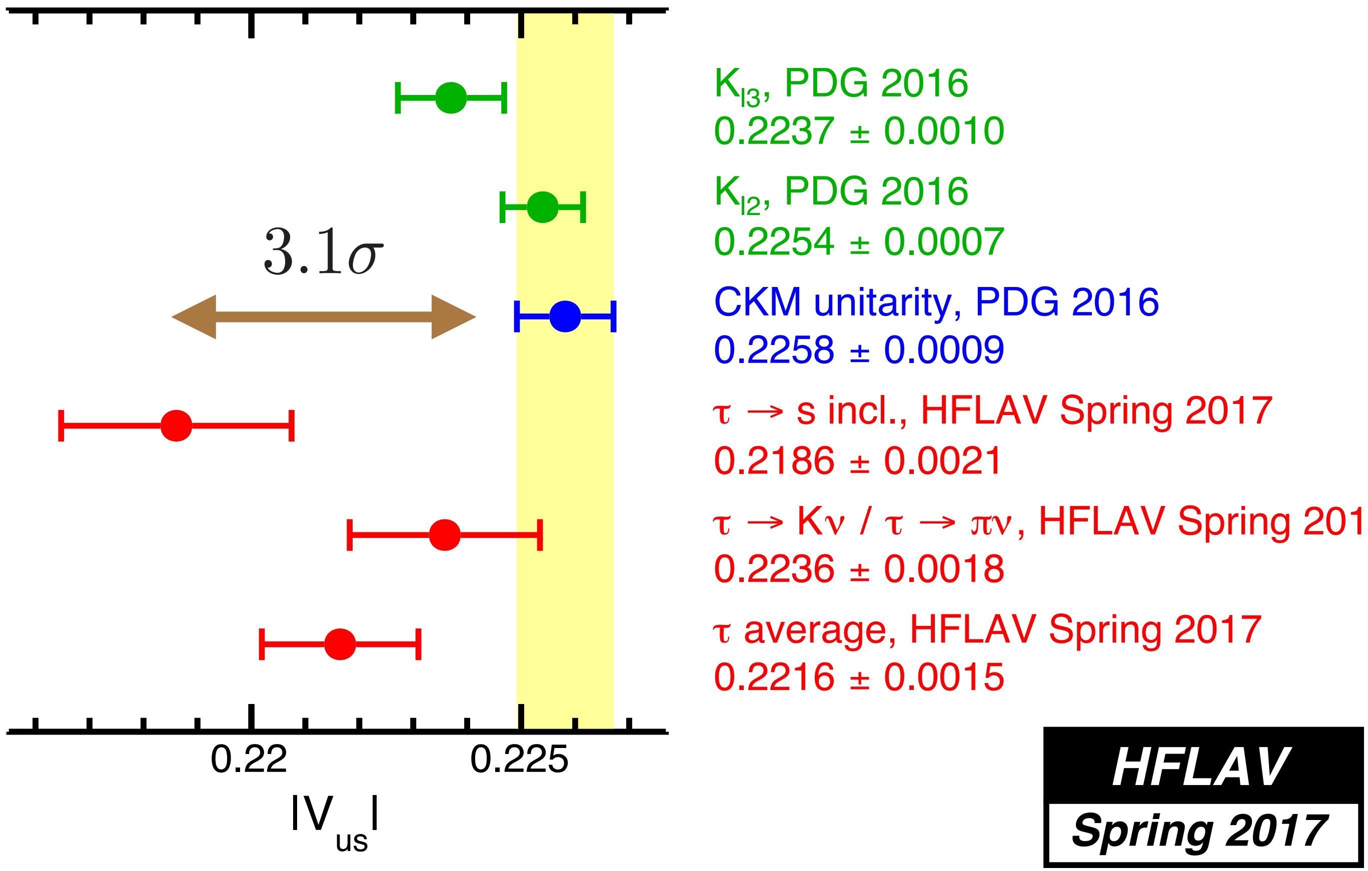
|Vus| determination from inclusive strange tau decay and lattice HVP

Hiroshi Ohki

Nara womens university

In collaboration with P. Boyle, R. J. Hudspith, T. Izubuchi, A. Juettner, C. Lehner, R. Lewis, K. Maltman, A. Portelli, and M. Spraggs (RBC/UKQCD collaboration)

@ Lattice 2017, Granada, Spain, June 23, 2017



- τ result v.s. non-τ result : more than 3 σ deviation : |Vus| puzzle
- new physics effect?
- incl. analysis uses Finite energy sum rule (FESR)
- pQCD and higher order OPE for FESR:

  underestimation of truncation error and/or non-perturbative effects ?

  (c.f. alternative FESR approach, R. Hudspith et. al arXiv:1702.01767)

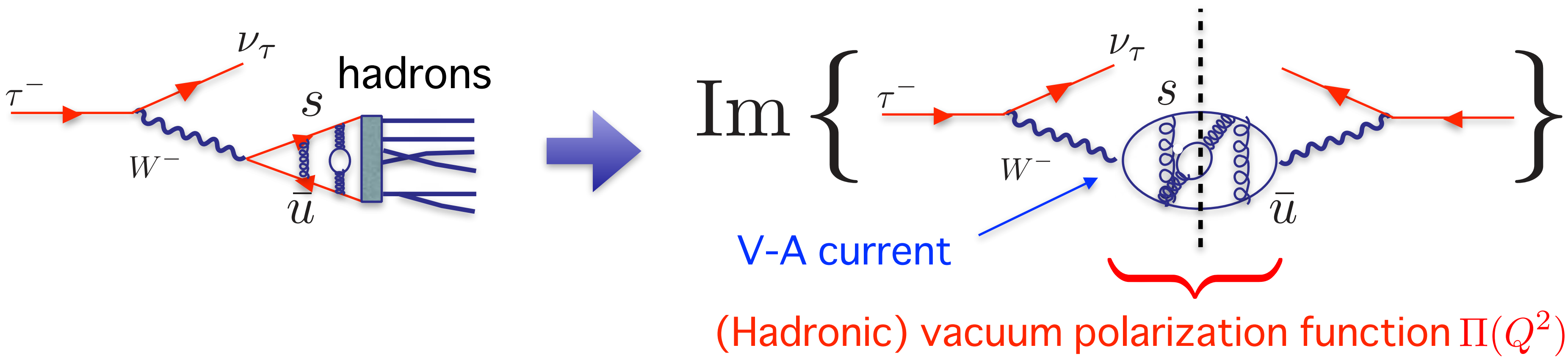
Tau decay experiment

$\tau \rightarrow \nu$ + hadrons decay through V-A current (weak decay)

R ratio(hadron/lepton) for the final states with strangeness -1

$$R_{ij;V/A} \equiv \frac{\Gamma[\tau^- \rightarrow \nu_\tau H_{ij;V/A}(\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu_e}(\gamma)]}$$

From unitarity of S matrix, invariant matrix elements are related to the total scattering cross section σ [Optical theorem]



The spin 0, and 1, hadronic vacuum polarization function for V/A current-current

$$\Pi^{(\mu\nu)}_{ij;V/A}(q^2) \equiv i \int d^4x e^{iqx} \langle 0 | T \left(J^{\mu}_{ij;V/A}(x) J^{\dagger\nu}_{ij;V/A}(0) \right) |0\rangle$$
$$= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^{(1)}_{ij;V/A}(Q^2) + q_\mu q_\nu \Pi^{(0)}_{ij;V/A}(Q^2)$$

Finite Energy Sum Rule (FESR)

[Shifman, Vainshtein, and Zakharov '79]

The finite energy sum rule (FESR)

$$\int_0^{s_0} \omega(s)\rho(s)ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} \omega(s)\Pi(s)ds, \quad (s_0 : \text{ finite energy})$$

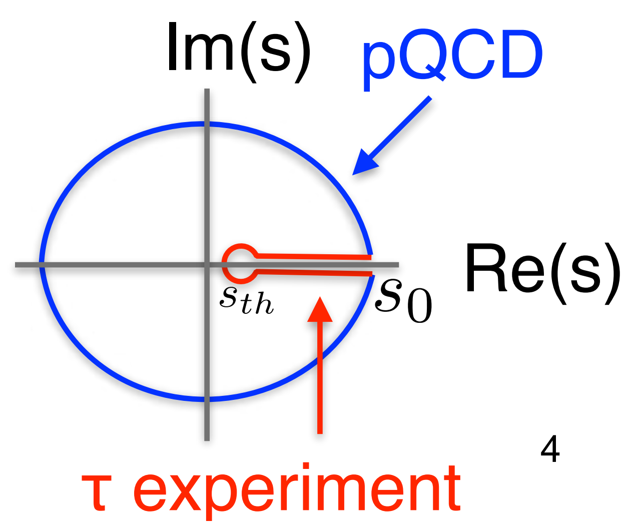
w(s) is an arbitrary regular function such as polynomial in s.

- LHS : spectral function ρ(s) is related to the experimental τ inclusive decays

$$\frac{dR_{us;V/A}}{ds} = \frac{12\pi^2|V_{us}|^2 S_{EW}}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\Pi^1(s)+\mathrm{Im}\Pi^0(s)\right]$$

$$\tilde{\rho}(s) \equiv |V_{us}|^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\Pi^1(s)+\mathrm{Im}\Pi^0(s)\right]$$

- RHS ⋯ Analytic calculation
  with perturbative QCD (pQCD) and OPE



|Vus| determination from FESR

- Conventional approach : use τ decay rates and OPEs [E. Gamiz, et al. JHEP 03 (2003) 060]

$$R^{\omega}_{ij;V/A}(s_0) \sim \int_{s_{th}}^{m_\tau^2} ds \frac{dR_{ij;V/A}}{ds} \frac{\omega(s/s_0)}{\omega_\tau(s/m_\tau^2)}$$

A flavor breaking combination with |Vud| input

$$\delta R \equiv \frac{R_{NS}}{|V_{ud}|^2} - \frac{R_S}{|V_{us}|^2}$$

$$|V_{us}| = \sqrt{\frac{R^{\omega}_{us;V+A}(s_0)}{\frac{R^{\omega}_{ud;V+A}(s_0)}{|V_{ud}|^2} - [\delta R^{\omega}(s_0)]^{OPE}}}$$

Use of perturbative OPE with D > 2 with vacuum saturation approximation

$$|V_{us}| = 0.2186(21)$$ 3.1 σ from SM unitarity [HFLAG, spring 2017]

- Alternative FESR approach: [R. Hudspith et al., 1702.01767] (HLMZ 17)

$$|V_{us}| = \begin{cases} 0.2204(23) \\ 0.2229(22) \end{cases}$$

2.1 σ (with HFAG16 $\tau \to K\pi$ exp. input)

1.1 σ (with alternative $\tau \to K\pi$ normalization) [M. Antonelli, et al. 2013])

No assumption on higher order OPE, lattice HVPs used to fit the high-dim. OPE.

Systematic study of the weight function (s_0) dependence :

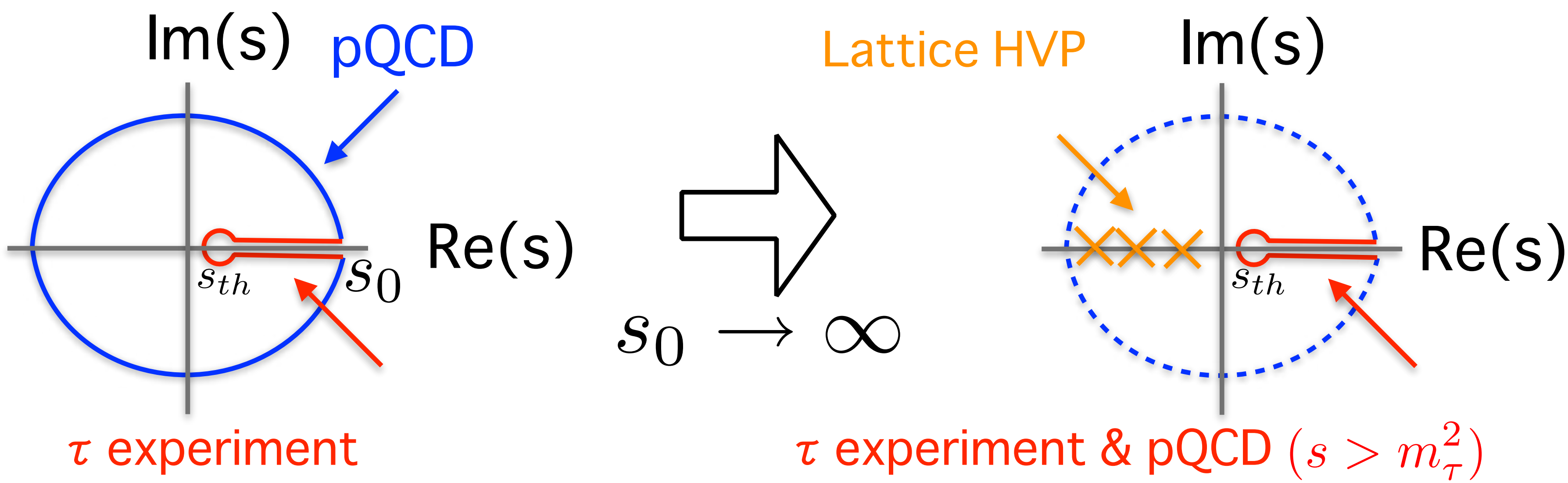
s_0-stable |Vus| result, shifting up, but larger error dominated by high-s experiments.

Our strategy

If we have a reliable estimate for Π(s) in Euclidean (space-like) points, $s = -Q_i^2 < 0$
we could extend the FESR with weight function w(s) to have N poles there,

$$\int_0^{\infty} \rho(s)\omega(s)ds = \sum_k^N \mathrm{Res}\left(\Pi(-Q_k^2)\omega(-Q_k^2)\right)$$

(generalized dispersion relation)



Advantages of weight function with poles

- Our choice of weight function

$$\omega(s) = \prod_k^N \frac{1}{(s+Q_k^2)} = \sum_k a_k \frac{1}{s+Q_k^2}, \qquad \text{(N: \# of poles)}$$

$$\sum_k (Q_k)^M a_k = 0, \quad (M = 0, 1, \cdots, N-2)$$

For N ≧ 3, the circle integral at |s|=∞ vanishes,

and the residue constraints automatically subtracts unphysical $\Pi^{(0,1)}(0)$

For experimental data, $\omega(s) \sim \frac{1}{s^n},\ n \geq 3$ can suppress

- larger error parts from higher multi hadron final states at larger $s < m_\tau^2$
- uncertainties from pQCD & OPE, and quark hadron duality violations at $s > m_\tau^2$

For lattice HVPs,

- Q_k^2 should be not too small to avoid finite size effect, and not too much larger to avoid large discretization error.

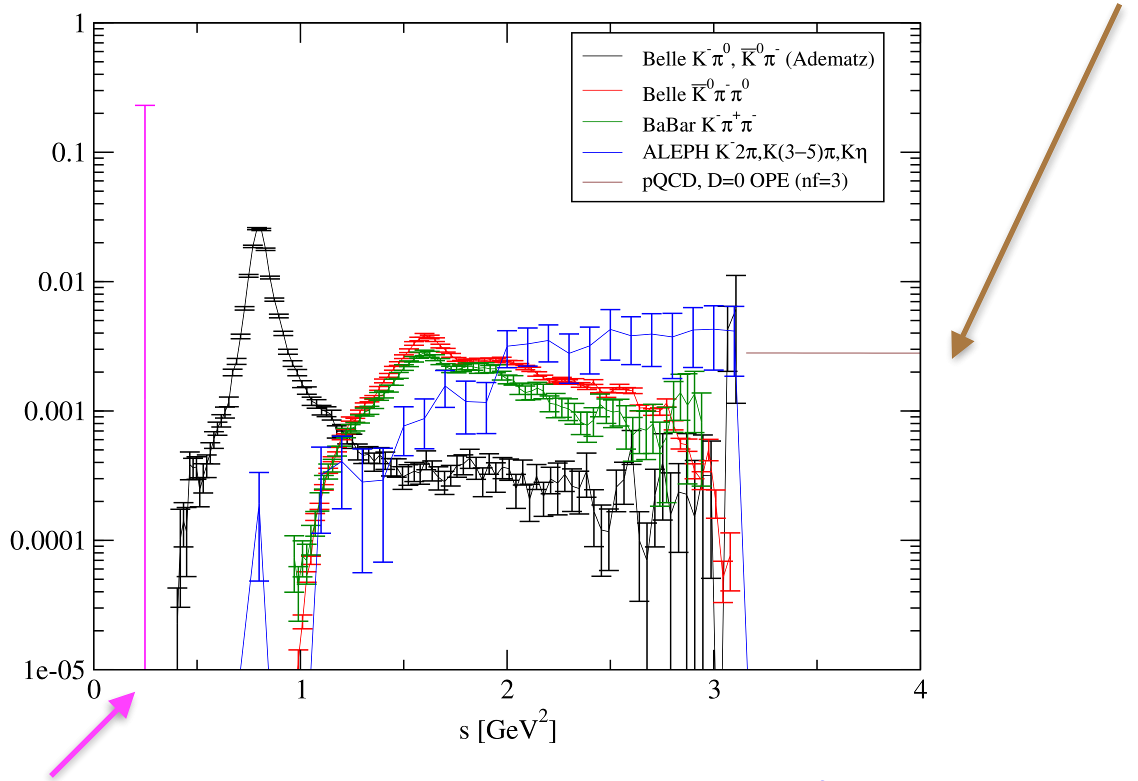
τ inclusive decay experiments

$$\tilde{\rho}(s) \equiv |V_{us}|^2 \left[\left(1 + 2\frac{s}{m_\tau^2} \right) \mathrm{Im}\Pi^1(s) + \mathrm{Im}\Pi^0(s) \right]$$

To compare with experiments,

a conventional value of |Vus|=0.2253 is used

$\tilde{\rho}(s)$



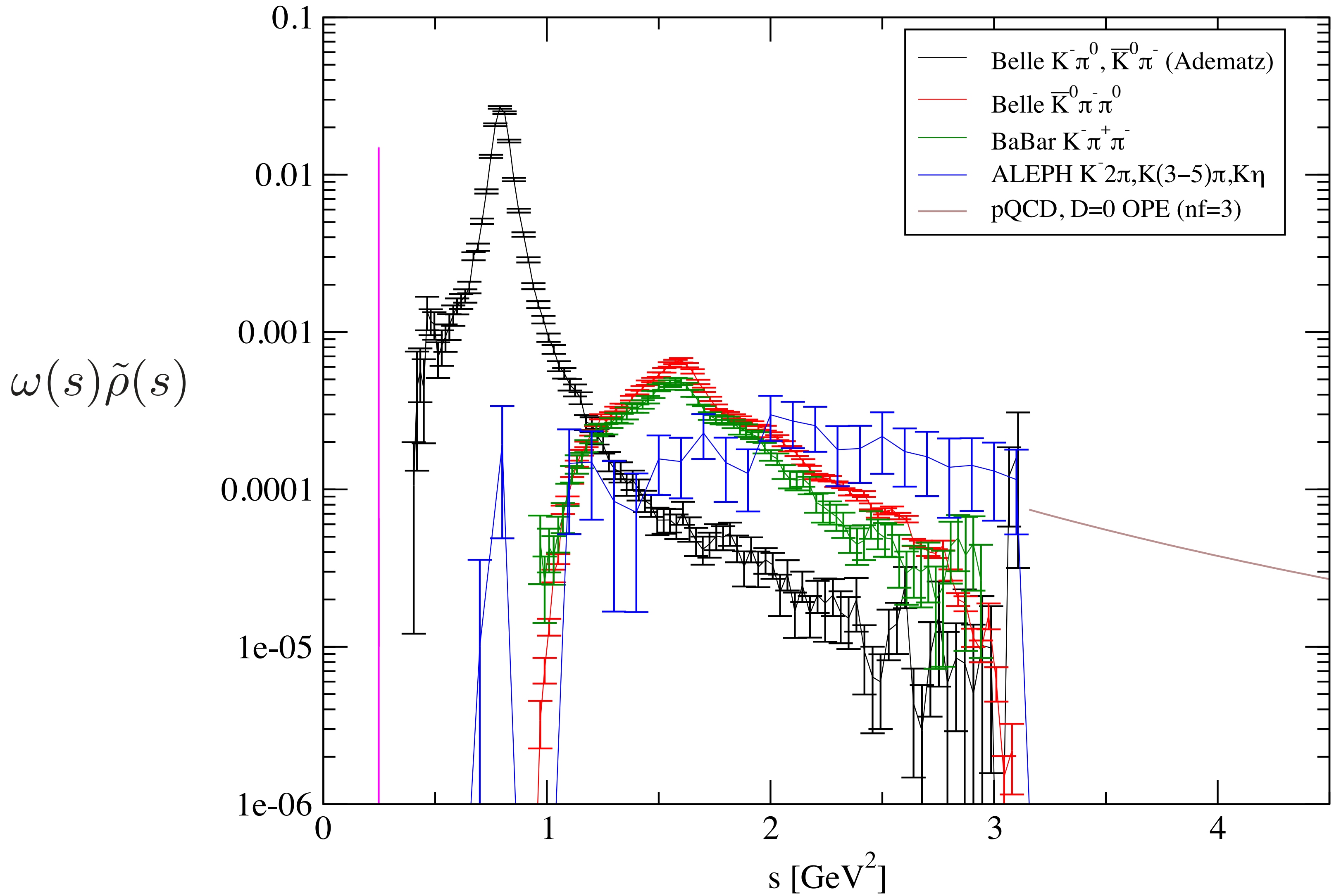
For K pole, we assume a delta function form $\gamma_K \omega(m_K^2)$

$\gamma_K \sim 2|V_{us}|^2 f_K^2$ obtained from either experimental value of K→μ or τ→k decay width.

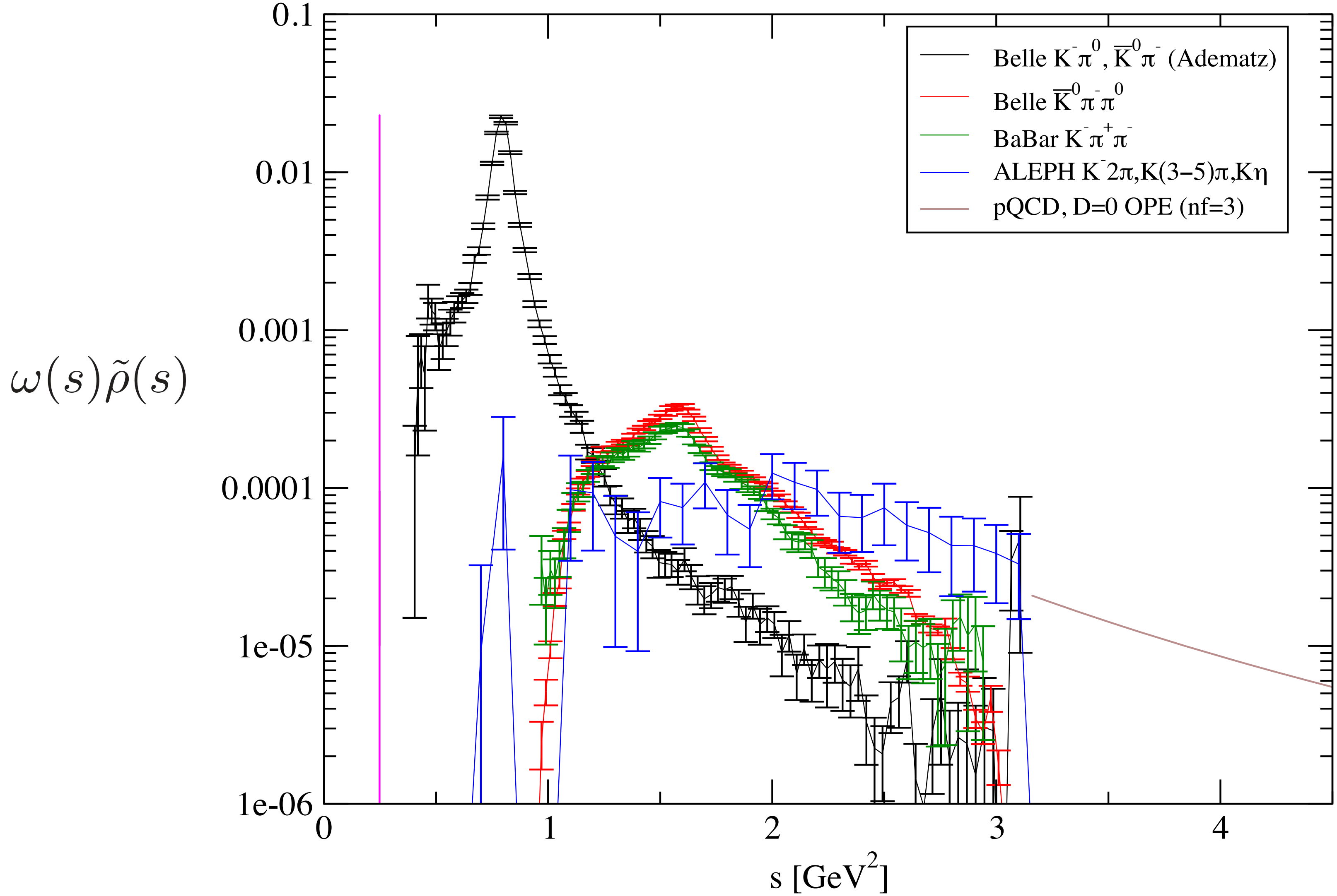
$$\gamma_K[\tau \to K\nu_\tau] = 0.0012061(167)_{exp}(13)_{IB} \ \text{[HFAG16]}$$

$$\gamma_K[K_{\mu 2}] = 0.0012347(\ \ 29)_{exp}(22)_{IB} \ \text{[PDG16]}$$

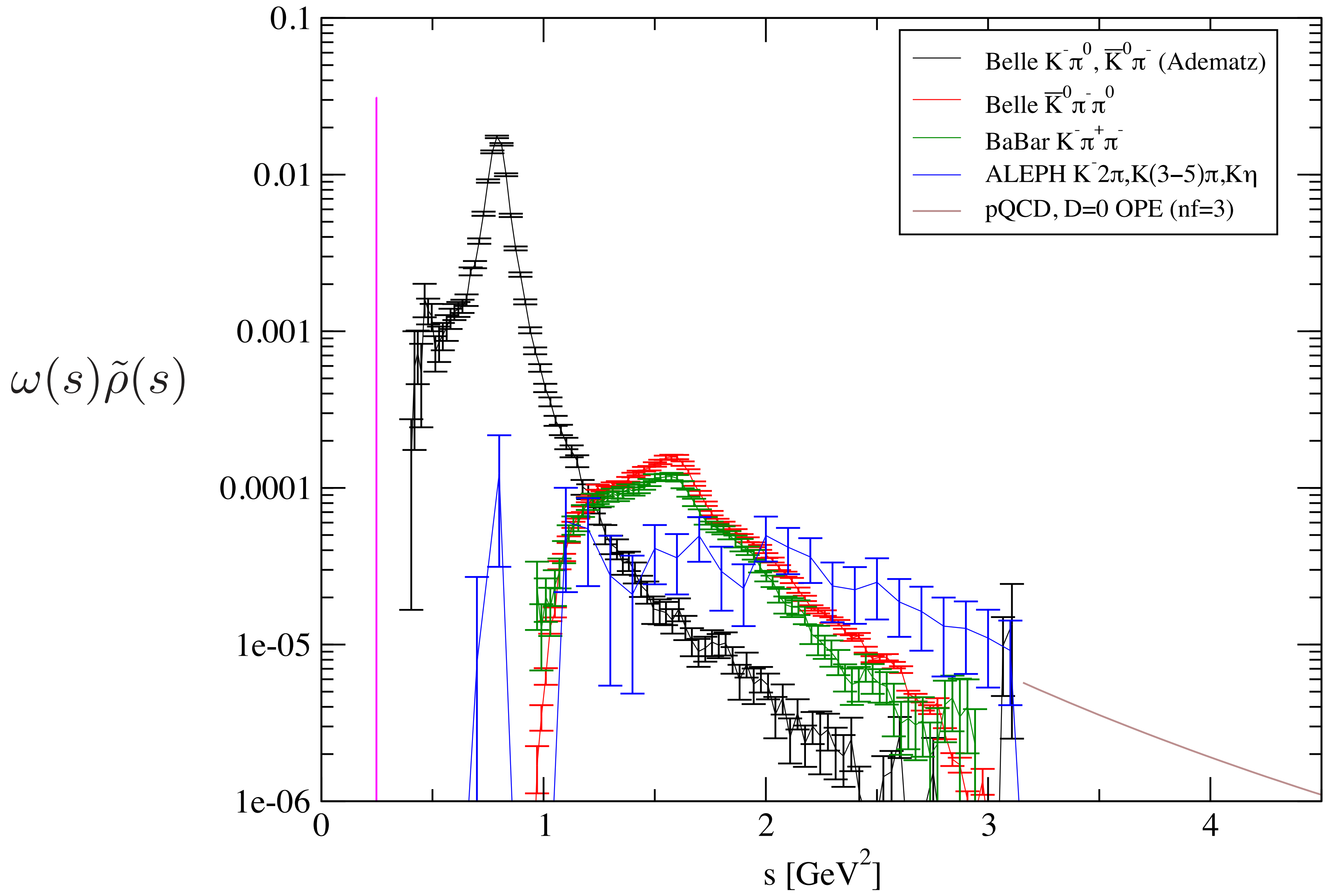
- example: N=3, $\{Q_1^2, Q_2^2, Q_3^2\} = \{0.1, 0.2, 0.3\}$ [GeV^2]



• example: N=4, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2\} = \{0.1, 0.2, 0.3, 0.4\}$ [GeV^2]



• example: N=5, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2, Q_5^2\} = \{0.1, 0.2, 0.3, 0.4, 0.5\}$ $[\mathrm{GeV}^2]$



Lattice calculation

Lattice QCD ensemble and parameters

2+1 flavor lattice chiral quark (domain wall fermion) gauge ensemble generated by RBC-UKQCD

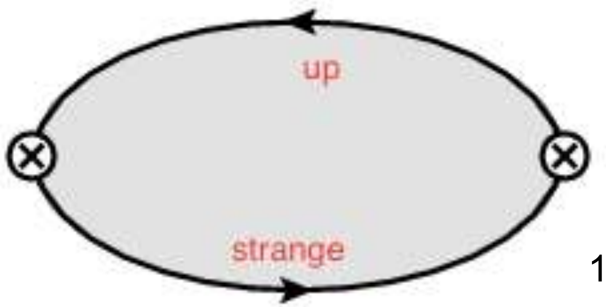
Vol.	a^{-1}[GeV]	m_π[GeV]	m_K[GeV]	stat.
$48^3 \times 96$	1.730(4)	0.139	0.499	88
		0.135	$0.496^\dagger$	5 PQ-correction (88)
$64^3 \times 128$	2.359(7)	0.139	0.508	80
		0.135	$0.496^\dagger$	5 PQ-correction (80)

- Lattice (axial) vector 2pt at near the physical quark mass, L=5 fm.

  Two lattice spacings for continuum extrapolation, a→0.
- PQ-correction: partially quench (PQ) corrected HVP data at the physical point (†)
- We reuse HVP data used for lattice (g-2)μ calculations.

  (No need for large additional computational effort)

$$\Pi^{\mu\nu}_{ij;V/A}(q^2) = i \int d^4x e^{iqx} \langle 0 | T J^\mu_{ij;V/A}(x) J^{\dagger\mu}_{ij;V/A}(0) | 0 \rangle$$
$$= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^{(1)}_{ij;V/A}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij;V/A}$$



A systematic study of weight function dependence

$$\omega(s) = \prod_{k}^{N} \frac{1}{(s+Q_k^2)}, \quad (Q_k^2 > 0)$$

- C (center value of weights),
- Δ (separation of the pole position),
- N (the number of the poles).

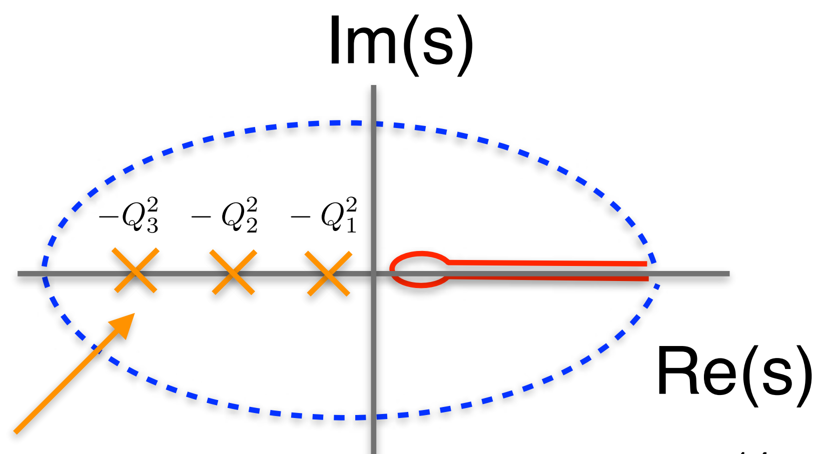
$$\{Q_1^2, Q_2^2, \cdots, Q_N^2\} = \{C-(N/2+1)\Delta, \cdots, C-\Delta, C, C+\Delta, \cdots, C+(N/2+1)\Delta\}$$

$$C = \frac{Q_1^2 + Q_2^2 + \cdots + Q_N^2}{N}$$

$$\Delta = 0.2/(N-1)[\mathrm{GeV}^2]$$

to ensure poles spanning the same Q^2 range

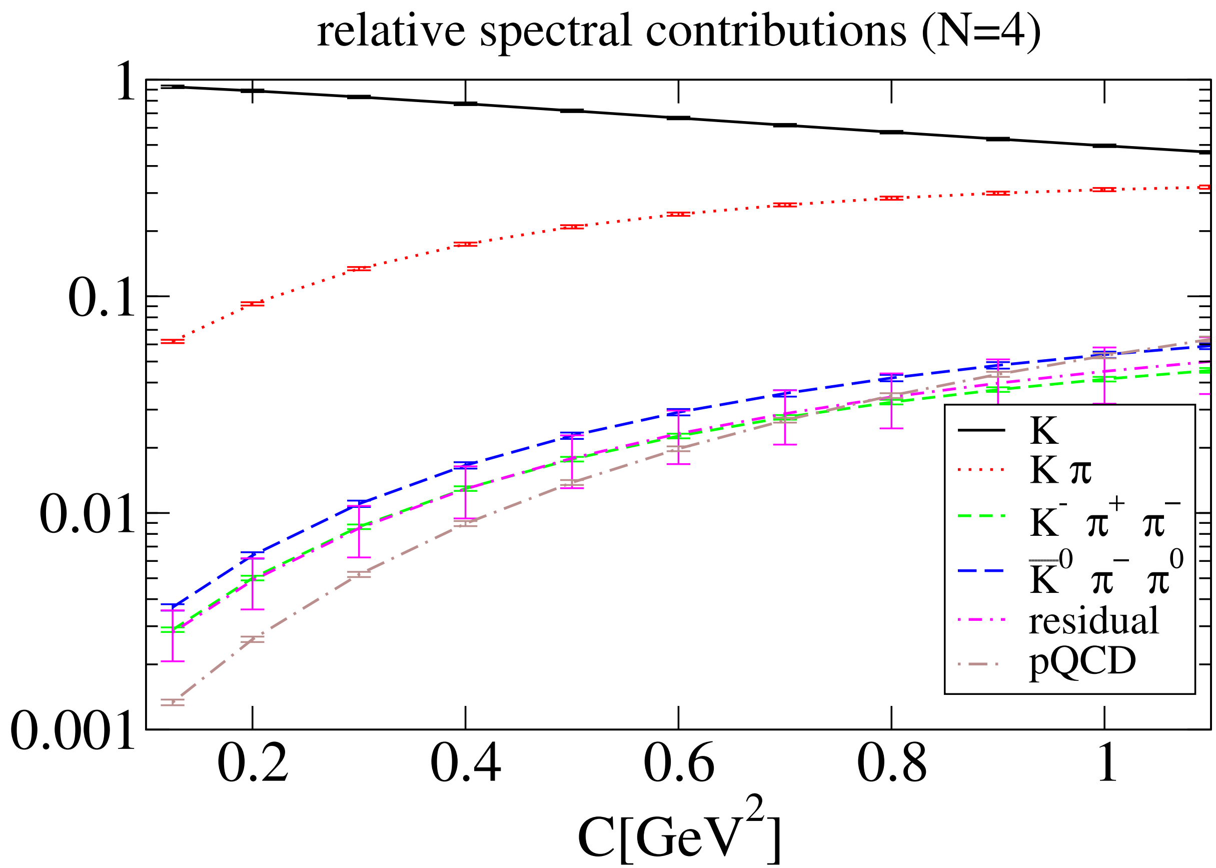
(Results are insensitive to modest change of Δ)



pole positions (N=3 case)

Tuning of the "inclusiveness" of experimental spectral integral

$$(N = 4,\ \Delta = 0.067\ \mathrm{GeV}^2)$$

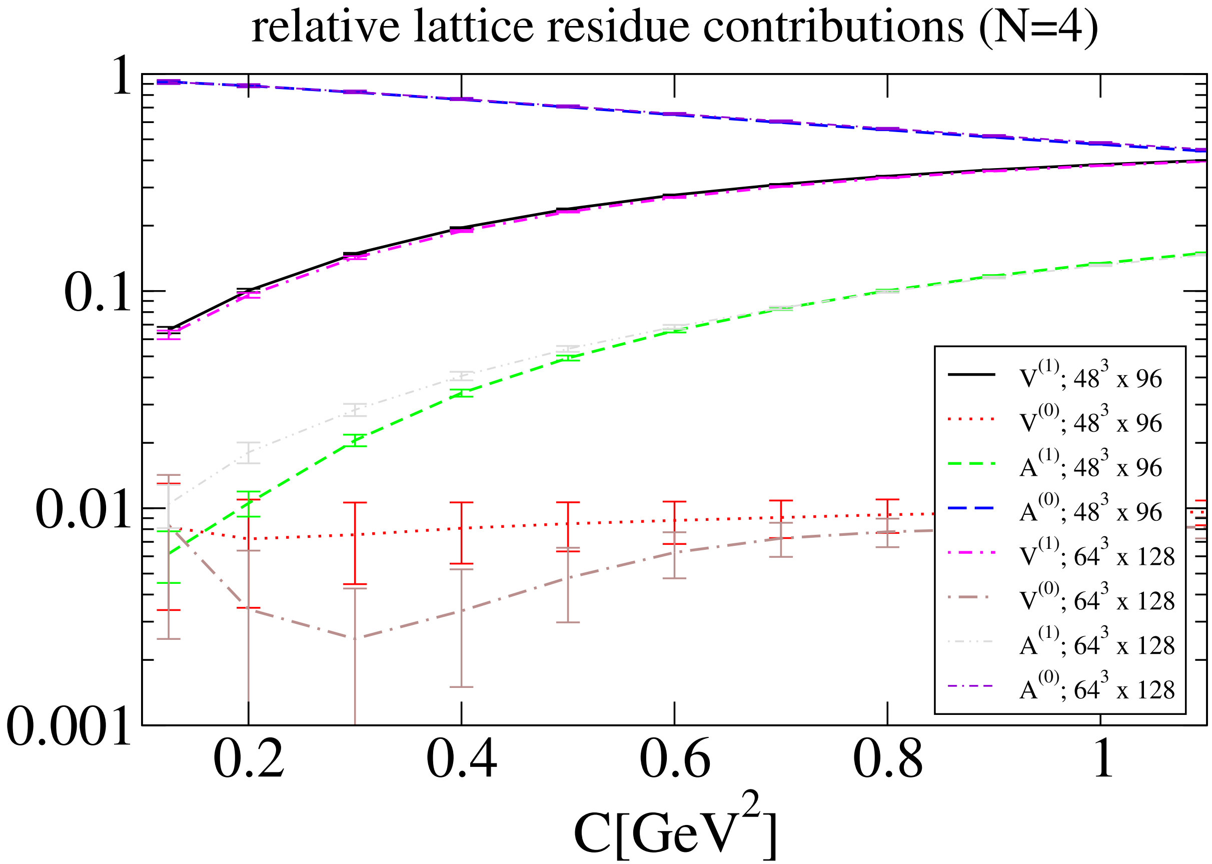


K, Kπ dominates spectral integrals,

high multiplicity modes and pQCD $(s > m_\tau^2)$ strongly suppressed

Lattice residue contributions

$$(N = 4,\ \Delta = 0.067\ \mathrm{GeV}^2)$$



Ratios of each contribution of V/A with spin=0, 1 to the total residue (Lattice)

$A^{(0)}$ dominance (K-pole)

|Vus| from inclusive decays

- 4 channels: Vector or Axial (V or A), spin 0 and 1
- A0 channel is dominated by K pole.

  → For the K pole contribution we use

  $f_K^{phys} = 0.15551(83)[\mathrm{GeV}]$ [RBC/UKQCD, 2014] instead of $A^{(0)}$
- Other channels :

A1, V1, V0 (& residual A0) → multi hadron states & pQCD ("other")

- We take the continuum limit using the data L=48 and 64

$$V_1 + V_0 + A_1 + A_0 \ : |V_{us}^{V_1+V_0+A_1+A_0}| = \sqrt{\frac{\rho_{exp}^{\mathrm{K-pole}} + \rho_{exp}^{others}}{(f_K^{phys})^2 \omega(m_K^2) + F_{lat}(\Pi_{\mathrm{others}}) - \rho_{\mathrm{pQCD}}}},$$

$$\rho_{exp}^{others} = |V_{us}|^2 \int_{s_{th}}^{m_\tau^2} ds \omega(s) \mathrm{Im}\Pi(\mathrm{s}) \qquad \rho_{pQCD} = \int_{m_\tau^2}^{\infty} ds \omega(s) \Pi_{OPE}(s)$$

$$F_{lat} = \sum_{k=1}^{N} \mathrm{Res}(\omega(-\mathrm{Q_k^2}))\Pi_{\mathrm{lat}}(-\mathrm{Q_k^2})$$

Systematic error estimate

- Higher order (a^4) discretization error for V1+V0+A1+(residual A0)

$$\mathcal{O}(C^2a^4) \sim 0.1Ca^2, \quad (a^{-1} = 2.37[\mathrm{GeV}])$$

Two lattice ensembles yield (less than)10% difference

→ We estimate 10% reduction of O(a^4) relative to O(a^2)

- Finite volume correction

1 loop ChPT analysis of current-current correlation function on finite volume for Kπ channel (V1).

- Isospin breaking effects

s-dependent strong isospin breaking corrected Kπ experimental data used.

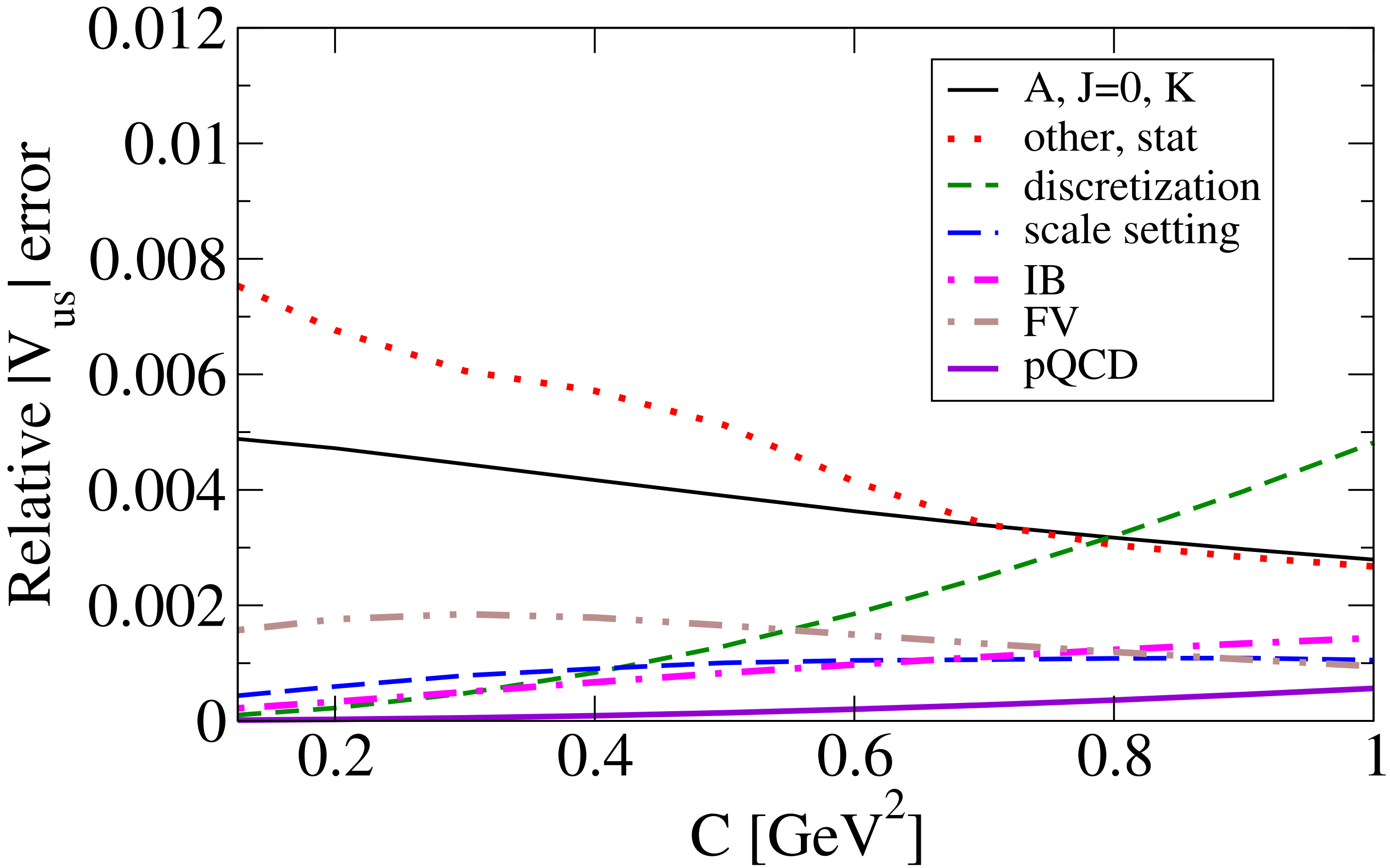
Theory error for dominant Kπ channels: 0.2 % for electromagnetic effects and ~ 1% strong isospin breaking effect on V1. [Ref: Antonelli, et al., JHEP10(2013)070]

- pQCD (OPE) uncertainty

2% for possible quark hadron duality-violation effect

$|V_{us}|$ systematic error of lattice residue contributions

$(N = 4,\ \Delta = 0.067\ \mathrm{GeV}^2)$

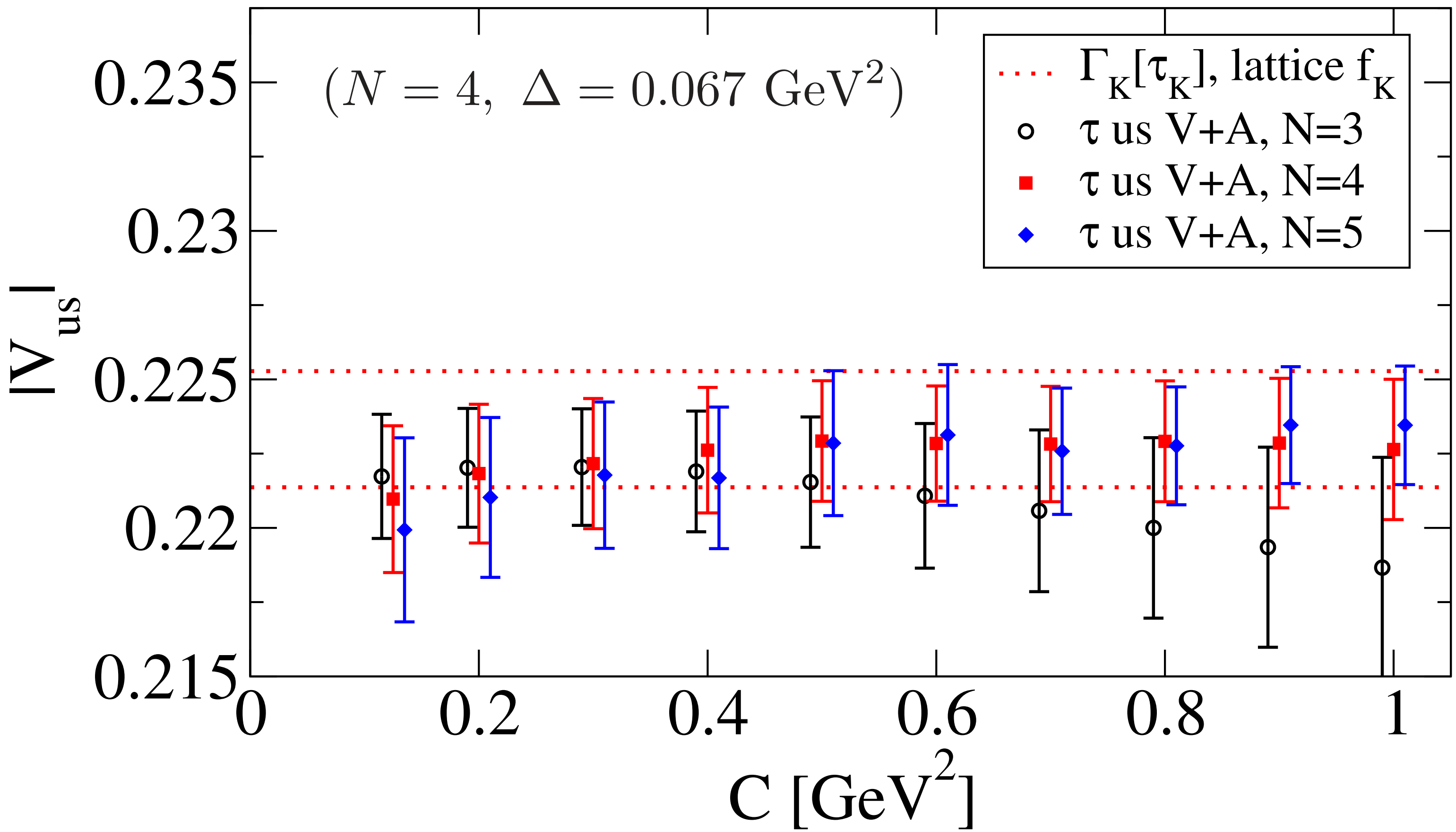


For small C, statistical error dominates.

For large C, discretization error becomes large.

We obtain optimal inclusive determinations around C=0.7.

Lattice Inclusive $|V_{us}|$ determinations

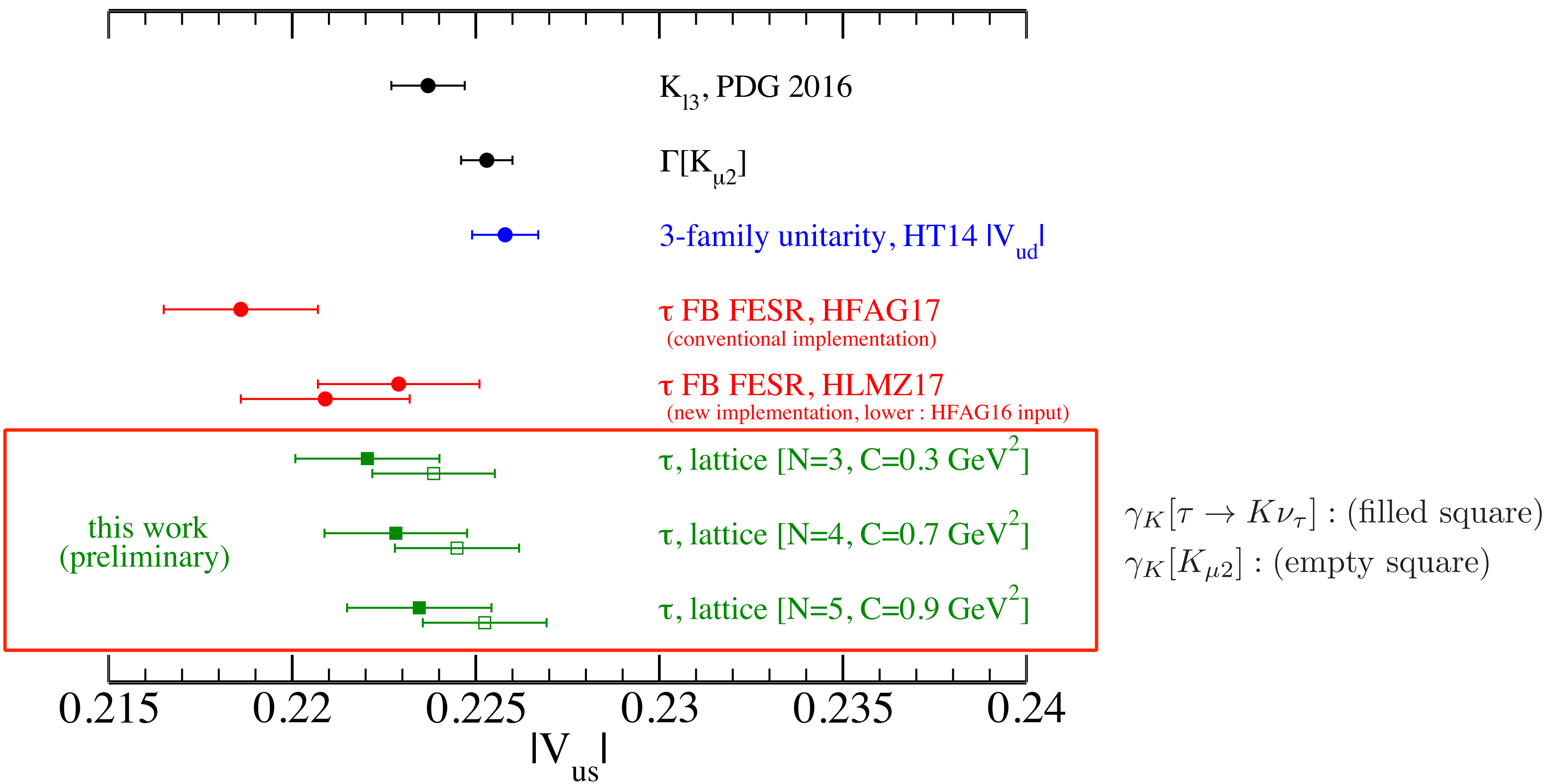


Theory and experimental errors are included.

The result is stable against changes of C and N.

$N = 4, C = 0.7[\text{GeV}^2] : |V_{us}| = 0.2228(15)_{exp}(13)_{th}$ (0.87% total error)

Comparison to $|V_{us}|$ from others



All our results (C<1, N=3, 4, 5) are consistent with each other within 1 σ error, as well as to CKM unitarity.

Approaches to determination of |Vus| from inclusive τ decays

Method	pQCD (OPE)	issues	Precision limit for lVusl
Conventional FESR	higher order OPE: vacuum saturation approximation	inconsistent OPE treatment ([Ref:HLMZ 17]) large contributions from high-s region contribution	3+σ discrepancy from CKM unitarity (uncontrolled QCD systematic errors?)
Alternative FESR [HLMZ 17]	higher order OPE: fit by experimental data, checked with lattice QCD data	large contributions from high-s region	dominant high multiplicity experimental data (residual modes : 25% error to the total contribution) [1.1% total error]
Our method (lattice-based inclusive analysis)	systematically suppressed uncertainties via first principle lattice QCD data		currently lattice and experimental errors are comparable (<1%) pQCD error is negligible. [0.87 % total error]

Summary

- We propose and apply a novel approach to determining $|V_{us}|$ which uses inclusive strange hadronic τ decay data and hadronic vacuum polarization functions (HVPs) computed on the lattice.
- The experimental and lattice data are related through dispersion relations which employ a class of weight functions having poles at space-like momentum.
- This approach is implemented using lattice data at physical quark mass with two lattice spacings generated by the RBC/UKQCD collaboration
- Weight functions are found that strongly suppress contributions from the part of the spectrum with large errors, or no data, at the same time as allowing accurate determinations of the required lattice HVP combinations and controlling the quark-hadron duality violation.
- Our $|V_{us}|$ agree well with K physics results and CKM unitarity expectations.
  (c.f. the $3 + \sigma$ discrepancy of the old conventional finite energy sum rule result).
- Key advantages of the lattice approach over the FESR approach to using the same data are:
  better control of theory systematics (lattice in place of the OPE) and
  better suppression of spectral contributions with large experimental errors.
- Other applications of dispersive analysis + Lattice :
  - Popular applications $(g\text{-}2)\mu$:
    Our lattice data is a byproduct of the lattice measurement of HVP.
  - Others (B-inclusive [c.f. S. Hashimoto Wed 12:20], Nucleon deep inelastic scattering [R.Young, Tue 15:50]

Thank you