Pseudo-PDFs

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WILLIAM & MARY
CHARTERED 1693

arXiv:1706.05373

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Introduction

- Goal: Compute hadron structure properties from QCD
  - Parton distribution functions (PDFs)
- Operator product: Mellin moments are local matrix elements that can be computed in Lattice QCD
  - Power divergent mixing limits us to few moments
- Recently X. Ji suggested an approach for obtaining PDFs from Lattice QCD
- First calculations already available

X. Ji, Phys.Rev.Lett. 110, (2013)

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

H.-W. Lin, J.-W. Chen, S. D. Cohen, and X. Ji, Phys.Rev. D91, 054510 (2015)

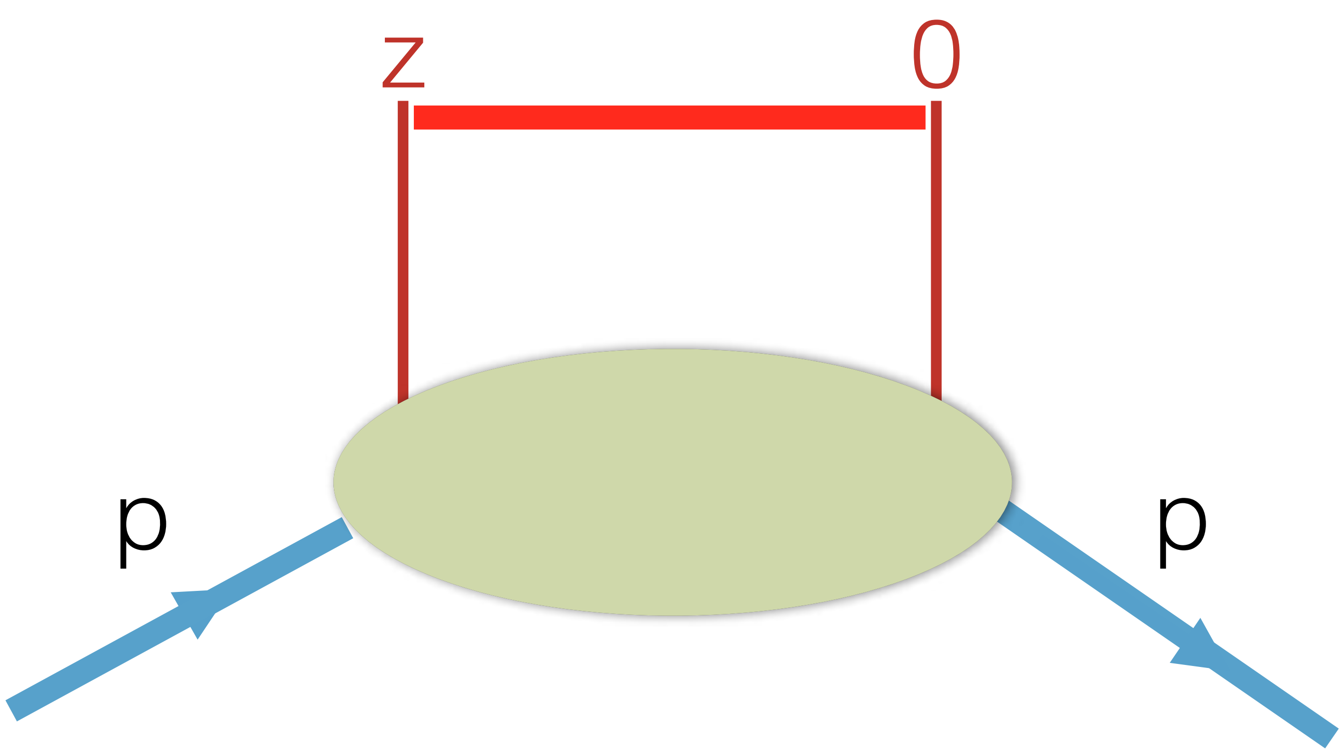
C. Alexandrou, et al, Phys. Rev. D92, 014502 (2015)

- A new approach for obtaining PDFs from LQCD introduced by A. Radyushkin is investigated numerically in this work

Unpolarized PDFs proton:

$$\mathcal{M}^{\alpha}(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^{\alpha}\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\hat{E}(0,z;A) = \mathcal{P}\exp\left[-ig\int_0^z \mathrm{d}z'_{\mu}\,A^{\mu}_{\alpha}(z')T_{\alpha}\right]$$



Lorentz decomposition:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha\mathcal{M}_p(-(zp),-z^2) + z^\alpha\mathcal{M}_z(-(zp),-z^2)$$

Collinear PDFs: Choose

$$z = (0, z_-, 0)$$

$$p = (p_+, 0, 0)$$

$$\gamma^+$$

$$\mathcal{M}^+(z,p) = 2p^+\mathcal{M}_p(-p_+z_-, 0)$$

Definition of PDF:

$$\mathcal{M}_p(-p_+z_-, 0) = \int_{-1}^{1} dx\, f(x)\, e^{-ixp_+z_-}$$

$\mathcal{M}_p(-pz, -z^2)$ is a Lorentz invariant therefore computable in any frame

$\nu = -zp$ ν is called Ioffe time B. L. Ioffe, Phys. Lett. 30B, 123 (1969)

$$\mathcal{M}_p(\nu, -z^2) \equiv \int_{-1}^{1} dx\, \mathcal{P}(x, -z^2) e^{ix\nu}$$

It can be shown that the domain of x is [-1, 1]

A. Radyushkin Phys.Lett. B767 (2017)

$\mathcal{M}_p(\nu, -z^2)$ at small z^2 is called Ioffe time PDF

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

$$-z^2 \leftrightarrow 1/\mu^2 \qquad \mathcal{P}(x, -z^2) \leftrightarrow f(x, \mu^2)$$

Ji Quasi-PDF

Choose

$$p = (0, 0, 0, p_3)$$

$$z = (0, 0, 0, z_3)$$

$$\gamma^3$$

$$h(z_3, p_3) = \frac{1}{2p_3}\mathcal{M}^3 = \mathcal{M}_p(-z_3 p_3, -z_3^2) + \frac{z^3}{2p_3}\mathcal{M}_z(-z_3 p_3, -z_3^2)$$

$$Q(y, p_3) = \frac{p_3}{2\pi}\int_{-\infty}^{\infty} dz_3 h(z_3, p_3) e^{iyp_3 z_3}$$

$\mathcal{M}^3$ On shell time local matrix element computable in Euclidean space

Briceno *et al* arXiv:1703.06072

A. Radyushkin Phys.Lett. B767 (2017)

$$Q(y,p_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu \left[\mathcal{M}_p(\nu,\nu^2/p_3^2) - \frac{\nu}{2p_3^2}\mathcal{M}_z(\nu,\nu^2/p_3^2)\right] e^{-iy\nu}$$

$$\nu = -p_3 z_3$$

$$\lim_{p_3\to\infty} Q(y,p_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\ e^{-iy\nu}\mathcal{M}_p(\nu,0) = f(y)$$

Light-cone limit $z^2 = 0$ achieved by $\frac{\nu^2}{p_3^2} = 0$

Singularity at $z^2 = 0$

$$Q(y,p_3) = \int_{-1}^{1} \frac{dx}{|x|} Z(\frac{y}{x},\frac{\mu}{p_3}) f(x,\mu) + \mathcal{O}(\frac{\Lambda_{qcd}^2}{p_3^2})$$

X. Xiong, X. Ji, J. H. Zhang, Y. Zhao, Phys. Rev. D 90, no. 1, 014051 (2014)
T. Ishikawa et al. arXiv:1609.02018 (2016)

Improvement to Ji's quasi-PDF

Choose

$$p = (0, 0, 0, p_3)$$

$$z = (0, 0, 0, z_3)$$

$$\gamma^0$$

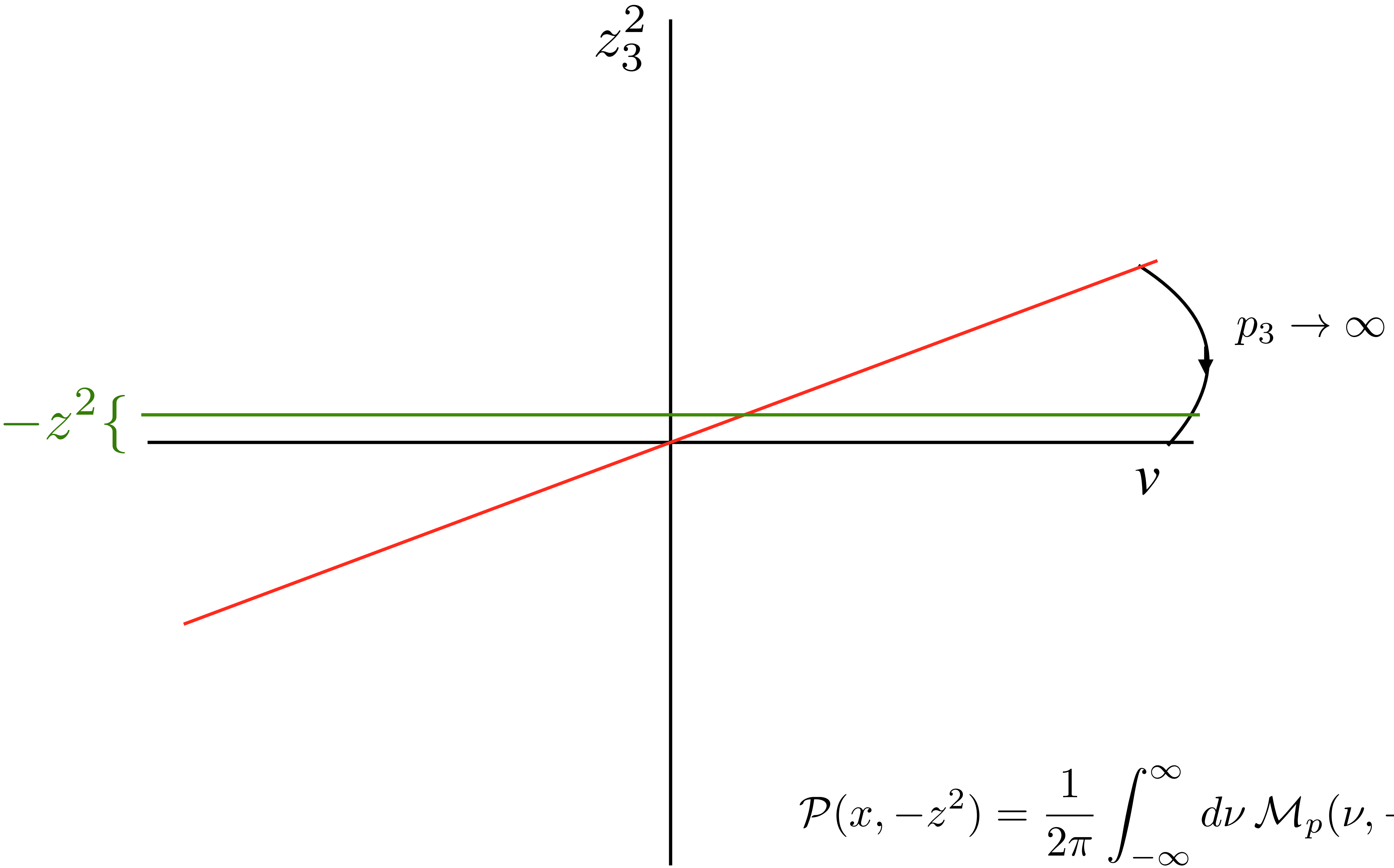
$$h(z_3, p_3) = \frac{1}{2p_3}\mathcal{M}^0 = \mathcal{M}_p(\nu, \nu^2/p_3^2)$$

$$Q(y, p_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu \mathcal{M}_p(\nu, \nu^2/p_3^2) e^{-iy\nu}$$

Suggested also by M. Constantinou GHP2017 based on a mixing argument

Alexandrou *et al* arXiv:1706.00265

$$Q(y, p_3) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \mathcal{M}_p(\nu, \nu^2/p_3^2) e^{-iy\nu}$$



$$\mathcal{P}(x, -z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, \mathcal{M}_p(\nu, -z^2) e^{-ix\nu}$$

Pseudo-PDF

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$$

UV divergences will cancel in this ratio

Denominator is regular at $z_3^2 \to 0$

$\mathcal{M}_p(0,0) = 1$ Isovector matrix element

Scaling violations to the Ioffe time PDF may be suppressed

B. U. Musch, *et al* Phys. Rev. D 83, 094507 (2011)

A. Radyushkin Phys.Lett. B767 (2017)

$$\mathcal{M}_p(\nu, z_3^2)) = \mathcal{Q}(\nu, z_3^2) + \mathcal{O}(z_3^2)$$

$\mathfrak{M}(\nu, z_3^2) = \mathcal{Q}(\nu, z_3^2) + \mathcal{O}(z_3^2)$ with smaller scaling violations

$\mu^2 = (2e^{-\gamma_E}/z_3)^2$ $\mathcal{Q}(\nu, z_3^2) \xrightarrow{\mathrm{F.T.}} f(x, \mu^2)$ $\overline{MS}$

$$\frac{d}{d\ln z_3^2}\,\mathcal{Q}(\nu, z_3^2) = -\frac{\alpha_s}{2\pi}\,C_F\int_0^1 du\,B(u)\,\mathcal{Q}(u\nu, z_3^2)$$

$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

DGLAP kernel in position space

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

$$\mathcal{Q}(\nu, {z'}_3^2) = \mathcal{Q}(\nu, z_3^2) \; - \frac{2}{3}\frac{\alpha_s}{\pi}\ln({z'_3}^2/z_3^2)\int_0^1 du\,B(u)\,\mathcal{Q}(u\nu, z_3^2)$$

Numerical Tests

- Quenched approximation β=6.0

$32^3 \times 64 \quad m_\pi \sim 600 MeV$

- Need series of small z_3
- Need a range of momenta to scan ν
- Goals:
  - Check scaling violations
  - Understand the systematics of the approach

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Matrix element calculation

$$C_P(t) = \langle \mathcal{N}_P(t) \overline{\mathcal{N}}_P(0) \rangle \qquad C_P^{\mathcal{O}^0(z)}(t) = \langle \mathcal{N}_P(t) \mathcal{O}^0(z) \overline{\mathcal{N}}_P(0) \rangle$$

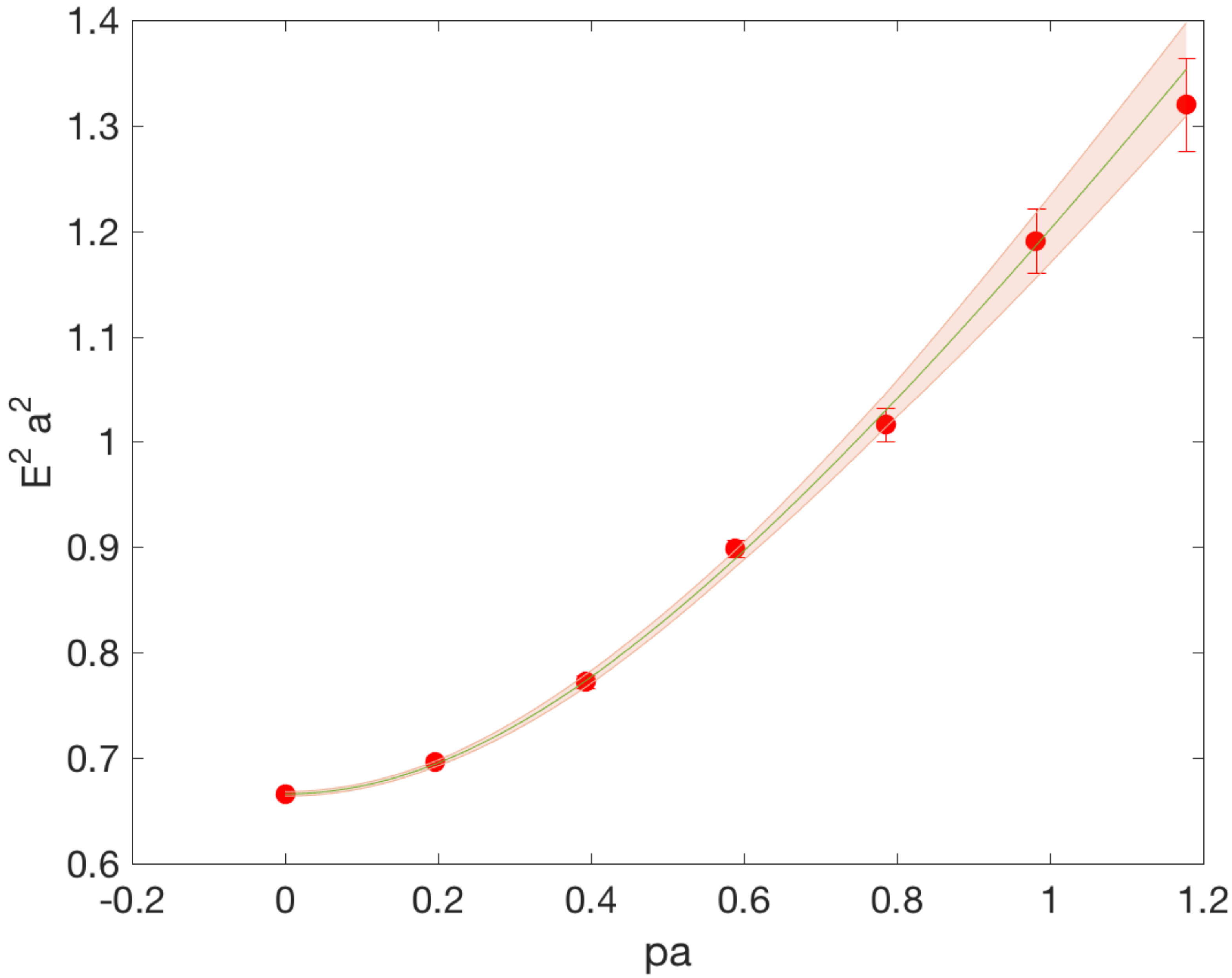
$$\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t) = \frac{C_P^{\mathcal{O}^0(z)}(t+1)}{C_P(t+1)} - \frac{C_P^{\mathcal{O}^0(z)}(t)}{C_P(t)}$$

C. Bouchard, et al arXiv:1612.06963 [hep-lat]

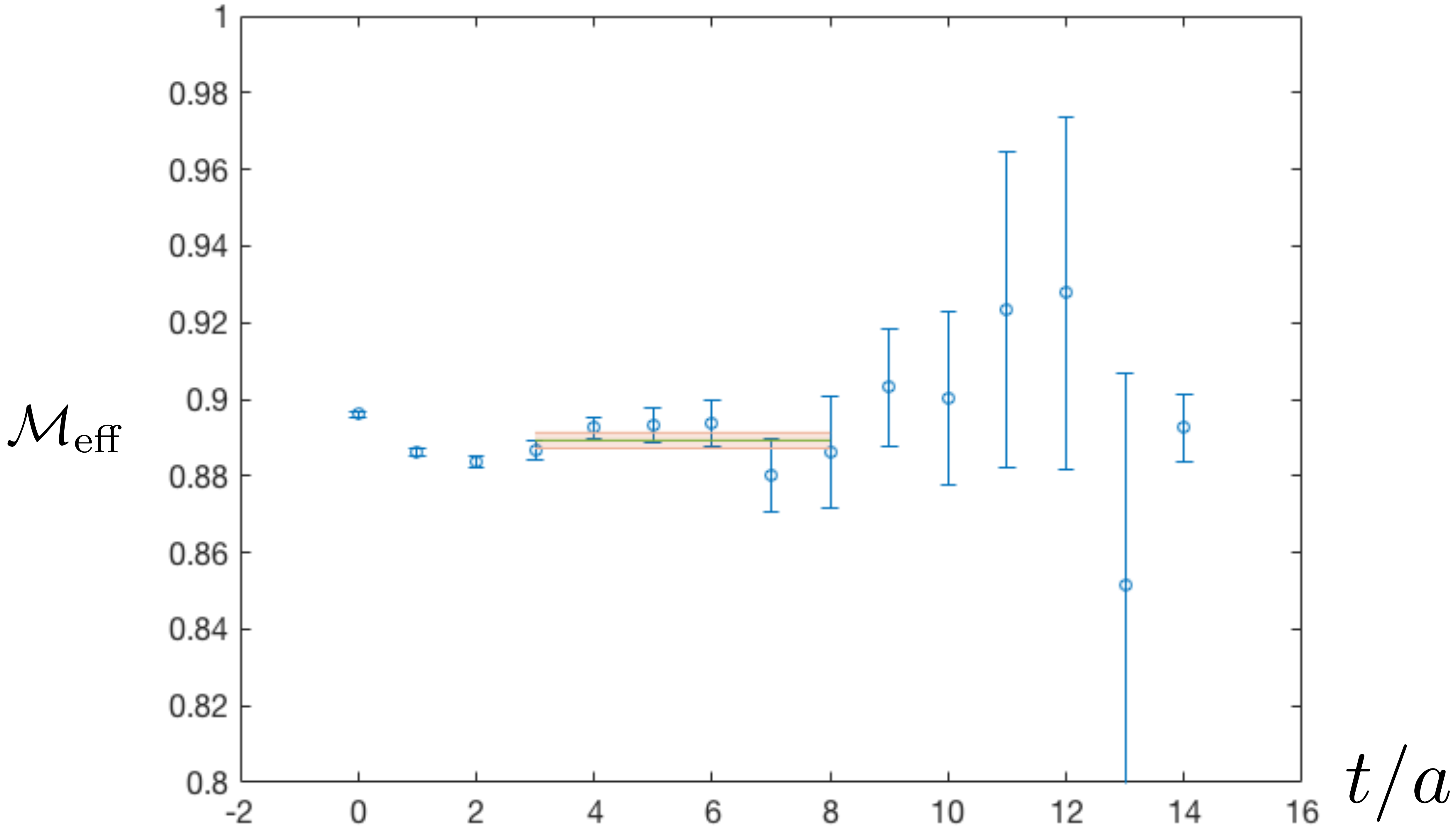
$$\mathfrak{M}(\nu, z_3^2) = \lim_{t \to \infty} \frac{\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t)}{\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t)\big|_{z_3=0}} \times \frac{\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t)\big|_{z_3=0}}{\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t)\big|_{P=0}}$$

Constructed to remove lattice spacing errors

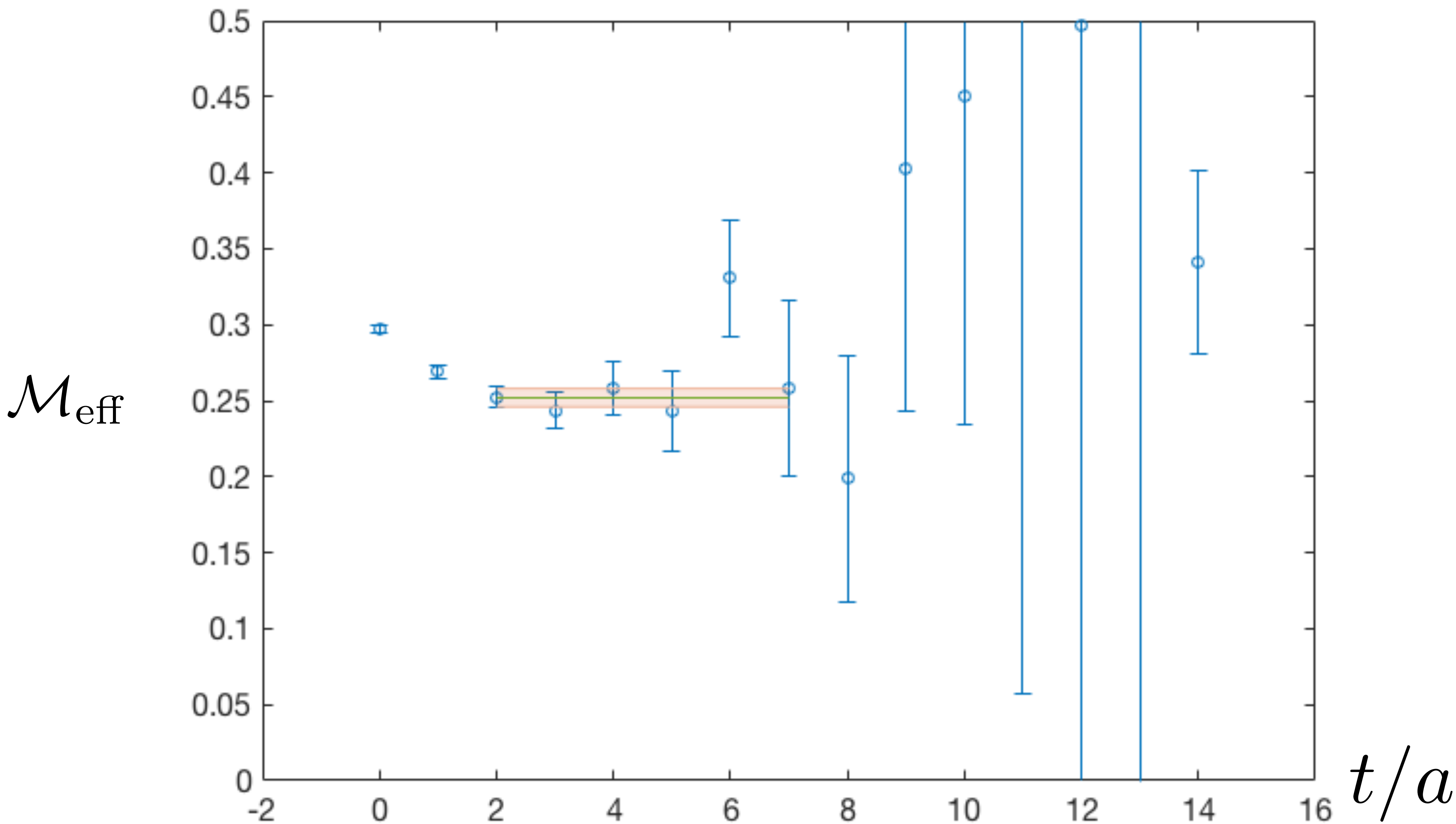
Dispersion relation



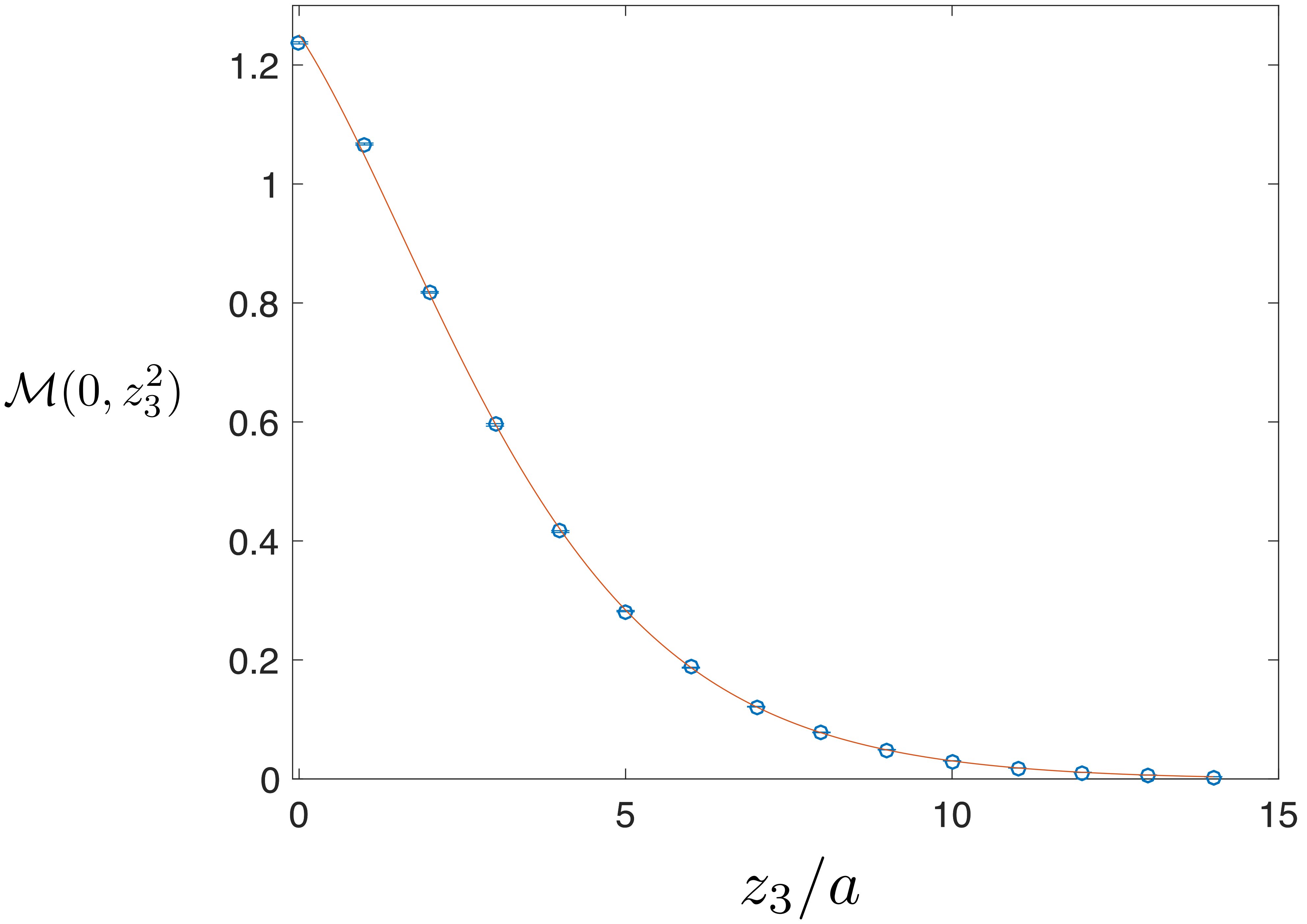
Gaussian smeared sources



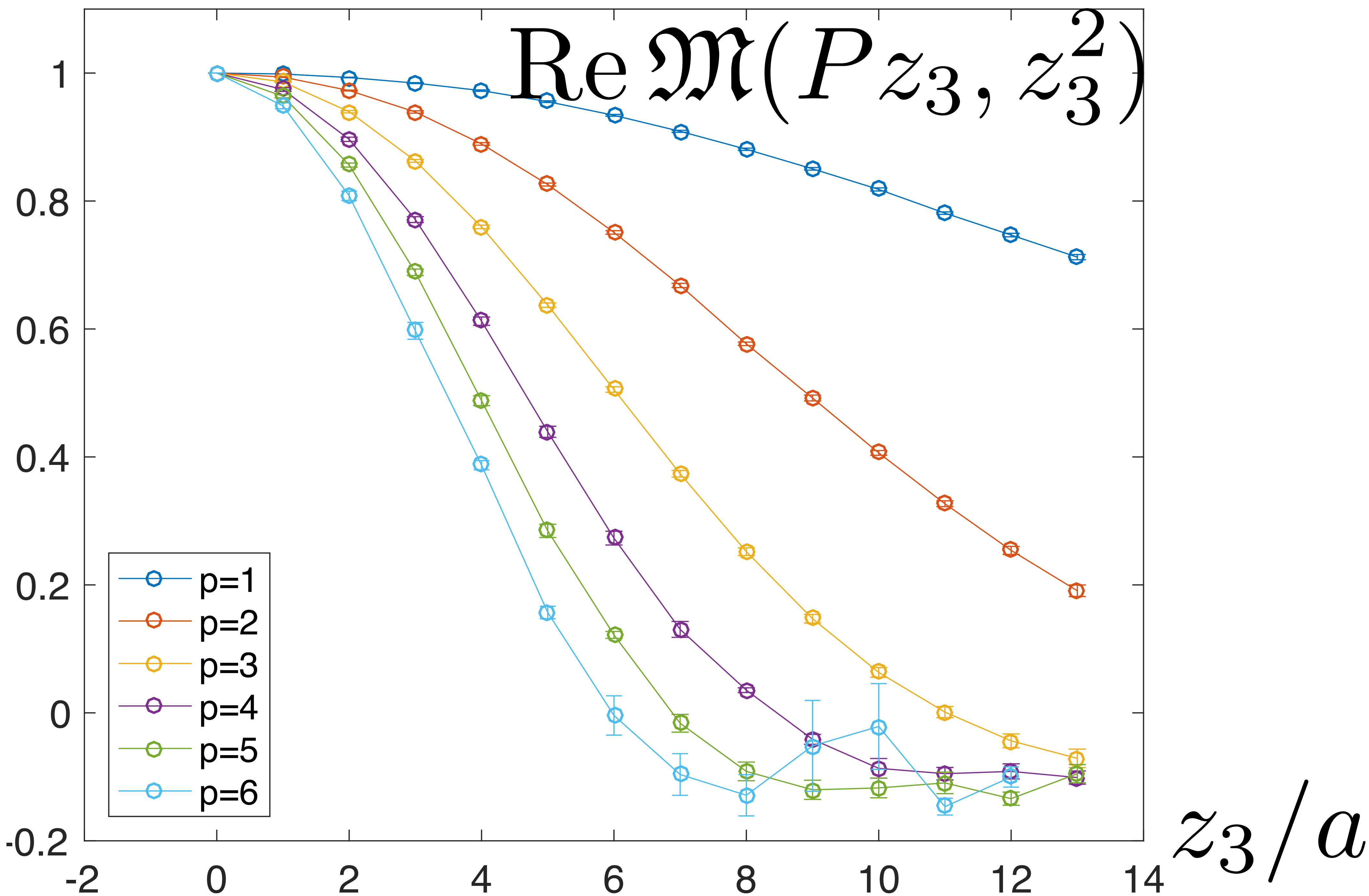
$\mathcal{M}_{\rm eff}$
t/a
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
-2
0
2
4
6
8
10
12
14
16



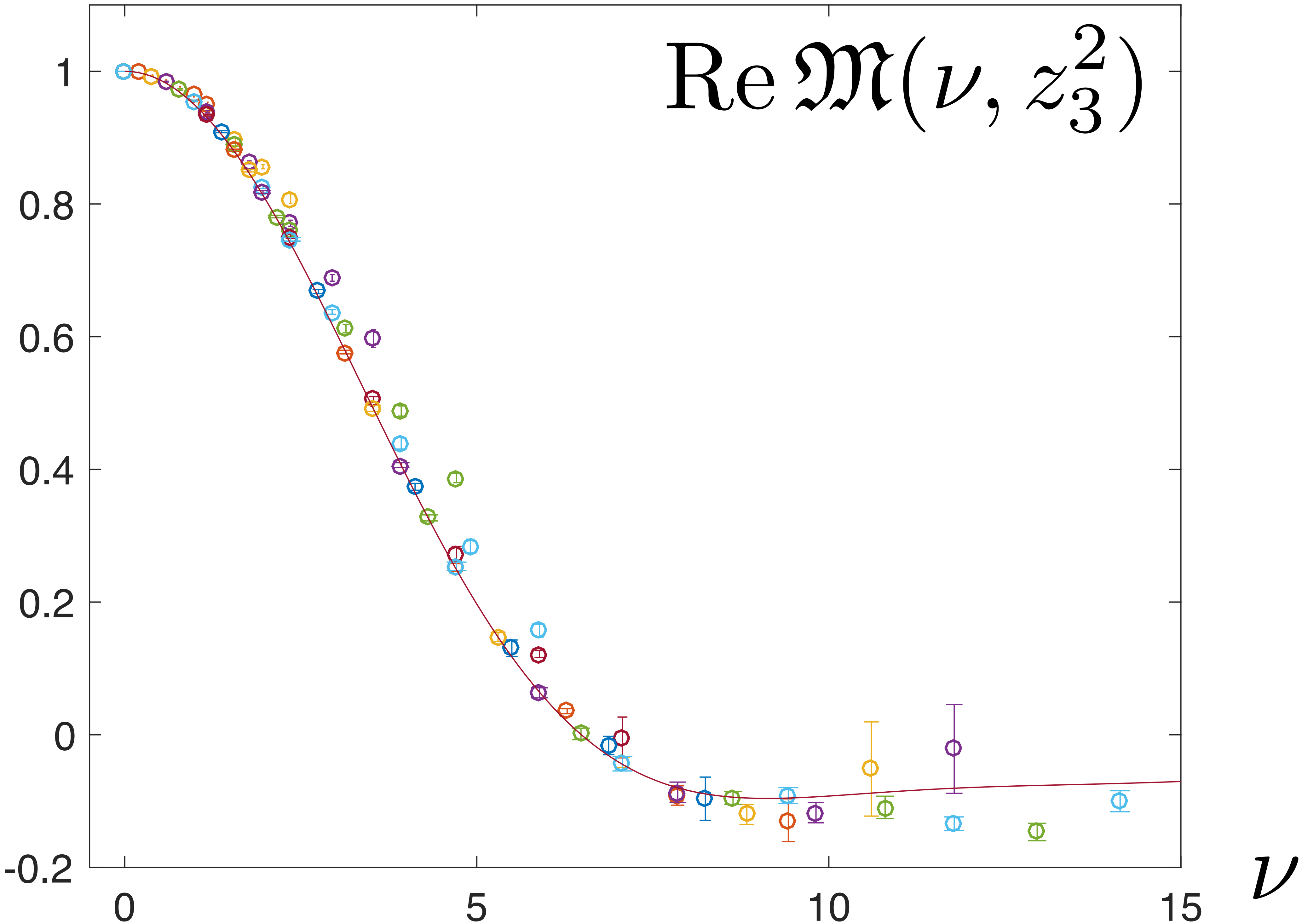
$\mathcal{M}_{\rm eff}$
t/a
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-2
0
2
4
6
8
10
12
14
16



Cusp indicates “linear” divergence of Wilson line

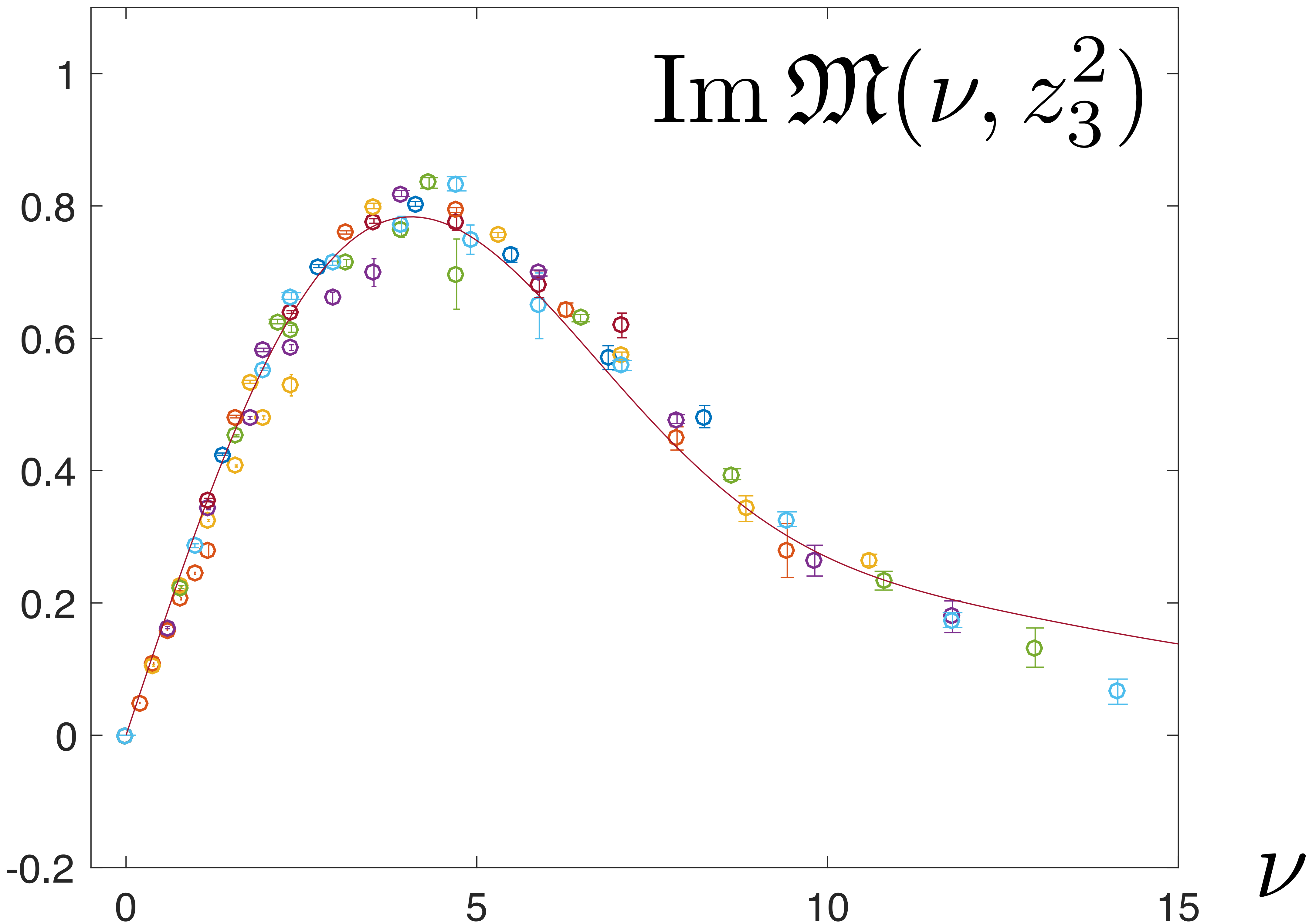


Ratio removes the linear” divergence of Wilson line



Points almost collapse on a universal curve

$$q_{-}(x) = \frac{315}{32}\sqrt{x}(1-x)^3$$



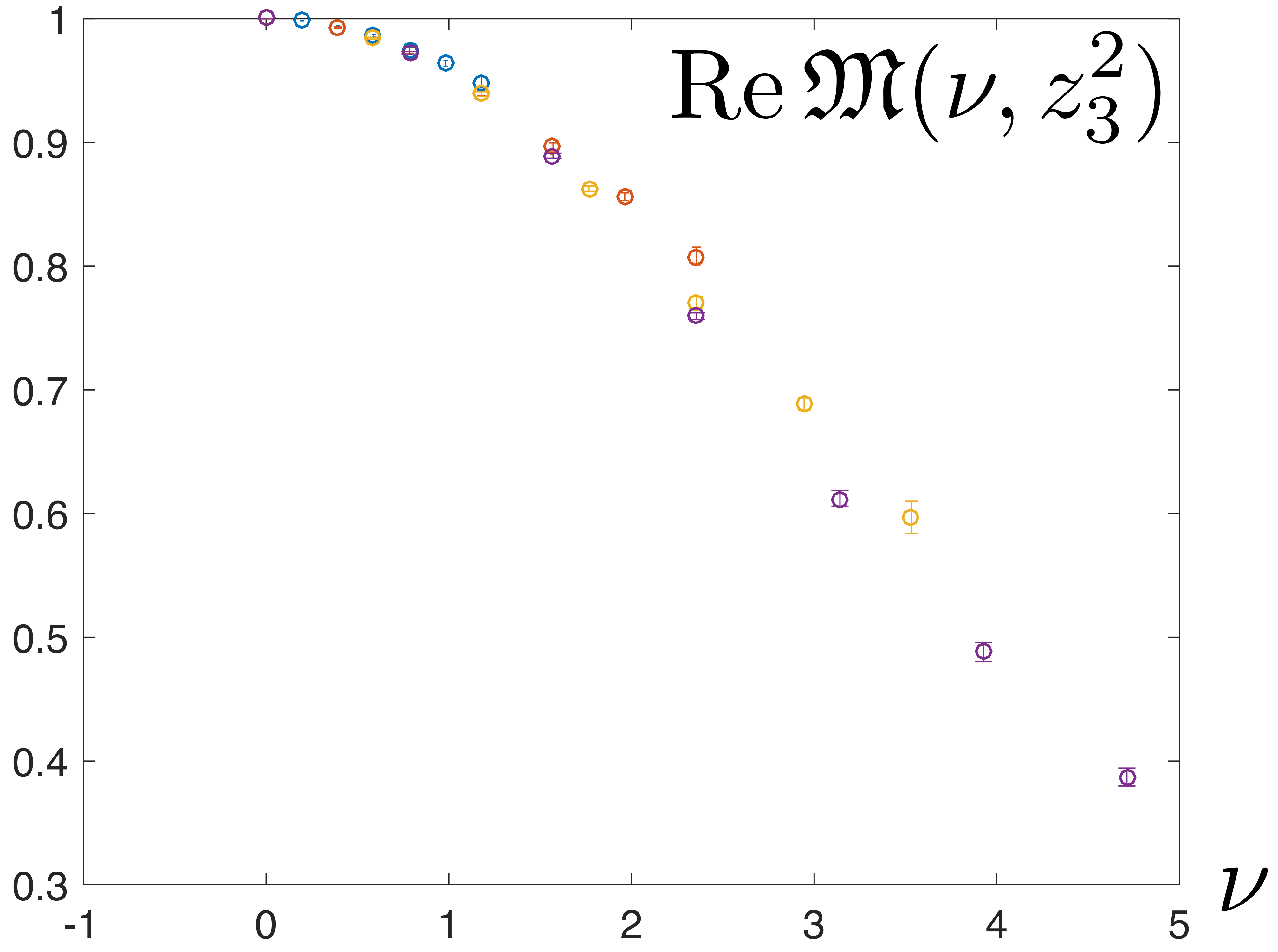
anti-quarks contribute to the imaginary part

Points in previous two plots obtained in with different z/a
i.e. correspond to the Ioffe time PDF at different scales!

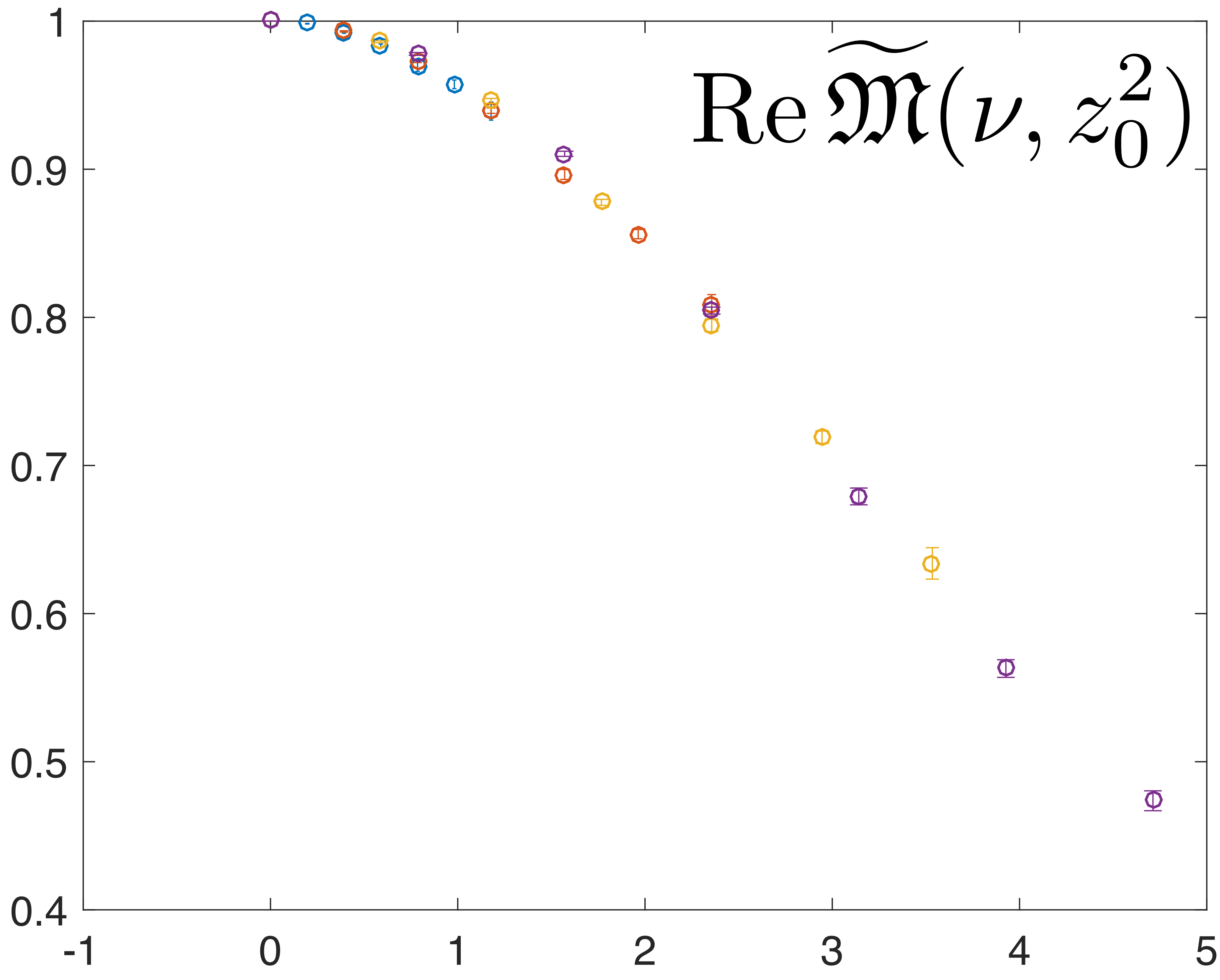
DGLAP evolution:

$$\mathcal{Q}(\nu, {z'}_3^2)=\mathcal{Q}(\nu, z_3^2) \ - \frac{2}{3}\frac{\alpha_s}{\pi}\ln({z'_3}^2/z_3^2)\int_0^1 du\, B(u)\, \mathcal{Q}(u\nu, z_3^2)$$

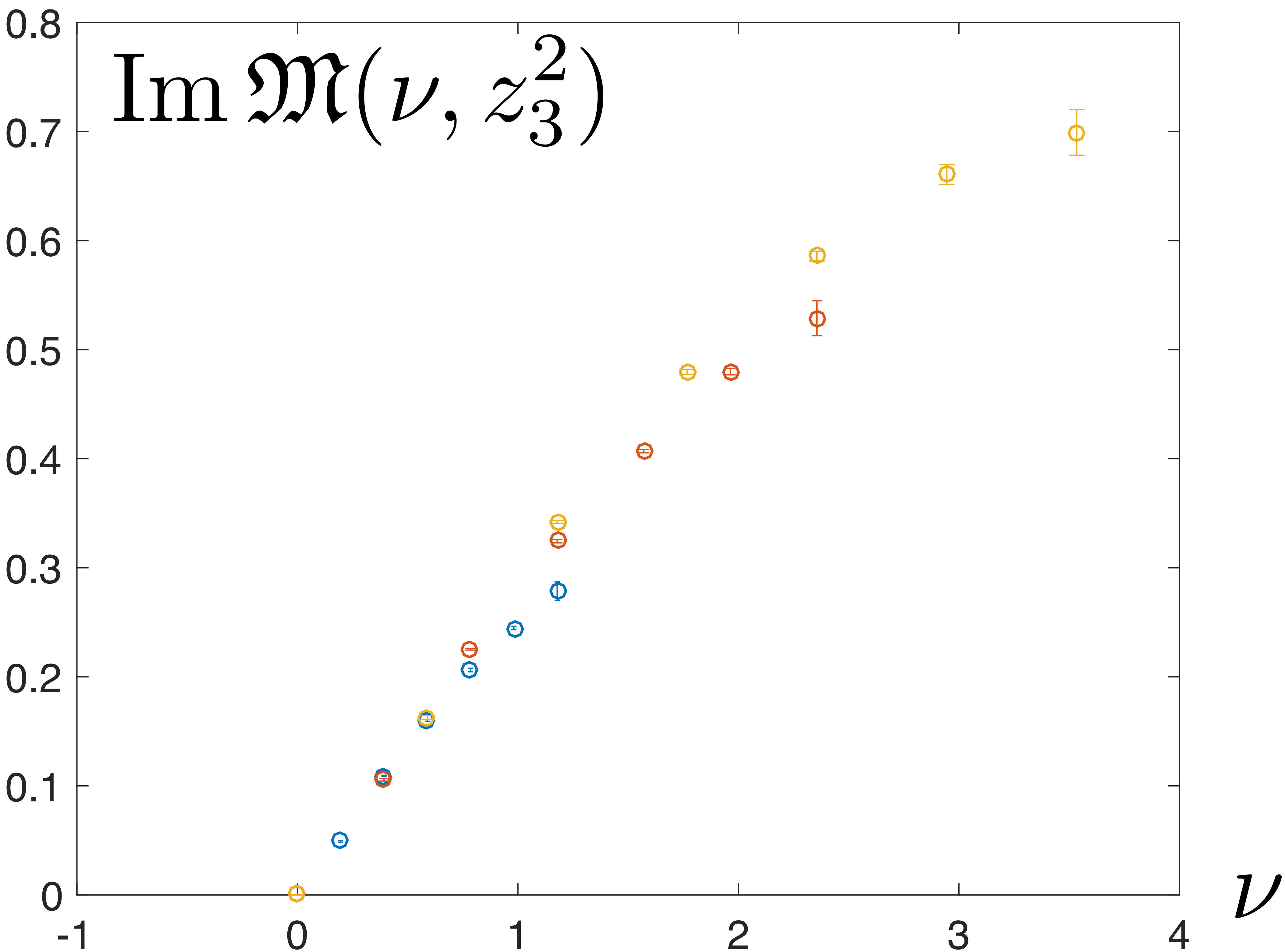
Apply evolution only at short distance points [~1GeV]



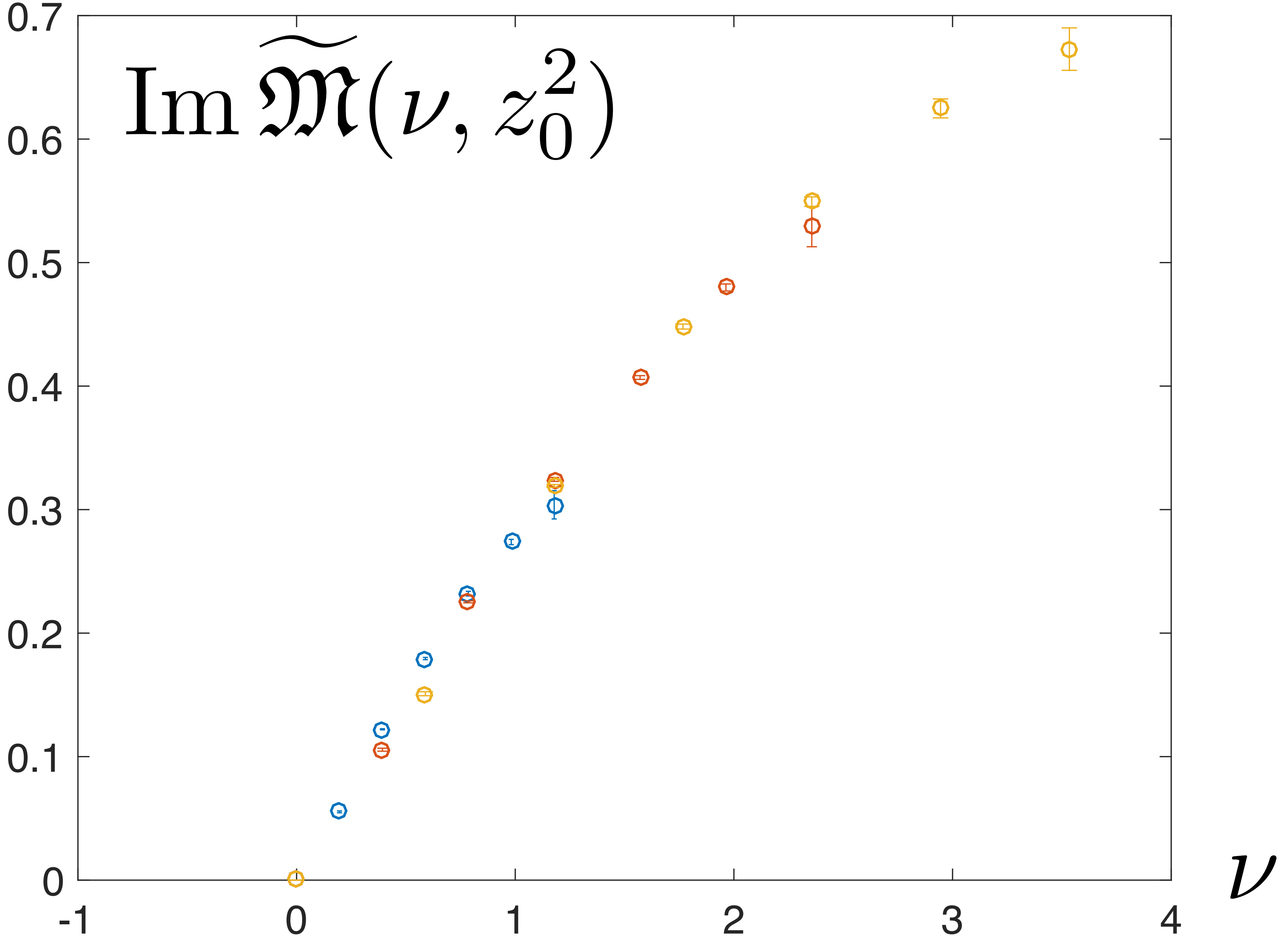
Data corresponding to z/a= 1, 2, 3, 4



Evolved to 1GeV



Data corresponding to z/a= 1, 2, 3, 4



Evolved to 1GeV

Summary

- Pseudo-PDFs are a viable new approach for obtaining collinear PDFs (Ioffe time PDFs) from LQCD calculations
- Use the hadron momentum to scan Ioffe time ν (which is the Fourier dual to momentum fraction x)
- Large hadron momentum is needed to access small x physics which corresponds to large ν
- Light cone is approached by sending z^2 to zero keeping ν fixed
- Scaling violations in z^2 are suppressed in pseudo-PDF ratio
- At short distances $z^2=0$ singularity leads to DGPLAP evolution
- At short distances, the observed z^2 dependence is compatible with DGLAP evolution