Computation of the entropy of SU(3) Yang-Mills theory using shifted boundary conditions

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We calculate the entropy of SU(3) Yang-Mills theory in a moving frame using the Symanzik improved action. By using shifted boundary conditions a moving frame can be realized on the lattice. In this setting the momentum density and the renormalization constant of the offdiagonal elements of the energy-momentum tensor are computed. The results are then used to compute the entropy density at temperatures $1.5T_c$ and $1.5\sqrt{2}T_c$.

Introduction

The field theory in a frame moving with an imaginary velocity $v = i\xi$ can be described by an Euclidean partition function with an action using shifted boundary conditions in the temporal direction $A_\mu(L_0, x) = A_\mu(0, x - L_0\xi)$ [1, 2].

Due to the invariance of the theory under transformations of the restricted Lorentz group, which becomes SO(4) invariance in the Euclidean metric, the partition function of a system with temporal extent L_0 and shift ξ equals the partition function of a system with zero shift and temporal extent $L_0\sqrt{1+\xi^2}$.

Because the field is now observed in a moving frame the expectation value of the momentum $\langle T_{0k}\rangle_\xi$ is non zero.

The equation of state

The entropy density at temperature $T = 1/(L_0\sqrt{1+\xi^2})$ is given by

$$s = -\frac{L_0(1+\xi^2)^{\frac{3}{2}}}{\xi_k} Z_T \langle T^{(1)}_{0k}\rangle_\xi,$$

where $\langle T^{(1)}_{0k}\rangle_\xi$ is the offdiagonal part of the energy-momentum tensor [1].

The renormalization constant Z_T is given as

$$Z_T = -\frac{1}{\langle T^{(1)}_{0k}\rangle_\xi}\frac{\partial f}{\partial \xi_k}.$$

$\partial f/\partial \xi_k$ can be approximated by the symmetric difference

$$\frac{\Delta f}{\Delta \xi_k} = \frac{1}{2aV}\ln\left[\frac{Z(L_0, \xi - \hat{e}_k a/L_0)}{Z(L_0, \xi + \hat{e}_k a/L_0)}\right],$$

which can be computed as [3]

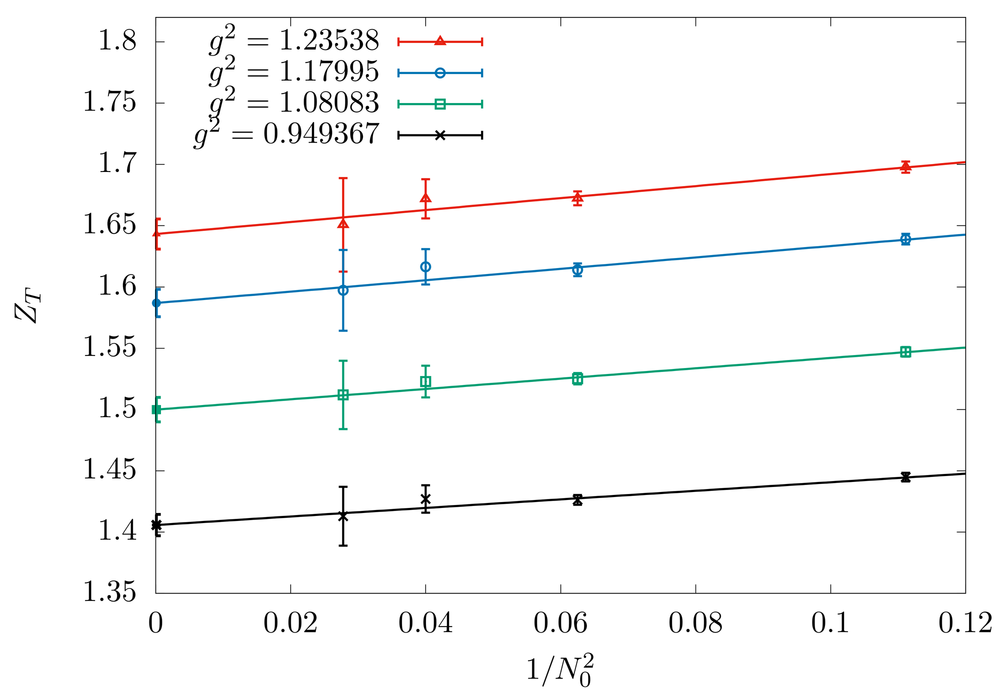
$$\frac{\Delta f}{\Delta \xi_k}(g^2) = \left.\frac{\Delta f}{\Delta \xi_k}\right|_{g^2=0} + \int_0^{g^2} dg'^2\, \frac{d}{dg'^2}\frac{\Delta f}{\Delta \xi_k}(g'^2),$$

with

$$\frac{d}{dg^2}\frac{\Delta f}{\Delta \xi_k} = \frac{1}{2aVg^2}\left(\langle S\rangle_{\xi - \hat{e}_k a/L_0} - \langle S\rangle_{\xi + \hat{e}_k a/L_0}\right).$$

Renormalization method I

In the approach suggested in [3], $Z_T(N_0, g^2)$ is computed on lattices with several different temporal extents N_0, all lying in the deconfined phase. Then $Z_T(g^2)$ is obtained through a zero temperature limit $1/N_0 \to 0$. The measured values of $\langle T^{(1)}_{0k}\rangle(N_0, g^2)$ are then renormalized with the N_0-independent $Z_T(g^2)$.



Renormalization method II

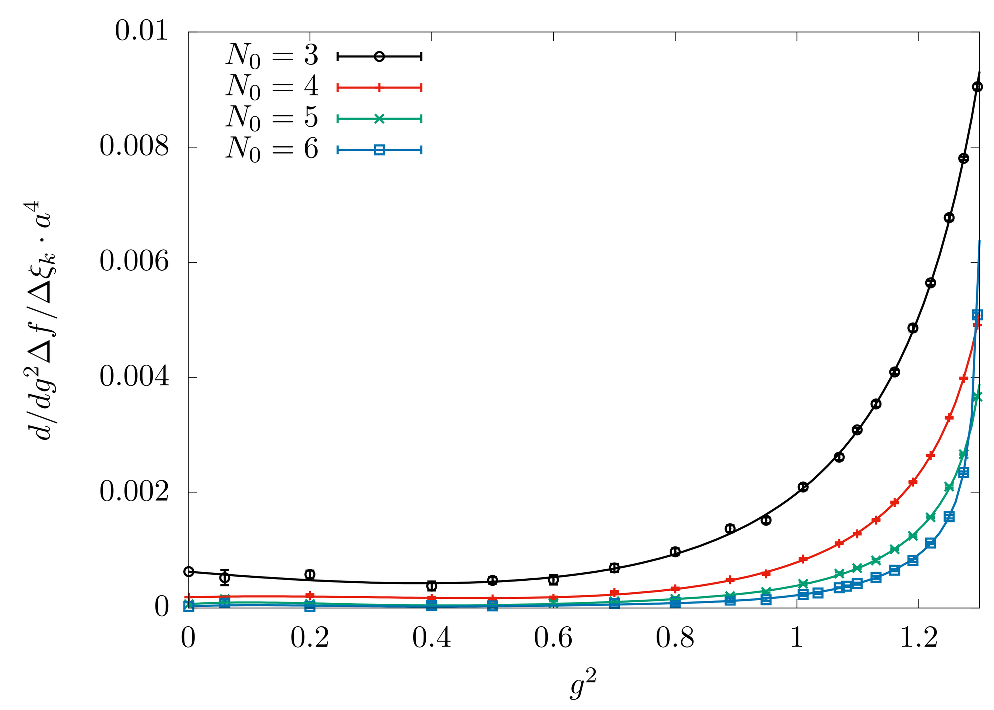
We suggest an alternative renormalization method, in which a separate zero temperature limit is not needed: $\langle T^{(1)}_{0k}\rangle_\xi(N_0, g^2)$ is renormalized using $Z_T(N_0/2, g^2)$, and the limit $\frac{\Delta f}{\Delta \xi} \to \frac{\partial f}{\partial \xi}$ is automatically taken care of during the continuum limit.

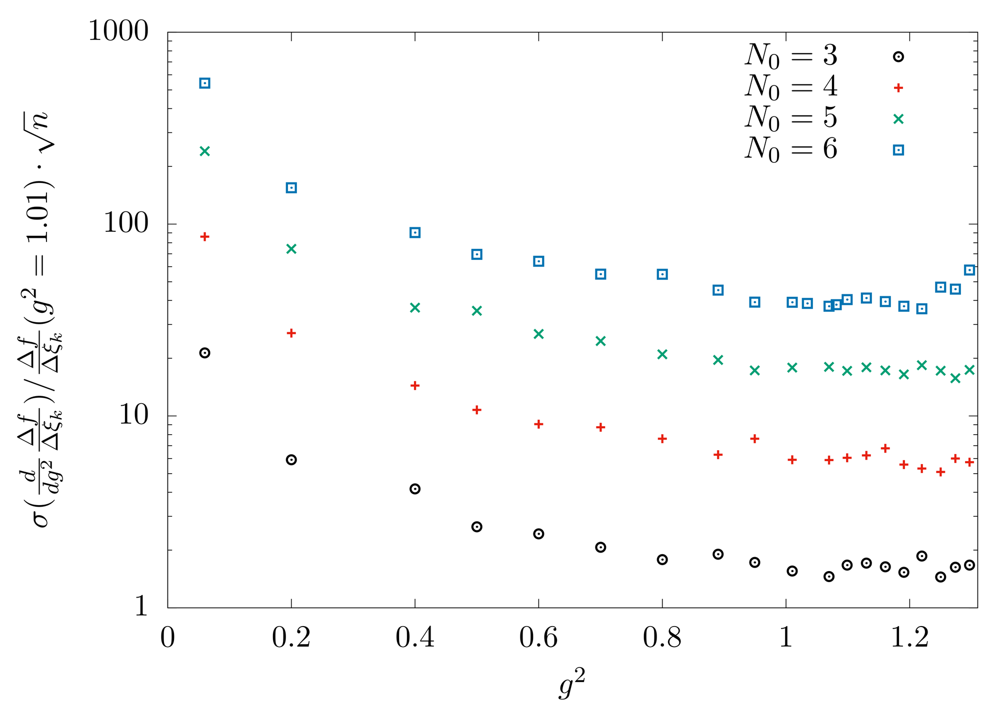
Computation of Z_T

$\frac{d}{dg^2}\frac{\Delta f}{\Delta \xi_k}(N_0, g^2)$ is computed for each N_0 at several different values of g^2, and then integrated with respect to g^2.

The data are fitted with several different Padé functions, linear splines, cubic splines and Akima splines. The variation in the integral is included in the systematic error, which is small compared to the statistical error.

We use lattices with a temporal extent of $N_0 = 3, 4, 5, 6$, a spatial extent of 48^3 and $\xi = (1, 0, 0)$.



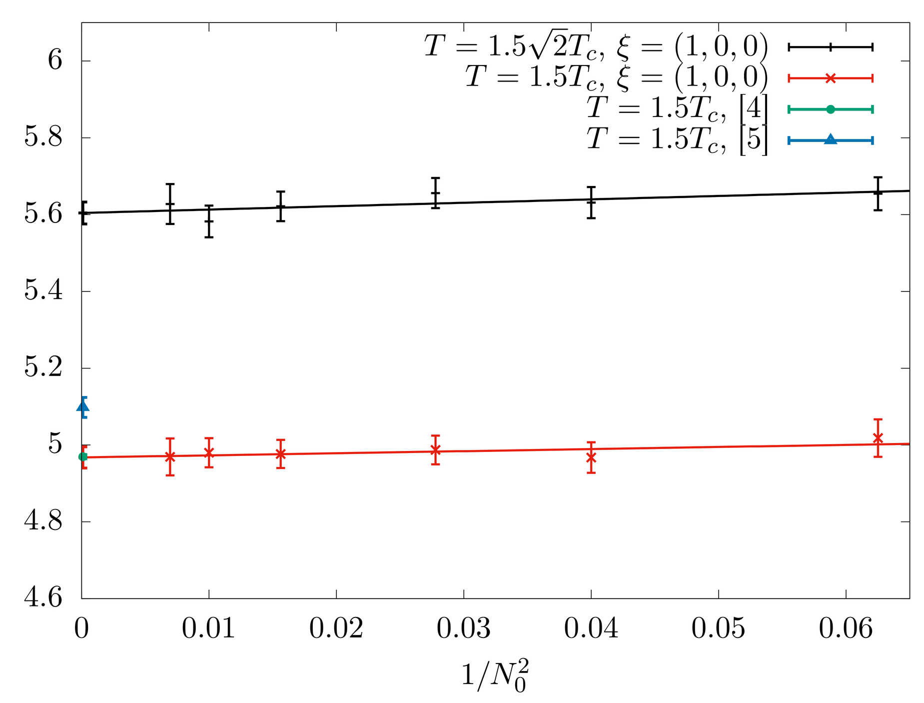


For the computation of $\frac{d}{dg^2}\frac{\Delta f}{\Delta \xi_k}$, the difference of $\langle S\rangle$ has to be measured between two ensembles at the same parameters but having slightly different shifts. The unavoidably occuring large cancellation, which is more prominent in the low g^2 region, makes the computation of $Z_T(N_0, g^2)$ particularly expensive.

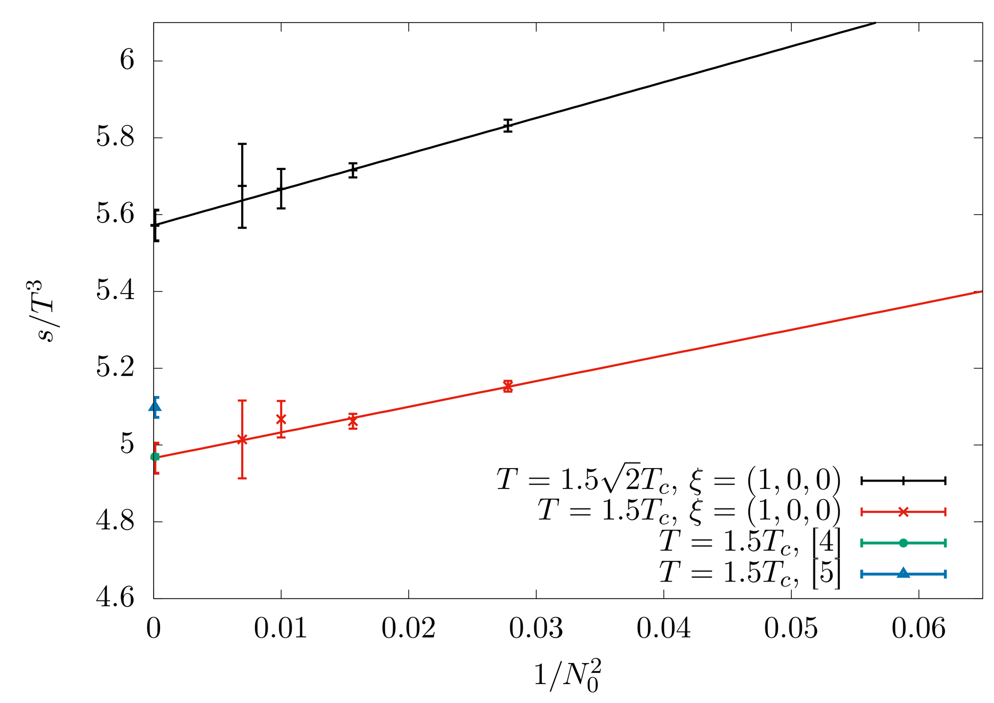
Results

The energy-momentum tensor is computed using the Symanzik improved action, on lattices with temporal extent $N_0 = 4, 5, 6, 8, 10, 12$ and aspect ratio 8, at temperatures $1.5T_c$ and $1.5\sqrt{2}T_c$.

Method I



Method II



The results of the two renormalization methods agree with each other and with the result from [4].

Conclusions

In this work we used a moving frame to compute the entropy at $1.5T_c$ and $1.5\sqrt{2}T_c$ with Symanzik improved action.

Therefore the renormalization constant of the offdiagonal components of the energy-momentum tensor was computed at several g^2. Due to the integration through the low g^2 region, the computation of Z_T is the most costly part of our analysis.

We used two different renormalization schemes and performed a continuum limit. Our findings agree with [4].

Acknowledgements

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