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Lattice study of continuity and finite-temperature transition in SU(N)x SU(N) Principal Chiral Model

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2d Principal Chiral Model

$$S = \frac{1}{g^2} \int d^2x \mathrm{Tr}\left[\partial_\mu U(x) \partial^\mu U^\dagger(x)\right] \quad U(x) \in SU(N)$$

A toy model for Yang-Mills theory:

- **Asymptotically free theory**
- **Integrable model**

$$M_r = M \frac{\sin(r\pi/N)}{\sin(\pi/N)}$$

- **Dynamically generated mass gap**

$$\Lambda^{\beta_0} = \mu^{\beta_0} e^{-\frac{4\pi}{g^2(\mu)}} \qquad \beta_0 = N$$

- **Matrix-like large N limit**
- **IR renormalon ambiguities**

(Fateev, Kazakov, Wiegmann)

$$\underbrace{e^{-\frac{8\pi}{g^2 N(Q)}}}_{1^{\mathrm{st}}\ \mathrm{IR-renormalon}} \gg \underbrace{e^{-\frac{16\pi}{g^2 N(Q)}}}_{2^{\mathrm{nd}}\ \mathrm{IR-renormalon}} \gg \dots$$

"Drawback": no topologically protected non-perturbative saddle points $\pi_2[SU(N)] = 0$

Compactification of PCM

PCM on $\mathbb{R}^2$: strongly coupled theory.

Evil toy model

Resurgence: what saturates IR renormalons?

Idea: $\mathbb{R}^2 \to \mathbb{R}^1 \times S^1$

$L \to 0$

Thanks to asymptotic freedom, at small L theory should be weakly coupled. Beware of "deconfinement" phase transition!

$$\mathcal{F} \sim (N^2 - 1)T^2 \xrightarrow{T \to 0} \mathcal{F} \to NT^2 \left(\frac{m}{2\pi T}\right)^{1/2} e^{-m/T}$$

How to avoid phase transition?

Twisted boundary conditions!

Phys. Rev. Lett. 112, 021601 (2014)

$$U(x_0 + L, x_1) = \Omega U(x_0, x_1) \Omega^\dagger \qquad \Omega = \mathrm{diag}\left\{1, e^{i\frac{2\pi}{N}}, \ldots, e^{i\frac{2\pi(N-1)}{N}}\right\}$$

"Maximal" destructive interference => many excited states eliminated

$$N e^{-ML} \to e^{-MLN}$$

Explicit demonstration: exactly solvable $\mathbb{CP}^{N-1}$ model

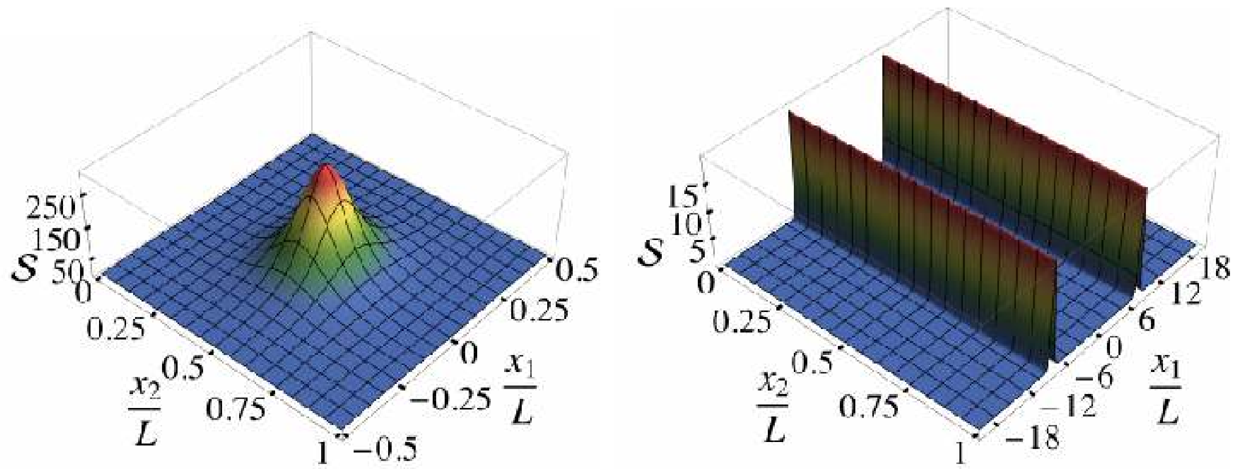
Phys. Rev. Lett. 118, 011601 (2017)

Non-perturbative saddle points

PCM on $\mathbb{R}^2$: unstable uniton saddle points

Harmonic maps $S^2 \to SU(N)$ $\quad$ $S_u = 8\pi/g^2$

Non-trivial effect of the twist in the small L limit:



A. Cherman, D. Dorigoni, M. Unsal, arXiv:1403.1277

Emergent topology => N stable fracton constituents at small L

$SU(N) \to U(1)^{N-1}$ **at energies smaller than** $1/(NL)$ $\quad$ $S_f = S_u/N$

Fractons are responsible for mass gap generation and IR renormalon ambiguity regularization via resurgence theory

Phys. Rev. Lett. 112, 021601 (2014) $\quad$ arXiv:1403.1277

Continuity conjecture

Periodic BC

"deconfinement"

strong coupling
"confinement"

Phase transition?

small L **large L**

Twisted BC

weak coupling
"confinement"
emergent topology

strong coupling
"confinement"

Continuity?

small L **large L**

Lattice PCM

$$S = -2\beta N \mathrm{Tr} \sum_{x,\mu} \left[U(x) U^\dagger(x+e_\mu) \right] \qquad \beta = 1/\lambda = 1/(g^2 N)$$

Simulation setup

Cabbibo-Marinari algorithm

- $N = 6, 9, 12, 18$ **+ fits** $O(L_0, N) = \tilde{O}(L_0) + c_1/N^2$
- **Lattice size:** $L_1 = 108$, $L_1 = 200, N = 18$ — $1 \leq L_0 \leq L_1$ compact direction
- **Boundary conditions: periodic and twisted**
- **Coupling:** $\beta = 0.332$ $\beta > \beta_c = 0.305$ **(weak coupling)**

Observables

$\xi_0 \equiv \xi(L_0 = L_1, PBC)$ scale

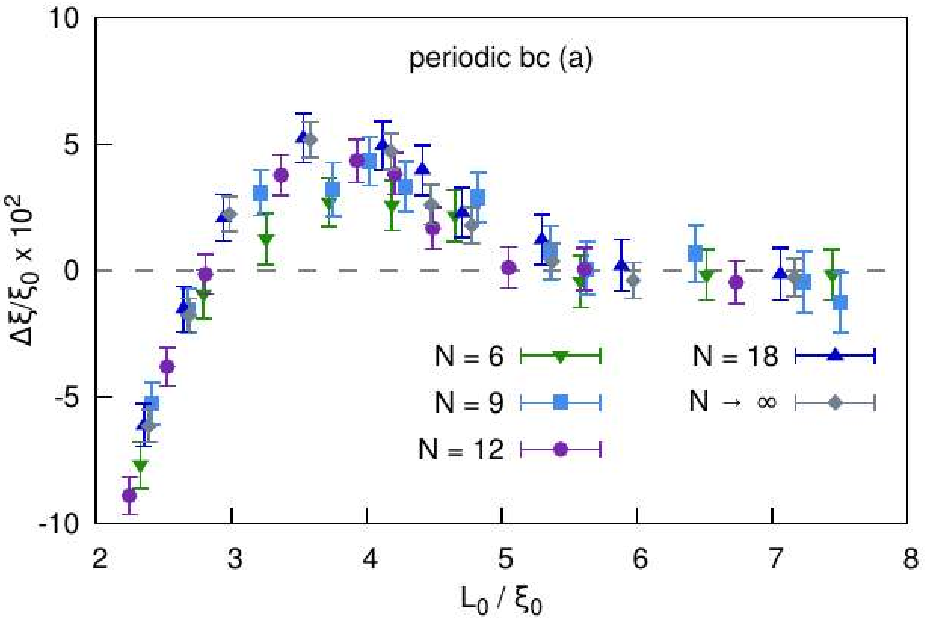
- **Static correlation length** $\xi(L_0)$
- **Mean energy** $E(L_0)$
- **Specific heat** $C(L_0)$

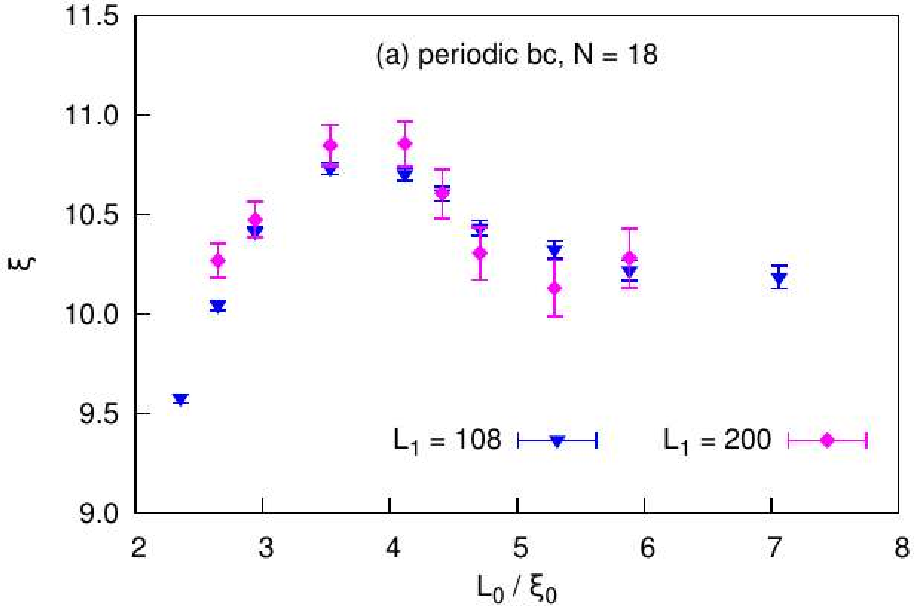
Notations:

$O_0 \equiv O(L_0 = L_1, PBC)$

$$\frac{\Delta O(L_0)}{O_0} \equiv \frac{O(L_0) - O_0}{O_0}$$

Periodic boundary conditions

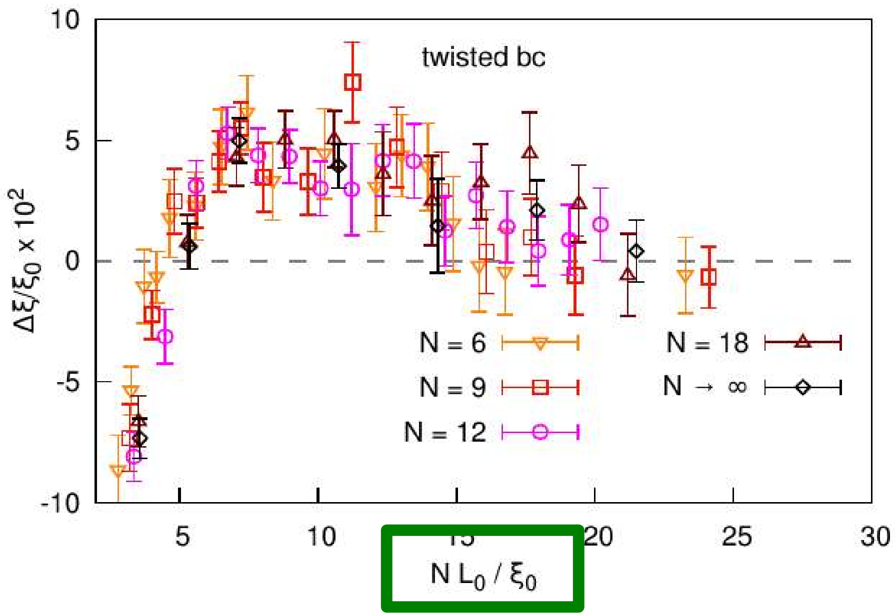


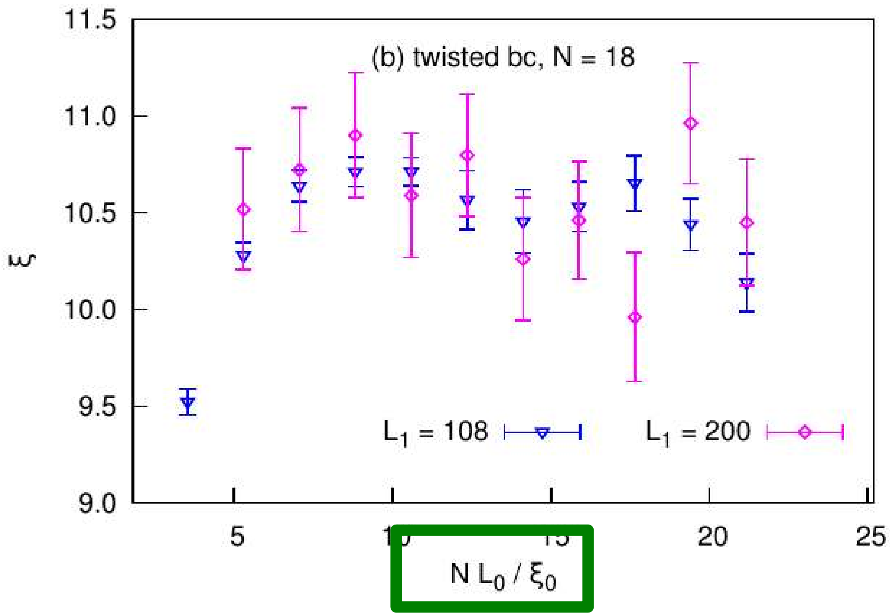


- **Weak enhancement in the region 3...5 (~ 5%)**
- **Slightly higher and narrower when N increases**
- **Infinite N extrapolation suggests that correlation length is finite**

  Compatible with DiagMC arXiv:1705.03368
- **Very mild volume dependence**
- **Large N volume independence in large volumes**

Twisted boundary conditions



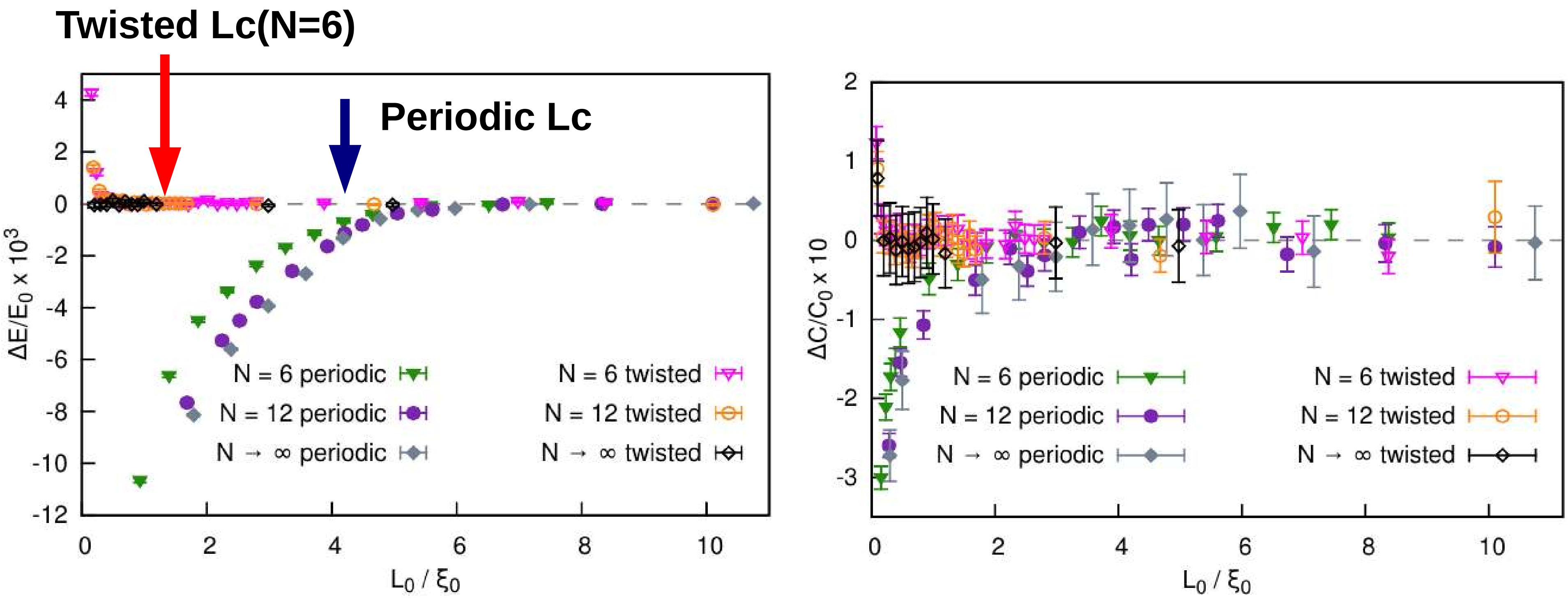


Difference compared to periodic BC:

$$Ne^{-ML} \to e^{-MLN}$$

- **Weak enhancement in the region NL = 5...15 (~ 5%)**
- **No evident change with N**

Mean energy and specific heat



- **Volume independence in large volumes**
- **Different behavior in small L limit**
- **Transition points agree with those for correlation length**
-
- **No signatures for phase co-existence**

Gradient flow: non-perturbative objects

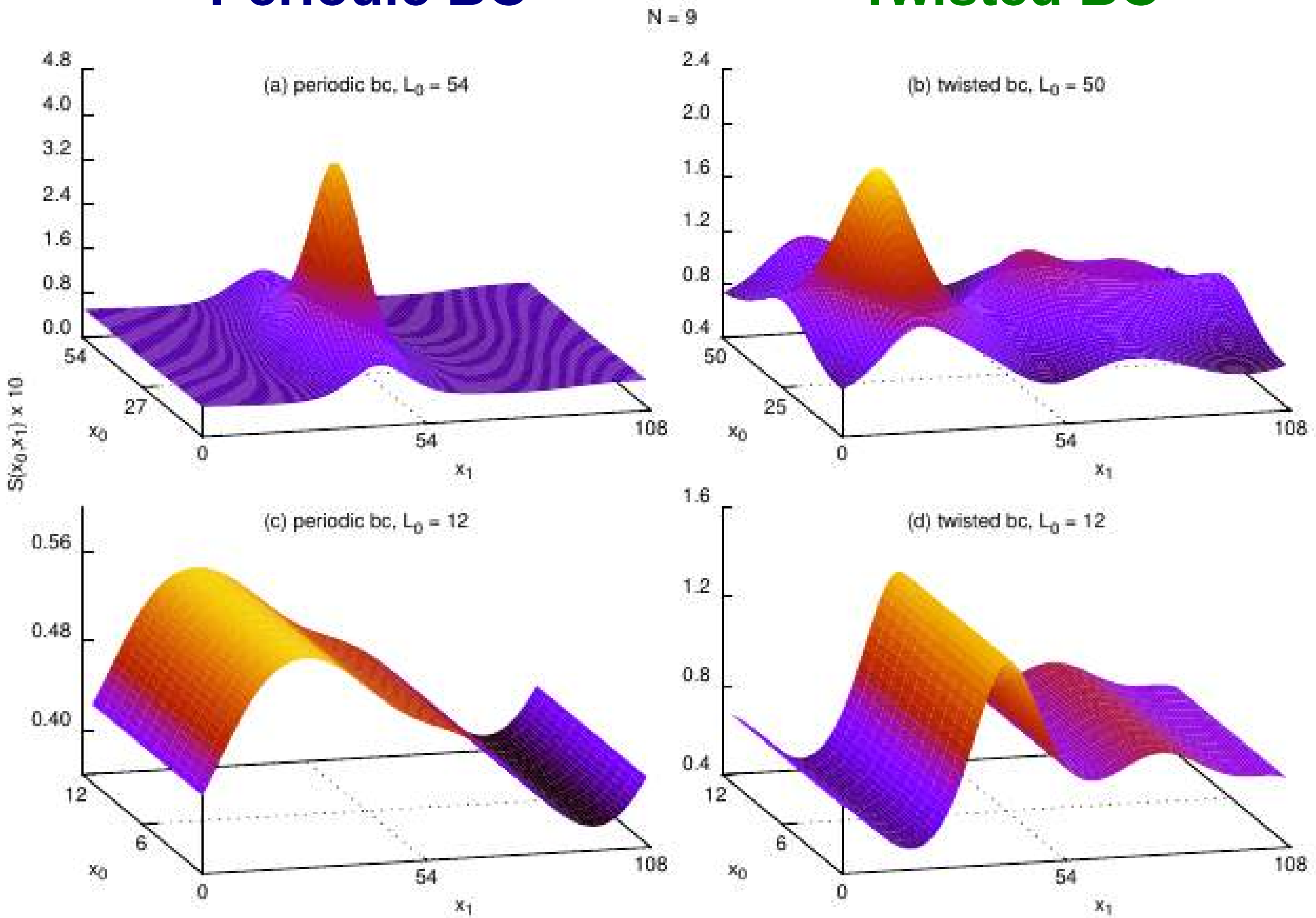
$$\frac{\partial U(\mathbf{x},\tau)}{\partial \tau} = -\frac{i}{\beta N}\nabla^a_{\mathbf{x}} S\left[U(\mathbf{x},\tau)\right] T_a U(\mathbf{x},\tau)$$
$$U(\mathbf{x},\tau=0) \equiv U(\mathbf{x})$$

$$\tau = 0 \ldots 1.5 \times 10^3$$

Periodic BC | **Twisted BC**

Large L

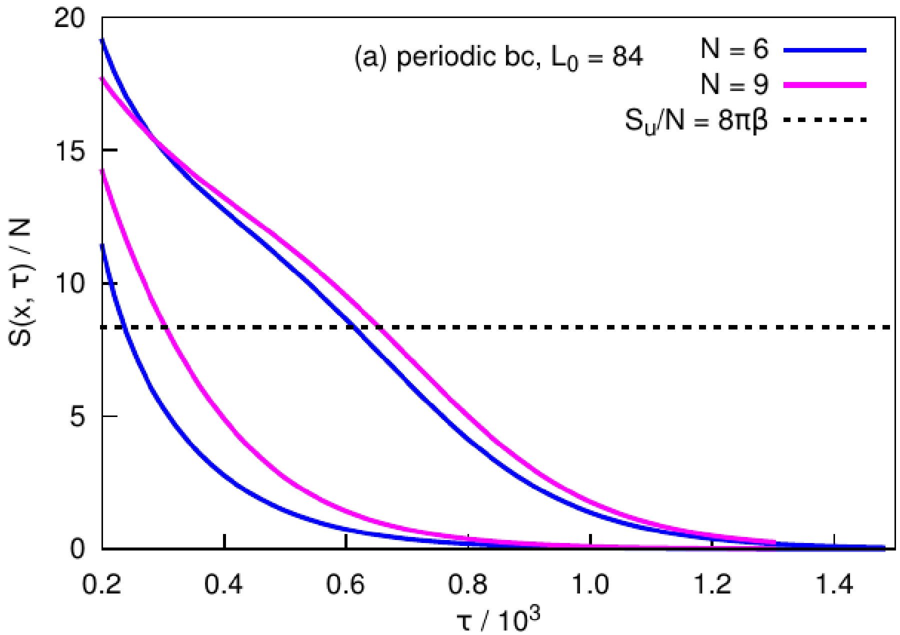
Small L



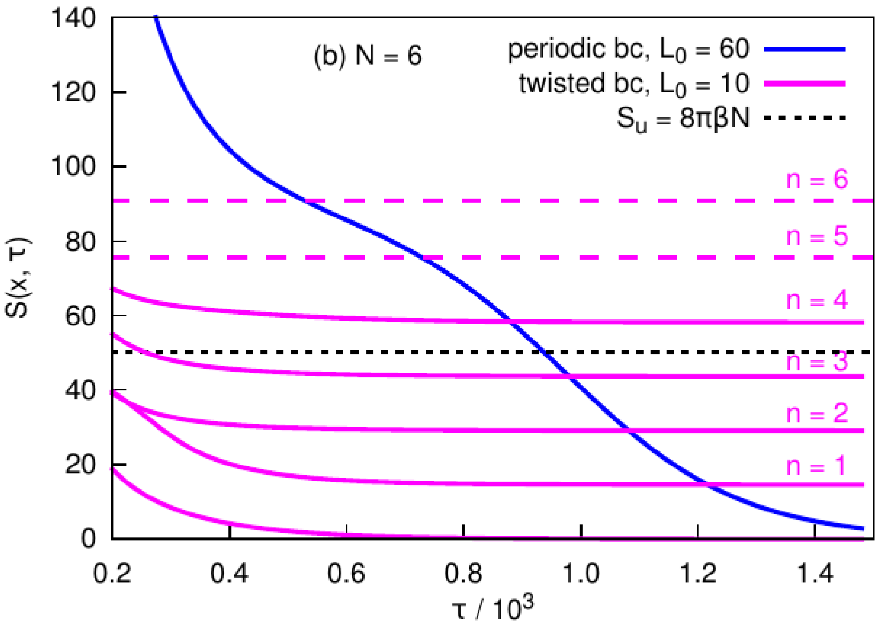
Gradient flow: the action

Uniton: $S_u = 8\pi\beta N$

Fracton: $S_f = 8\pi\beta$



Unitons for N=6,9

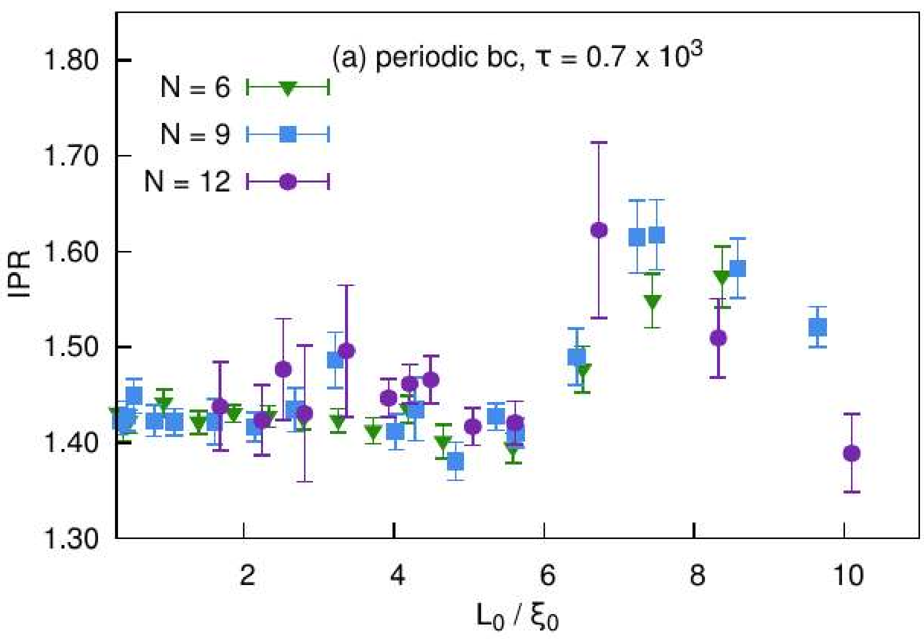


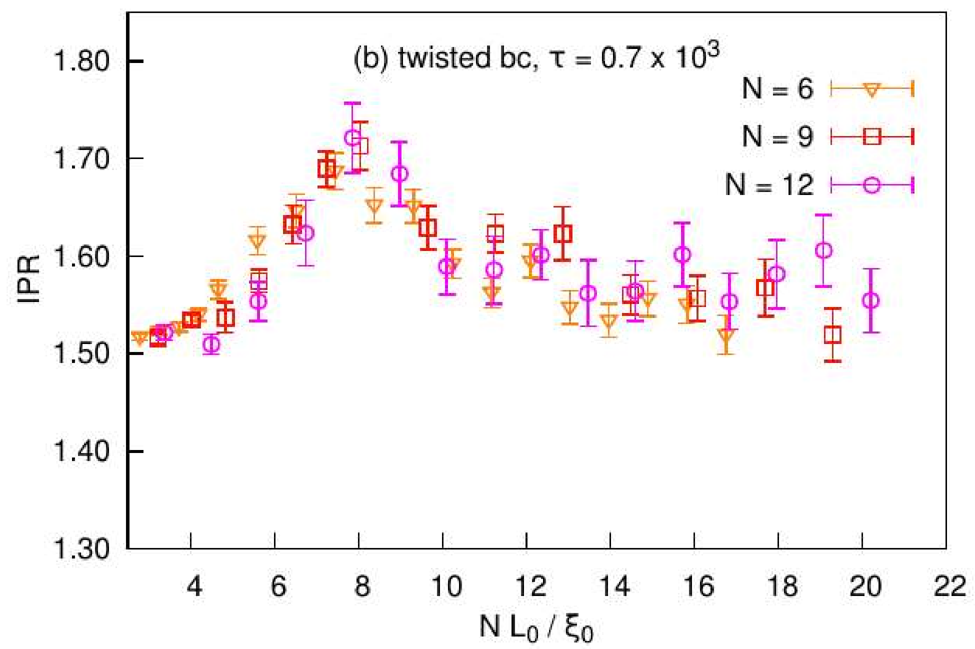
Stable fractons compared to uniton with $N L^{\rm TBC} = L^{\rm PBC}$

- **Very stable saddle points with twist in small L limit, evidence for emergent topology**

- **Presumably, the plateaus can be associated with unitons and fractons**

Inverse Participation Ratio (IPR)





$$\mathrm{IPR}\,(\tau) = V \left\langle \frac{\sum_{\mathbf{x}} \tilde{S}^2(\mathbf{x},\tau)}{\left(\sum_{\mathbf{x}} \tilde{S}(\mathbf{x},\tau)\right)^2} \right\rangle$$

$$\tilde{S}(\mathbf{x},\tau) = S(\mathbf{x},\tau) - \min_{\mathbf{x}} S(\mathbf{x},\tau)$$

We use IPR as a measure of action density localization

- **Interesting peak in twisted case which coincide with twisted NLc**

Conclusions

- **We find evidences compatible with a weak crossover or phase transitions for both types of boundary conditions**

- **For periodic BC, correlation length enhancement become larger and narrower as N increases**

- **For twisted BC, correlation length enhancement is N independent if considered as a function NL**

- **Volume scaling seems to be very mild in both cases.**

- **Using Gradient flow equations, we find an evidence for emergent topology in small L limit with twisted BC.**

- **More work is needed: combined study of volume and N scaling, continuum limit.**

- **Might be a challenge for resurgence theory if phase transition (possibly of infinite order) is confirmed**