Investigating BSM models with large scale separation

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based on
R. Brower, A. Hasenfratz, C. Rebbi, E. Weinberg, O. Witzel
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Assume we have unlimited computational power 

We simulate our theories on lattices of arbitrary large size, for whatever value of the parameters, and with unlimited accuracy.

Ultimately we go to the continuum limit: in this limit the value, in lattice units, of any observable whose dimension is a positive power of mass will have to approach zero.

Consider first SU(3) with 4 flavors of mass m.

SU(3), 4 flavors

We simulate with definite β and, initially, with $m = 0$. The theory is well defined. Correlation functions exhibit an exponential decay and we can measure observables, in particular F_π.

For $\beta \to \infty, \quad F_\pi \to 0$.

To go to the continuum limit we let $\beta \to \infty$: ratios of masses and other observables over F_π go to well defined limits.

We can now take one or two masses, $m_1, m_2 \neq 0$. As $\beta \to \infty$ we also send $m_1, m_2 \to 0$ in a controlled manner. The spectrum of masses and other observables will depend on the way m_1, m_2 go to zero with β and the ratios of observables to F_π will tend to finite limits. We recover QCD with massless up and down quarks and non-vanishing strange and charm masses.

Nihil sub sole novi. (Nothing new here.)

SU(3), 12 massless flavors

We simulate the SU(3) theory with 12 fermions of mass $m = 0$.

I assume that theory is conformal. We can take any $\beta > \beta_{\rm cr}$: the correlations functions will exhibit power law behavior at large distances (we simulate lattice of arbitrary large extent.)

The behavior of the correlation functions for large distances has power law behavior, but is non-trivial. If we rescale lengths appropriately, the long range behavior with any two values of β will be the same, because under renormalization group transformations the theory runs to to an IRFP, in the space of infinite couplings. (The location of the IRFP will depend on the specifics of the renormalization.)

If we do the tricky step of going first to a lattice that is so large that we are able consider sublattices of much smaller size, but still sufficiently large that they can represent the short distance behavior in the continuum, then the short distance behavior will be trivial because the theory is asymptotically free in the UV.

SU(3), 4 massless and 8 massive flavors

We keep 4 fermion massless and give a mass m to the other 8 fermions. The observables (in lattice units) will now depend on β and m.

At large distances the correlation functions will exhibit exponential behavior and we can measure a non-vanishing F_π as well as non-vanishing masses and other observables.

In renormalization transformation, in the space of infinite couplings, the system will move away from the IRFP with $m = 0$ because m is a relevant parameter. The recover the continuum limit with $F_\pi \to 0$ we must let m approach $m = 0$, i.e. its fixed-point value, but, and here is the crucial point, contrary to the case of the four flavor theory, now we do not need to let $\beta \to \infty$, since, as $m = 0$, the theory will flow closer and closer the IRFP no matter what is the original β.

If we take m sufficiently small, the ratios of observables, whether built out of the massless fermions, massive fermions or both, and in particular the ratios of observables to F_π, will be independent of m and β.

SU(3), 4 massless and 8 massive flavors, continued

If we give two masses, m_1, m_2, to the massive fermions, or if we give a small mass m_ℓ to the massless fermions, hyperscaling arguments (A. Hasenfratz, Lattice 2016) show that the ratios of observables to F_π or among themselves only depend on the ratios of the Lagrangian masses.

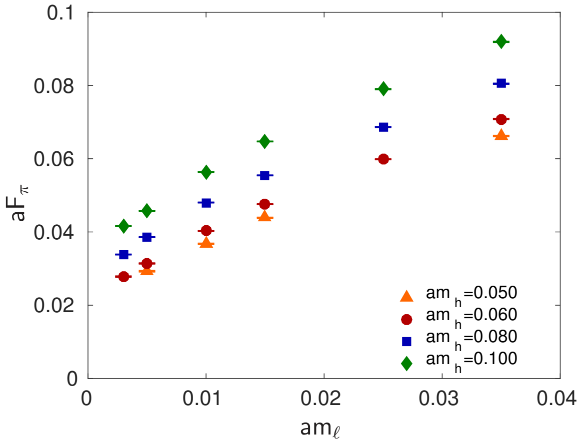
In conclusion, we can take F_π to set the scale, and then the continuum theory is fully defined by the ratio of Lagrangian masses which must all go to zero. If we only have a common mass parameter m for the 8 massive fermions, while the 4 light fermions are kept at zero mass, then the theory is fully defined without any free parameter. In this case m plays a role similar to β for massless QCD.

We do not have unlimited computational power 

We simulated a theory with:

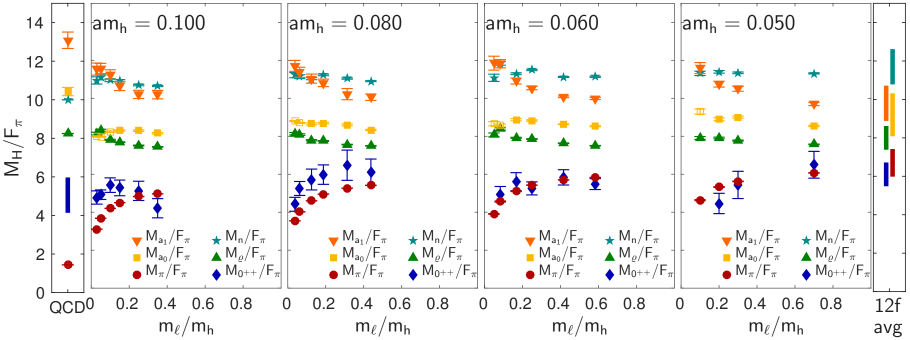
- 1 staggered field (= 4 flavors) with light mass m_ℓ plus 2 staggered fields (= 8 flavors) with heavy mass m_h. Simulations done with $am_\ell = 0.003, 0.005, 0.010, 0.015, 0.025, 0.035$, $am_h = 0.050, 0.060, 0.080, 0.100$.
- fundamental-adjoint gauge action with $\beta = 4, \beta_a = -\beta/4$ [Cheng et al. 2013][Cheng et al. 2014], nHYP smeared staggered fermions [Hasenfratz et al. 2007].
- lattice sizes mostly $24^3 \times 48$ and $32^3 \times 64$, but also $16^3 \times 32$ (exploratory), $36^3 \times 64$ and $48^3 \times 96$.
- We also simulated a system with $\beta = 4.4$, $am_h = 0.070$, $am_\ell = 0.009, 0, 013125, 0.0175, 0.0245$, on a $32^3 \times 64$ lattice.

The pseudoscalar decay constant in lattice units



As expected, aF_π (F_π in lattice units) goes to zero as $am_h \to 0$ approaching the fixed-point value.

Spectrum of light fermion composite states



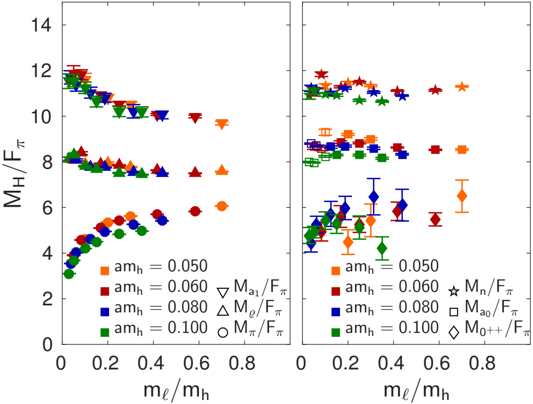
Shown are the ratios of the masses of π, ρ, a_0, a_1, N, and of the 0^{++} over F_π.

The left panel shows results for QCD, the right panel for a (mass deformed) 12 flavor theory.

The 0^{++} is light ($M_{0^{++}} < M_\rho$), tracking the pion.

M_π/F_π decreases towards zero for $m_\ell/m_h \to 0$ giving evidence of chiral symmetry breaking.

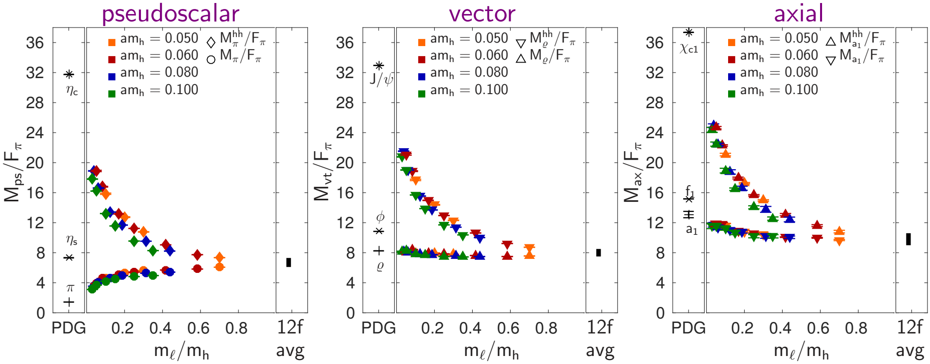
Hyperscaling: light hadron masses



- $M_n/F_\pi \approx 11$
- $M_\varrho/F_\pi \approx 8$
- $M_{0^{++}}/F_\pi \approx 4-5$ (taking the chiral limit is difficult but 0^{++} well separated from the ϱ)

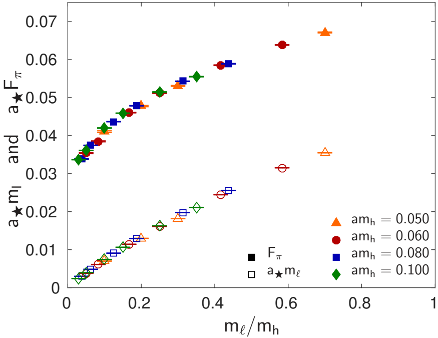
Superimposing the data for different m_h shows strong evidence of hyperscaling, as one would expect in the neighborhood of an IR fixed-point (cfr. Anna Hasenfratz, Lattice 2016.)

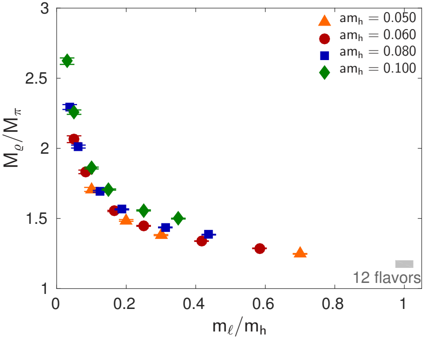
Light-light and heavy-heavy masses



The heavy-heavy spectrum also gives clear evidence of hyperscaling.
The upward bend is due to the fact that F_π shows a marked decrease for $m_\ell/m_h \to 0$, but still tends to a finite limit.
Note the large separation of scales.

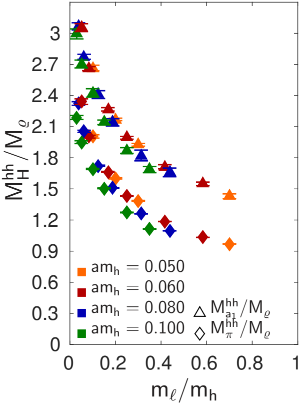
The system is chirally broken

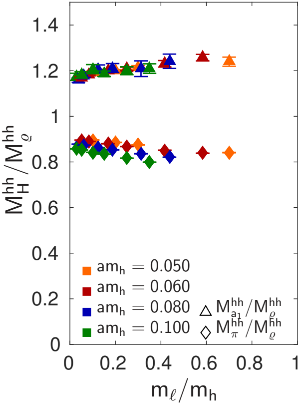




(In the left graph we use a different method to set the scale, based on the Wilson flow.)

Ratios over M_ϱ and M_ϱ^{hh}

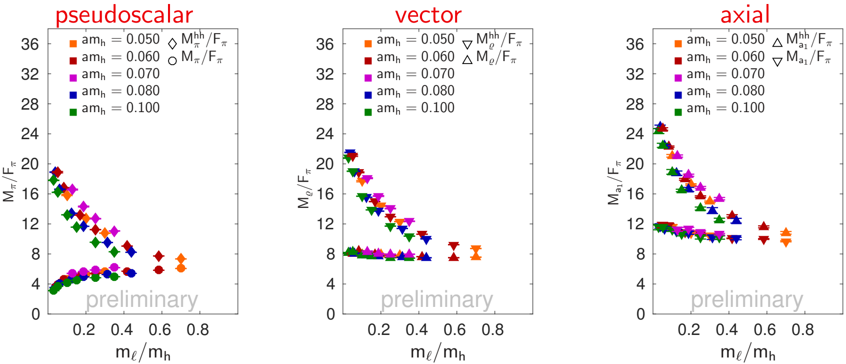




The increase of the ratios in the left graph is due to the decrease of the light-light M_ρ in the denominator. Ratios against heavy-heavy M_ρ show little variation.

We did not calculate the spectrum of heavy-light states, but expect hyperscaling there also.

Irrelevance of β



The coupling constant β is an irrelevant parameter. Ratios of physical quantities should be largely independent of β.

To verify this we performed additional simulations with $\beta = 4.4, am_h = 0.070$ (arXiv:1611.07427); data showed in the previous slide were obtained with $\beta = 4.0, am_h = 0.100, 0.080, 0.060, 0.050$.

The new results confirm hyperscaling and the irrelevance of β.

Conclusions (so far)

- Our results give evidence for hyperscaling in the heavy fermion masses and the irrelevance of the gauge coupling. For sufficiently small m_h ratios of physical quantities will depend only on m_ℓ/m_h (and will be independent of β.)

- The continuum limit would be reached for $m_\ell, m_h \to 0$ with m_ℓ/m_h kept fixed. In particular, in the chiral limit $m_\ell = 0$, m_h would only serve to set the scale. There is some analogy between m_h in our model and the bare coupling constant g in QCD: near the fixed-point the physical values of masses and other observables do not depend on them.

- In principle, in the chiral limit, the theory built in the neighborhood of the IR fixed-point would be a self-consistent, parameter free theory, very much like QCD with massless quarks.

SU(3) with 4 massless and 6 massive fermions

- Even if a theory built on the IR fixed-point were self-consistent, like QCD it would have to be embedded into a more general BSM framework.

- Then, on various phenomenological grounds (scaling dimensions of the condensate, of baryonic operators etc.), it can be argued that the theory should be strongly coupled: thus, within the conformal window, but as close as possible to its beginning.

- There is some evidence (T-W Chiu, Proceedings, lattice 2016, Anna Hasenfratz, Friday talk) that an SU(3) theory with 10 massless flavors has an IR fixed point at a stronger coupling than SU(3) with 12 flavors.

- The Lattice Strong Dynamics Collaboration (http://lsd.physics.yale.edu/) has embarked in an investigation of SU(3) with 4 massless and 6 massive fermions, using the domain-wall fermion discretization.

Additional slides

Computational details

- 1 staggered field (= 4 flavors) with light mass m_ℓ plus 2 staggered fields (= 8 flavors) with heavy mass m_h. Simulations done with $am_\ell = 0.003, 0.005, 0.010, 0.015, 0.025, 0.035$, $am_h = 0.050, 0.060, 0.080, 0.100$.
- fundamental-adjoint gauge action with $\beta = 4, \beta_a = -\beta/4$ [Cheng et al. 2013][Cheng et al. 2014], nHYP smeared staggered fermions [Hasenfratz et al. 2007].
- lattice sizes mostly $24^3 \times 48$ and $32^3 \times 64$, but also $16^3 \times 32$ (exploratory), $36^3 \times 64$ and $48^3 \times 96$.
- Also $\beta = 4.4$, $am_h = 0.070$, $am_\ell = 0.009, 0, 013125, 0.0175, 0.0245$, on a $32^3 \times 64$ lattice.
- lattice generations with hybrid MC with one Hasenbusch intermediate mass; most simulations/measurements performed with FUEL [J. Osborn]; most calculations done with USQCD SciDAC software on USQCD computers at Fermilab and NSF-MRI computers at MGHPCC.
- disconnected diagrams (for 0^{++}) computed with stochastic sources (6 sources, full color and time dilution, even-odd space dilution.)

Hyperscaling: Wilson RG

Scale change: $\mu \to \mu' = \mu/b$, with $b > 1$, transforms the bare masses $\widehat{m} = am$: $\widehat{m}(\mu) \to \widehat{m}(\mu') = b^{y_m}\widehat{m}(\mu)$ (increase).
The bare coupling approaches its fixed point $g \to g'$: Any 2-point correlator $C_H(t; g, \widehat{m}_i, \mu) \to b^{-2y_H} C_H(t/b; g', b^{y_m}\widehat{m}_i, \mu)$.
But $C_H(t) \propto exp(-M_H t)$, thus $aM_H \propto (\widehat{m})^{1/y_m}$ (hyperscaling)
Amplitudes also show hyperscaling, thus M_H/F_{PS} stays constant.

Hyperscaling with a mass-split system:

If some of the masses remain massless, Wilson RG arguments do not change!
$C_H(t; g, \widehat{m}_i, \mu) \to b^{-2y_H} C_H(t/b; g', b^{y_m}\widehat{m}_i, \widehat{m}_\ell = 0, \mu)$.
Mass-split systems show the same hyperscaling in the $m_\ell = 0$ limit: $aM_H \propto (\widehat{m})^{1/y_m}$.
H can be build up of only light, light and heavy, or only heavy flavors: M_H/F_π ratios are independent of m_h for $m_h \ll 1$.