Determination of QCD phase transition line in the canonical approach

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D.L. Boyda - speaker

Mission of this talk

This talk is about:

- Calculation of observables at nonzero chemical potential using LQCD simulations at imaginary μ

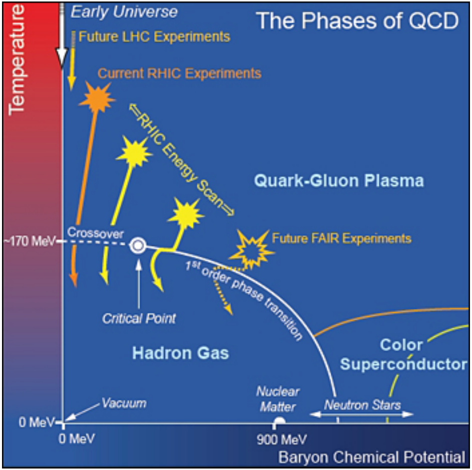
Main goals:

- Reliability range of our results for real chemical potential values
- Propagation of errors from imaginary μ calculations to real μ values
- *(Preliminary)* crossover line

The outline

- Introduction: QCD Phase Diagram, Sign Problem, Canonical Approach, Number Density Formulation
- Lattice Setup
- Number density fits and extrapolation to real μ
- Canonical Approach: Integration method
- Comparison of our results with RHIC data
- Temperature dependence
- Crossover line
- Conclusion

Introductory (1): QCD Phase Diagram



Experiments on heavy ion collisions:
RHIC, LHC, J-PARC, NICA

Theory
LQCD is only tool based on first principles calculation to study QCD Phase Diagram

but ...

Introductory (2): Sign problem

Standard Monte Carlo techniques

$$Z_{GC}(\mu, T, V) = \int \mathcal{D}U (\det \Delta(\mu))^{N_f} e^{-S_G}$$

Fermion determinant satisfies relation

$$[\det \Delta(\mu)]^* = \det \Delta(-\mu^*)$$

- μ is real $\rightarrow$ $\det \Delta(\mu)$ is complex
- μ is imaginary $\rightarrow$ $\det \Delta()$ is real

!!!

Monte Carlo calculations for imaginary μ is free from Sign problem

But how can we extract data for real μ?

Introductory (3): Canonical approach

By definition

$$Z_{GC}(\mu, T, V) = Tr(e^{-\frac{\hat{H}-\mu\hat{N}}{T}}) = \sum_{n=-\infty}^{\infty} < n|e^{-\frac{\hat{H}}{T}}|n > e^{\frac{\mu n}{T}} =$$

$$= \sum_{n=-\infty}^{\infty} Z_C(n, T, V) e^{\frac{\mu n}{T}} = \sum_{n=-\infty}^{\infty} Z_n \xi^n$$

$Z_C(n, T, V)$ - canonical partition function (Z_n in following)
$\xi = e^{\mu/T}$ - fugacity
$\hat{N}$ - operator of any conserved quantum number (baryon, charge, etc.)

For imaginary μ we can calculate Z_n by inverse Fourier transformation (A. Hasenfratz and D. Toussaint, Nucl. Phys. B 371 (1992))

$$Z_n = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-in\theta} Z_{GC}(\mu = i\theta T, T, V).$$

Introductory (4): Number Density Formulation

Number density on a **lattice**:

$$n_B^{lat} = \frac{1}{3N_sN_t}\frac{\partial \ln Z_{GC}}{\partial(a\mu)} = \frac{N_f}{3N_s^3N_t}\int \mathcal{D}Ue^{-S_G}\mathrm{tr}\left[\Delta^{-1}\frac{\partial\Delta}{\partial(a\mu)}\right](det\Delta(\mu))^{N_f}$$

Number density using **canonical formula** ($\frac{\mu}{T} = i\frac{\mu_I}{T} = i\theta$)

$$n_B = \frac{1}{3V}T\frac{\partial}{\partial\mu}\ln\left(\sum_{n=-\infty}^{\infty} Z_n e^{(\mu/T)n}\right) = C\frac{\sum_{n>0}^{\infty} nZ_n(e^{i\theta n} - e^{-i\theta n})}{Z_0 + \sum_{n>0}^{\infty} Z_n(e^{i\theta n} + e^{-i\theta n})}$$

$$= iC\frac{2\sum_{n>0} nZ_n\sin(n\theta)}{1 + 2\sum_{n>0} Z_n\cos(n\theta)}$$

where we used $Z_n = Z_{-n}$
(we write Z_n/Z_0 as Z_n)

Lattice Setup

- clover improved Wilson action
- Iwasaki gauge action
- Lattice $4x16^3$ ($L \approx 3.2fm$, $a \approx 0.2fm$)
- $\frac{m_\pi}{m_\rho} = 0.8 \quad (m_\pi = 0.7 GeV)$
- $T/T_c = 0.93(5)$ (0.84, 0.99, 1.08, 1.20, 1.35)
- 40 values μ_I, 1800 - 3800 configurations (10 trajectories separated)

Parameters were taken from
S. Ejiri et. al., PRD 82, 014508 (2010)

Our cluster: Vostok1 (20 GPU K40)

Number density fits and Extrapolation to real μ

Idea: *number density - $n_B(\mu/T)$ can be parametrized by some function $f^{pol}(x)/f^{sin}(x)$ at imagine μ_I and extrapolated to real region*

Taylor series (Polynomial fit)

$$f^{pol}(x) = \sum_k^{N_{max}} a_k\, x^{2k+1}$$

Fourier series

$$f^{sin}(x) = \sum_k^{N_{max}} f_k\, sin(k\, x), \quad x = \mu_I/T$$

Extrapolation $(x- > ix)$:

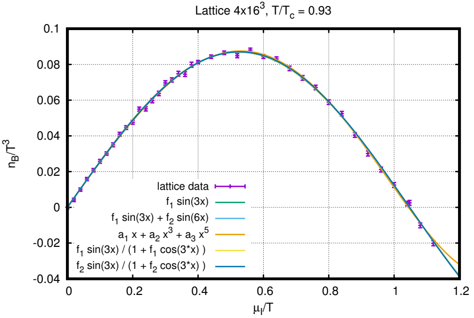
$$\to \sum_k^{N_{max}} (-1)^k\, a_k\, x^{2k+1}$$

$$\to \sum_k^{N_{max}} f_k\, sinh(k\, x)$$

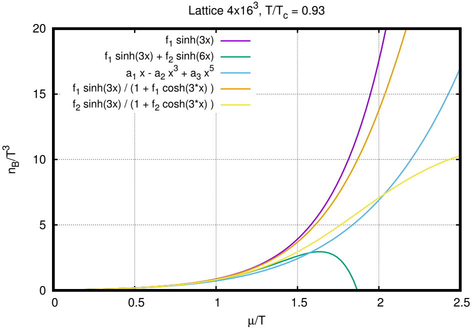
M. D'Elia, et. al., Phys. Rev. D 95, 094503 (2017)
J. Gunther, et. al., arXiv:1607.02493 (2016)
J. Takahashi et. al., Phys. Rev. D91, 014501 (2015)
J. Takahashi et. al., arXiv:1002.0890 (2010)
M. D'Elia, et. al., Phys. Rev. D 80, 014502 (2009)

Number density fits and Extrapolation to real μ (2)

Fit Imagine μ



Extrapolation to Real μ



Questions:

- How can we estimate **reliability range of** μ?
- Is this method able to **find transition** line ?

Canonical approach: Integration method

Idea: *Using observable - baryon number density calculate Grand Canonical Partition Function -* $Z_{GC}(\mu/T)$

(V. Bornyakov, et. al. Phys. Rev. D 95, 094506 (2017))

working at pure imaginary chemical potential $\mu = i\mu_I$*:*

$$n_B = \frac{1}{V}\frac{\partial(lnZ_G)}{\partial(\mu/T)} \quad \rightarrow \quad lnZ_{GC}(\theta) - lnZ_{GC}(0) = V\int_0^\theta d(i\tilde{\theta})\, i\, Im[n_B(\tilde{\theta})]$$

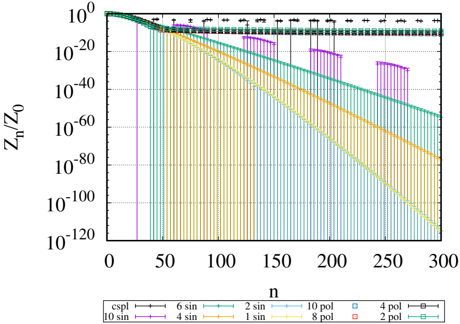
$$\frac{Z_{GC}(\theta)}{Z_{GC}(0)} = exp\left(-V\int_0^\theta dx\, Im[n_B(x)]\right)$$

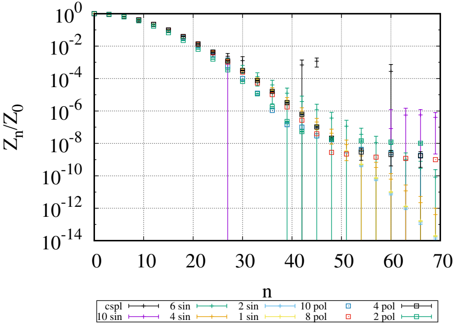
- numerical integration (quadrature formula/cubic spline)
- use of some parametrization ($f^{pol}(x)$ or $f^{sin}(x)$) for $n_B(x)$

Strategy:

1. Calculate $n_B(\mu/T)$ using LQCD simulation at imaginary μ
2. Calculate Z_{GC}
3. Calculate Z_n as Fourier transformation of Z_{GC}
4. Using Z_n calculate any thermodynamic observable

Canonical approach: Integration method (2)



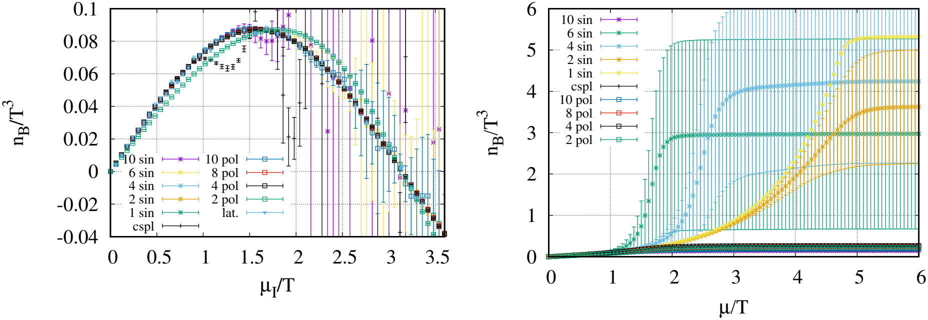


Z_n must be positive!

$$n_B = \sum_k^{N_{max}} sin(k\,x)$$

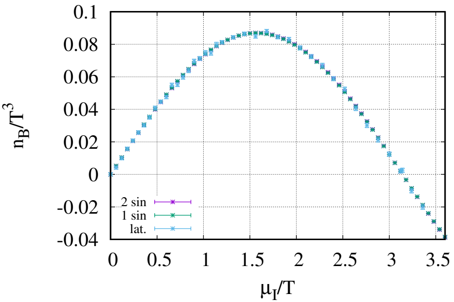
N_{max}	$f_1 * 10^6$	$f_2 * 10^6$	$f_3 * 10^6$	$f_4 * 10^6$	$\chi^2/d.o.f.$
1	1359(4)	-	-	-	0.94
2	1360(4)	-5(4)	-	-	0.93
4	1360(4)	-4(4)	-3(4)	-2(4)	0.96

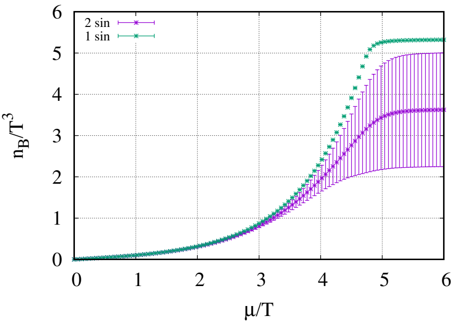
Canonical approach: Integration method (3)



$$n_B = C\frac{2\sum_n^{N_{max}} nZ_n \sinh(n\theta)}{1+2\sum_n^{N_{max}} Z_n \cosh(n\theta)} \xrightarrow{\theta\to\infty} C\frac{2N_{max}Z_{N_{max}} sinh(N_{max}\theta)}{2Z_{N_{max}} cosh(N_{max}\theta)} \sim C\, N_{max}$$

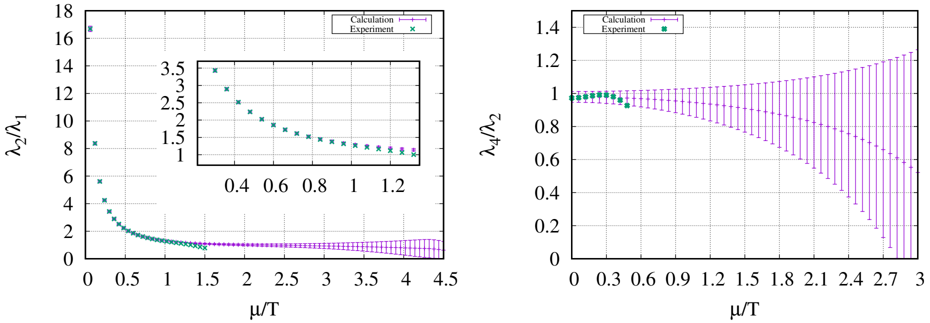
Canonical approach: Integration method (4)





Comparison of our results with RHIC data

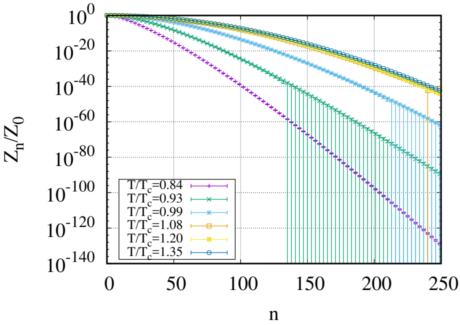
$$\lambda_k(\mu) = \left(T\frac{\partial}{\partial\mu}\right)^k \log\left(\sum_n Z_n \xi^n\right)$$

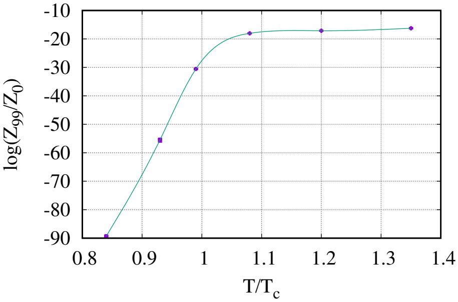


Z_n constructed for proton multiplicity RHIC data ($\sqrt{s_{NN}} = 62.4$ GeV) were taken from *A. Nakamura, K. Nagata, PTEP 3, 33D01 (2016)*

Temperature dependence

Using Z_n dependence on temperature we can calculate observables for any temperature and study crossover(transition) line

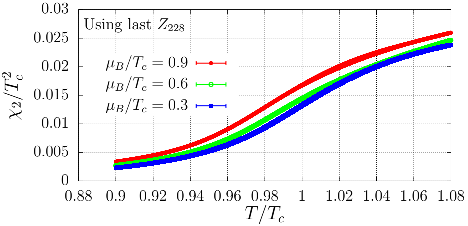


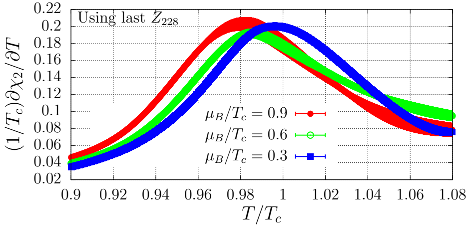


~~See talk by Vladimir Goy~~ → next 2 slides

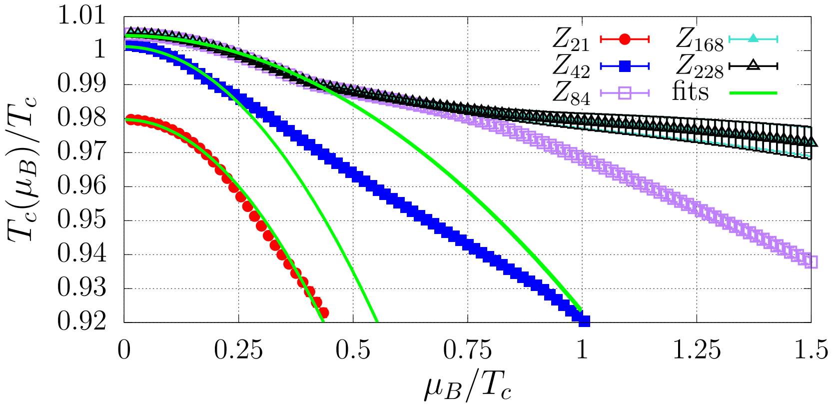
(Preliminary) Dependence of susceptibility on T

$$\chi_2(\mu, T) = \frac{\partial}{\partial\mu} n_B(\mu, T) = \frac{T}{V}\left(\frac{\partial}{\partial\mu}\right)^2 log\left(\sum_n^{N_{max}} Z_n(T)\xi^n\right), \ \xi = e^{\mu/T}$$





(Preliminary) Crossover line



for small μ crossover line can be fitted by $f(x) = a - bx^2$, a = 1.0049(3), b = 0.033(10)

Conclusions

- Canonical approach and analysis with Z_n allow to choose the best parametrization function and find reliability range
- λ_2/λ_1 and λ_4/λ_2 were calculated and agreed with RHIC data
- Using Z_n dependence on temperature we can study any temperature
- *(Preliminary)* Crossover line was calculated
- Integration method allows study QCD at finite density without any anzatz (using numerical integration for calculation Z_{GC})