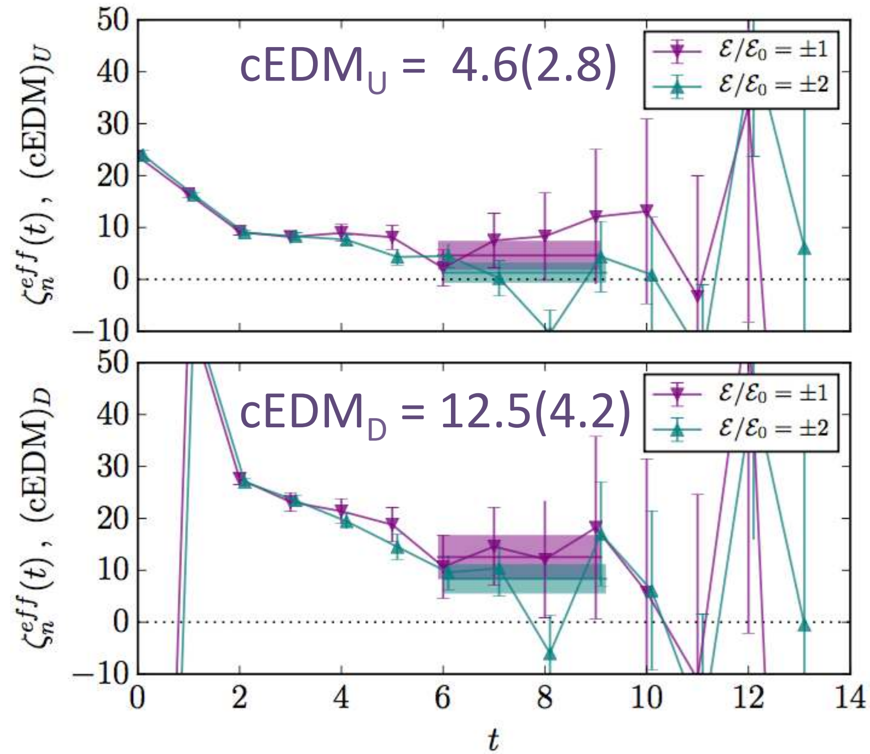
- In the presence of uniform electric field [redacted]

$$E^{\text{cEDM}}_{\vec{S}} - E^{\text{cEDM}}_{-\vec{S}} \approx 2 d_N \vec{S} \cdot$$ [redacted]

- Neutron correlator with CEDM term at leading order

$\left\langle N\bar{N} \right\rangle_{cEDM}$ ([redacted]$,t) = \left\langle N(t)\bar{N}(0) \right\rangle$[redacted] $+ \tilde{d}_q \left\langle N(t)\bar{N}(0)\, O_{\text{cEDM}} \right\rangle$[redacted] $+ O\left(\tilde{d}_q^2\right)$

- DWF
- a = 0.11fm
- m_π=340 MeV
- $\mathcal{E}_0$ = 0.039 GeV^2



[Syritsyn@ Wed 9:00]

Abramczyk, et al., arXiv:1701.07792

Schwinger Source Method

- Quark chromo EDM operator is a quark bilinear

$$i\bar{q}(\sigma \cdot G)\gamma_5 q$$

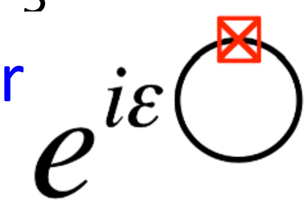
- Include cEDM term in valence quark propagators by changing Dirac op inversion routine

$$D_{\text{clov}} \to D_{\text{clov}} + i\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}$$

  Equivalently,

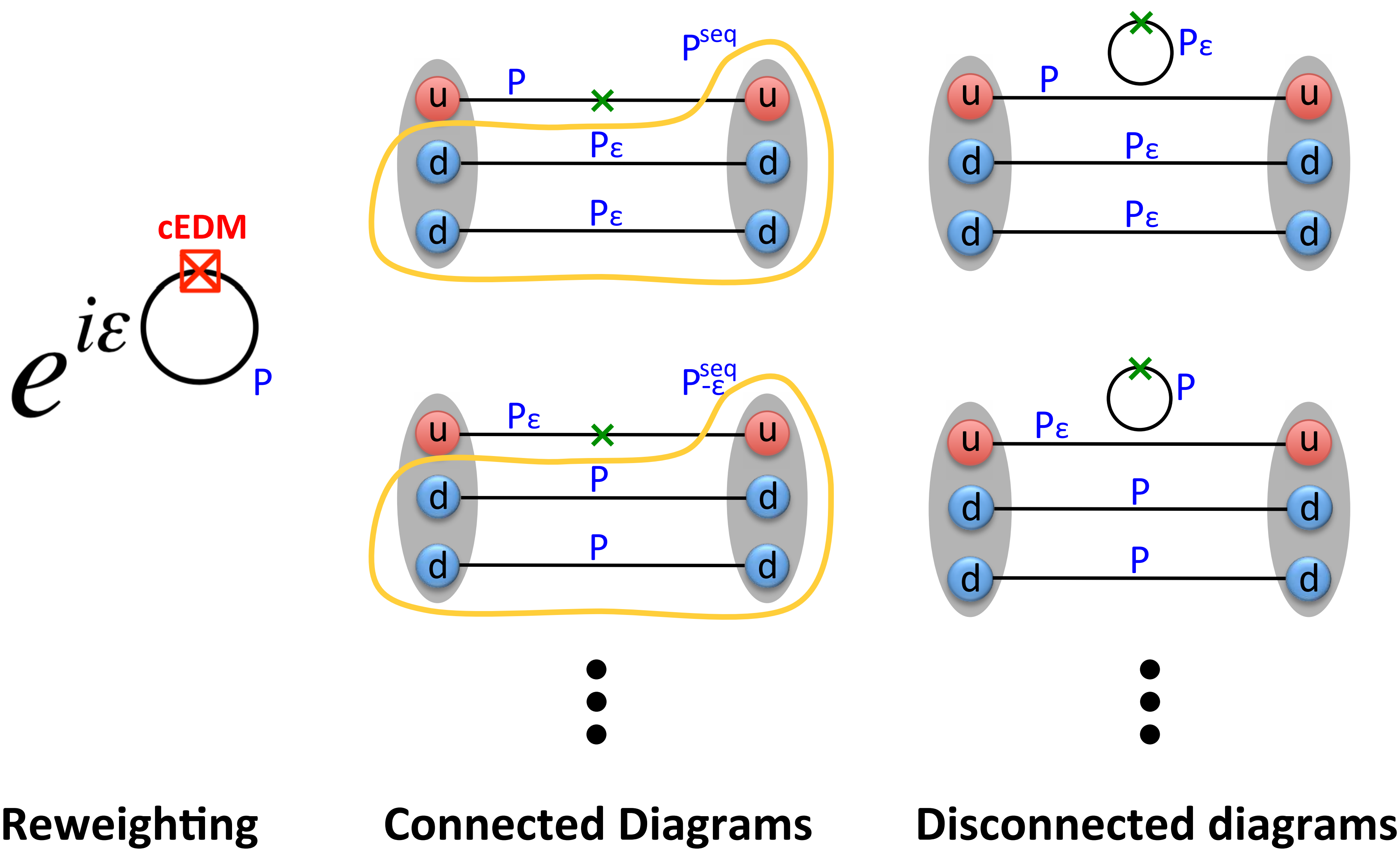
$$c_{sw}\sigma^{\mu\nu}G_{\mu\nu} \to \sigma^{\mu\nu}(c_{sw} + i\varepsilon\gamma_5)G_{\mu\nu}$$

- No four-point correlators; d_N extracted from F_3
- Fermion determinant gives reweighting factor



$$\frac{\det\left(D_{\text{clov}} + i\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}\right)}{\det\left(D_{\text{clov}}\right)} \approx \exp\left[i\varepsilon\,\mathrm{Tr}\left(\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}\, D_{\text{clov}}^{-1}\right)\right]$$

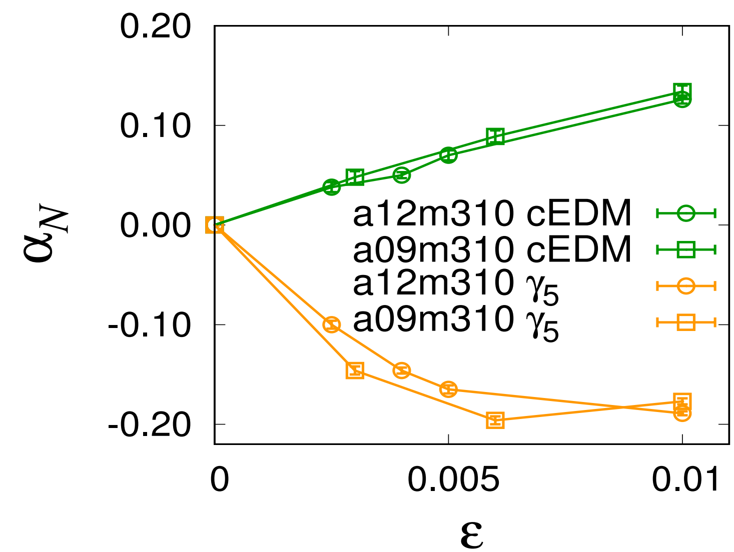
Schwinger Source Method



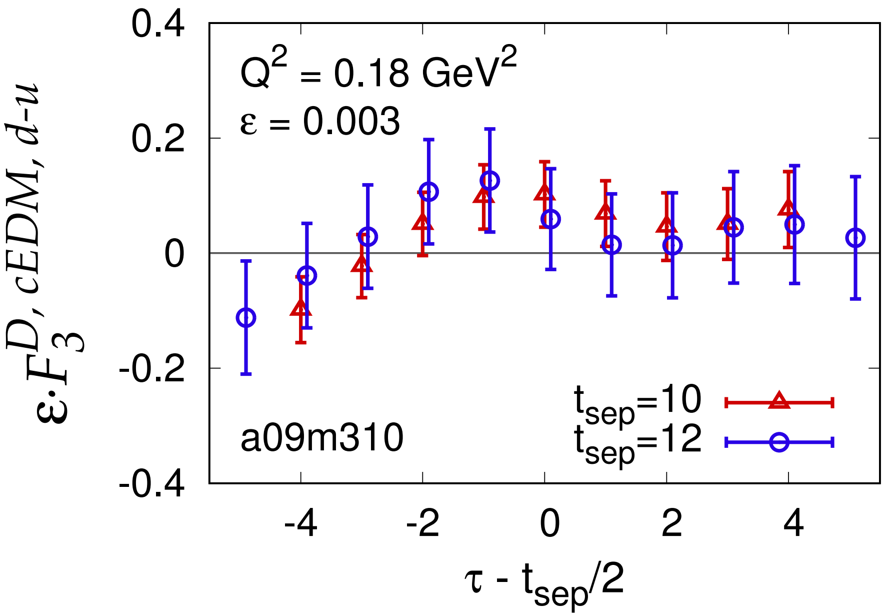
Schwinger Source Method

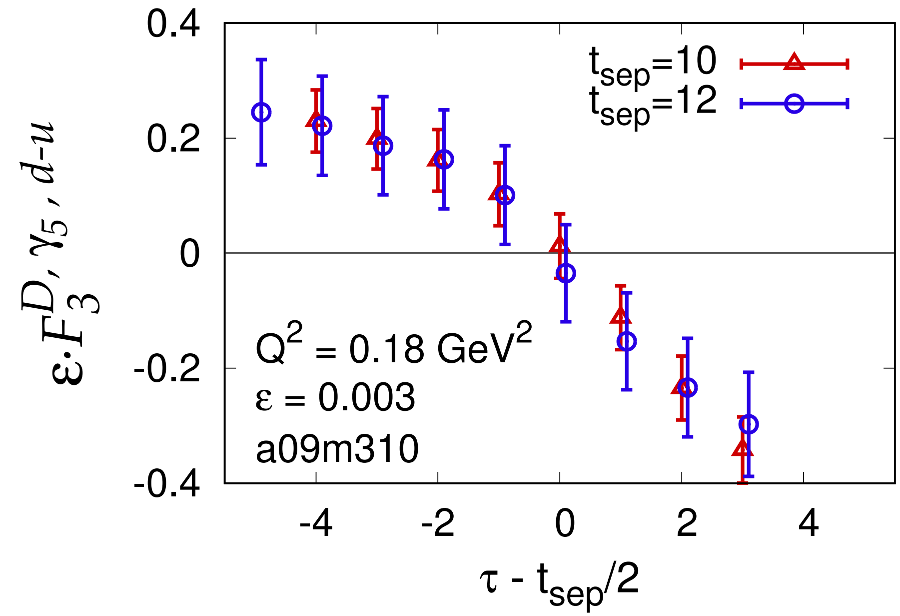
Bhattacharya, et al., arXiv:1612.08438

- Calculation performed at small ε so that results are linear in ε
- cEDM mixes with γ_5, so investigated both operators



- Test at *a = 0.09* fm, $m_\pi=310$ MeV





Renormalization

- Renormalization of cEDM operator is studied
  - 1-loop perturbation on twisted-mass fermion
    [Constantinou, et al, 2015]
  - Nonperturbative $\text{RI-}\tilde{\text{S}}\text{MOM}$
    [Bhattacharya, et al, 2015]

- Mixing with lower-dimensional operator

$$O_{\text{cEDM}} = a^2 \bar{q}\sigma^{\mu\nu}\gamma_5 G_{\mu\nu} q$$

$$O_{\text{P}} = \bar{q}\gamma_5 q$$

  - Introduce *1/a*^{2} mixing

Other Related Results

- Weinberg Three-gluon Operator $d_w \frac{g_s}{6} G\tilde{G}G$

[Dragos @ Wed 9:20]

- Lattice QCD spectroscopy for hadronic CPV

Vries, et al., PLB (2017)

- ChPT for Neutron-Antineutron oscillations

[Bijnens@ Thur 16:00]

Summary

- **QCD θ-term**

  Actively being calculated; need better precision

- **Quark EDM**

  $g_T^{u,d}$ given 10% error; better g_T^s precision needed

- **Quark Chromo EDM**

  Exploratory studies started; need to address disconnected diagrams & renormalization/mixing

- **Weinberg Three-gluon Operator**

  Just started

- **Four-quark Operators**

  Not explored

Acknowledgement: Computing Resources

- USQCD
- Oak Ridge Leadership Computing Facility at the Oak Ridge National Laboratory, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC05-00OR22725.
- National Energy Research Scientific Computing Center, a DOE Office of Science User Facility supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231
- Institutional Computing at the Los Alamos National Laboratory