Eigenvalue statistics in chirally symmetric toy model

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Based on a work in collaboration with:
S. Capitani, P. Dimopoulus, G.M. de Divitiies,R. Frezzotti, M. Garofalo, B. Knippschild, B. Kostrzewa, G.C. Rossi, C. Urbach

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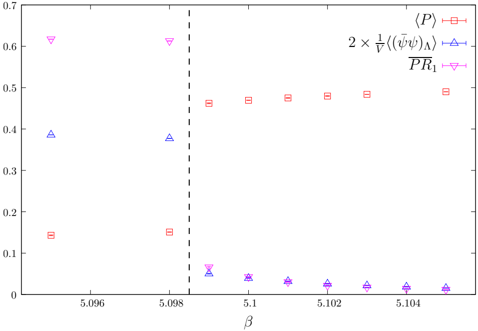
Chiral transition at high temperature

$N_t = 4$ $N_f = 3$ unimproved staggered fermions ($m = 0.001$)

M.Giordano,S.D.Katz,T.G.Kovács and FP(2017)

- Order parameter related to the low modes: $\langle\bar{\psi}\psi\rangle \propto \rho(0)$
- The nature of low D eigenmodes also changes.
- Low modes becomes localized
- Deconfinement, chiral and localization transition occurs at the same point

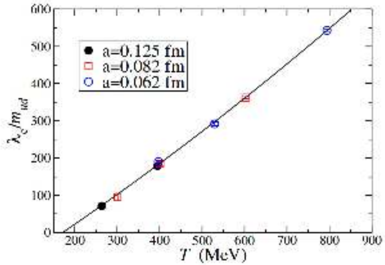


- $\langle P\rangle = \langle Tr \prod_{t=0}^{N_t-1} U_4(t,\vec{x})\rangle$
- $\langle\bar{\psi}\psi\rangle_\Lambda = \int_0^\Lambda \frac{2m}{\lambda^2+m^2}\rho(\lambda)\,\mathrm{d}\lambda$
- $PR_1 = \frac{1}{N_t V}\left[\sum_{\vec{x},t} |\psi_\alpha^\dagger(t,\vec{x})\,\psi_\alpha(t,\vec{x})|^2\right]$

Chiral transition at high temperature

Appearance of localized modes, and the mobility edge

- Evidence for
  - staggered T.G.Kovács and FP(2012)
  - overlap F.Bruckmann,T.G.Kovács, S.Schierenberg(2011)
  - domain wall G. Cossu,S. Hashimoto(2016)
- Real feature of QCD
- Appearantly goes together with chiral symmetry breaking



Mechanism behind localization (M.Giordano, T.G.Kovács and FP(2015))

- High temperature the time slices of the Dirac eigenfunctions are correlated
- Spectrum is shifted up proportionally to T (Matsubara frequency, anti-periodic bc.)
- For example: (unit spatial links, temporal gauge $U_4(\vec{x},t) = 1$, except at the boundary $U_4(\vec{x}, N_t - 1) = P(\vec{x})$ (Polyakov line)

$$\psi(N_T, \vec{x}) = -P(\vec{x})\,\psi(0, \vec{x})$$

$$i\partial\!\!\!/\,\psi_n = \lambda_k \psi_n$$

- Temporal momentum will depend on the on the phases of the Polyakov line($\phi(k)$).

$$\omega_k = aT(\pi + \phi_k)$$

- Polyakov loop completely ordered $\omega_k = aT\pi$
- When locally not ordered, eigenvalue can be smaller

Consequences of localization

Mobility edge as an effective mass gap

Typical correlator for calculating hadron masses

$$\mathrm{Tr}\Lambda D^{-1}(x,y)\Lambda D^{-1}(y,x) \tag{1}$$

- Writing the inverse in terms of the Dirac eigenmodes

$$D^{-1}(x,y) = \sum_j \frac{1}{i\lambda_j + m_{bare}} |\psi_j(x)\rangle\langle\psi_j(y)|$$

- The contribution of one localized eigenmode to the correlator is proportional to

- $\propto |\langle\psi_j(x)|\Lambda|\psi_j(y)\rangle|^2$
- $\propto \frac{1}{\lambda_j^2 + m_{bare}^2}$

- They do not interact neither with the delocalized nor with the other localized modes
- For large distance correlations they do not contribute $\rightarrow$ They push the delocalized modes further away from the origin

Toy model with chiral symmetry at zero temperature

Talk by:M.Garofalo,P. Dimopoulus(Monday)

Frezzotti, Rossi: Non-perturbative mass generation for elementary fermions(Phys. Rev. D92 (2015) 054505)

- Fermion isodoublet $Q_L = (u_L\ d_L)^T$ and $Q_R = (u_R\ d_R)^T$ interacting through $SU(3)$ gauge fields (U) and an ($SU(2)$) doublet of scalar field (Φ) via the Lagrangian:

$$\mathcal{L} = \overbrace{\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{Q}_L \not{D} Q_L + \overline{Q}_R \not{D} Q_R + \frac{1}{2} \mathrm{Tr} \partial \Phi^\dagger \partial \Phi}^{\mathcal{L}_{kin}}$$

$$\overbrace{\eta(\overline{Q}_L \Phi Q_R + h.c.)}^{\mathcal{L}_{Yukawa}} + \overbrace{\frac{a^2}{2} \rho \left(\overline{Q}_L \overleftarrow{D} \Phi D Q_R + \overline{Q}_R \overleftarrow{D} \Phi^\dagger D Q_L \right)}^{\mathcal{L}_{Wilson\ like}}$$

$$\overbrace{\frac{1}{2} \mu_{scalar} \mathrm{Tr} \left(\Phi^\dagger \Phi \right) + \frac{1}{4} \lambda \left(\mathrm{Tr} \left[\Phi^\dagger \Phi \right] \right)^2}^{\mathcal{L}_{scalar}}$$

- We simulate the quenched model on the lattice with finite twisted mass μ.
- In this talk we will see how the quark eigenmodes of the Dirac in this model look like

Symmetries of the model

- Extended chiral symmetry (for all values of η and ρ)

χ global $SU(2)_L \times SU(2)_R$ transformations

- $\tilde{\chi}_L \bigotimes (\Phi \to \Omega_L \Phi)$
- $$\tilde{\chi}_L = \begin{cases} Q_L \to \Omega_L Q_L \\ \overline{Q}_L \to \overline{Q}_L \Omega_L^\dagger \end{cases} \quad \Omega_L \in SU(2)$$
- $\tilde{\chi}_R \bigotimes \left(\Phi \to \Phi \Omega_R^\dagger\right)$
- $$\tilde{\chi}_R = \begin{cases} Q_R \to \Omega_R Q_R \\ \overline{Q}_R \to \overline{Q}_R \Omega_R^\dagger \end{cases} \quad \Omega_R \in SU(2)$$

- General (η, ρ) values only fermionic symmetry is violated

Restoration of $\tilde{\chi}$ symmetry

Ward identities

- Consider the local infinitesimal transformations:
  - $\delta Q_L(x) = i\varepsilon_i(x)\tau_i Q(x)$
  - $\delta \overline{Q}_L(x) = -\overline{Q}(x)\varepsilon_i(x)\tau_i$, where τ_i-s are the Pauli matrices
- We are looking for an η value, where the correspondig Ward identity becomes a conservation law for the current of $\tilde{\chi}$ tranformation.
- In the lattice discretization we are computing $\langle J(x)D(\tilde{y})\rangle$:

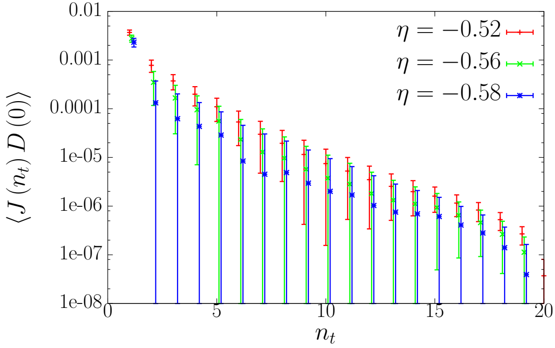
$$J = \frac{1}{2}\left[\overline{Q}_L\left(x-\hat{0}\right)\frac{\tau_i}{2}\gamma_0 U_0\left(x-\hat{0}\right)Q_L(x) + \overline{Q}_L(x)\frac{\tau_i}{2}\gamma_0 U_0^\dagger\left(x-\hat{0}\right)Q_L\left(x-\hat{0}\right)\right]$$

- $$D = \overline{Q}_L(\tilde{y})\frac{\tau_i}{2}\Phi(\tilde{y})Q_R(\tilde{y}) - \overline{Q}_R(\tilde{y})\Phi^\dagger(\tilde{y})\frac{\tau_i}{2}Q_L(\tilde{y})$$

Results: η_{cr} determination

Parameters

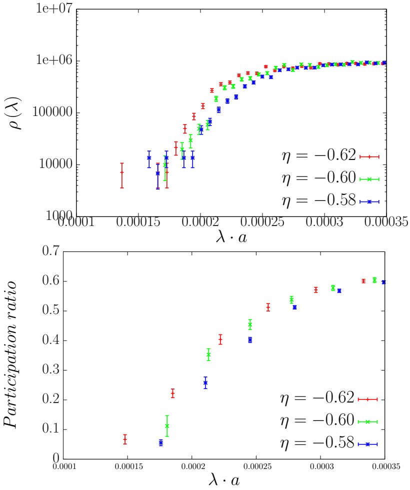
- Wilson plaquette action
- $\rho = 1$
- $\beta = 5.85$
- $N_s = 16$
- $N_t = 40$
- $\langle \Phi \rangle = 0$
- Point sources



Spectral density and participation ratio for $D_{lat} \cdot D_{lat}^{\dagger}$

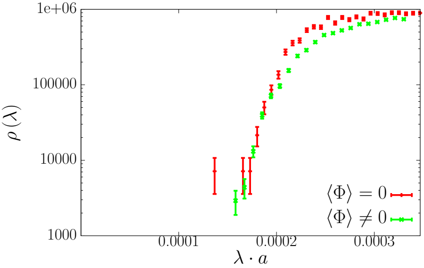
Preliminary results for determining η_{cr} from the spectrum

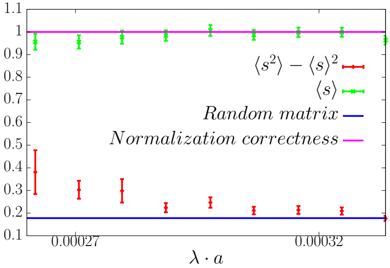
- There is a spectral gap
- Gap gets smaller towards η_{cr}
- This gap is proportional to the Ward identity breaking
- For quantitative results $V \to \infty$ and $\mu \to 0$ limits are needed
- Modes near the gap seems to be localized



Preliminary results in the $\langle\Phi\rangle \neq 0$ phase

- Non-perturbatively generated mass might appear
- Are there more localized modes in this case?





- Most probably the dynamical mass (if exist) is due to the some properties of the eigenvectors (currently being checked)

Conclusion, outlook

Conclusion

- The gap in the spectrum of the toy model shrinks towards the symmetry restoration
- First few modes modes in the Wigner phase might be localized
- Finite volume effect

Outlook

- Appearantly only the very few low modes are localized
- Determining the correlation function on the subspace of the low modes might significantly improve the signal
- Point source propagators can be replaced by all-to-all
- Currently under investigation

Thank you for your attention!