

# Temperature dependence of bulk viscosity of SU(3) gluodynamics

V.V. Braguta

ITEP

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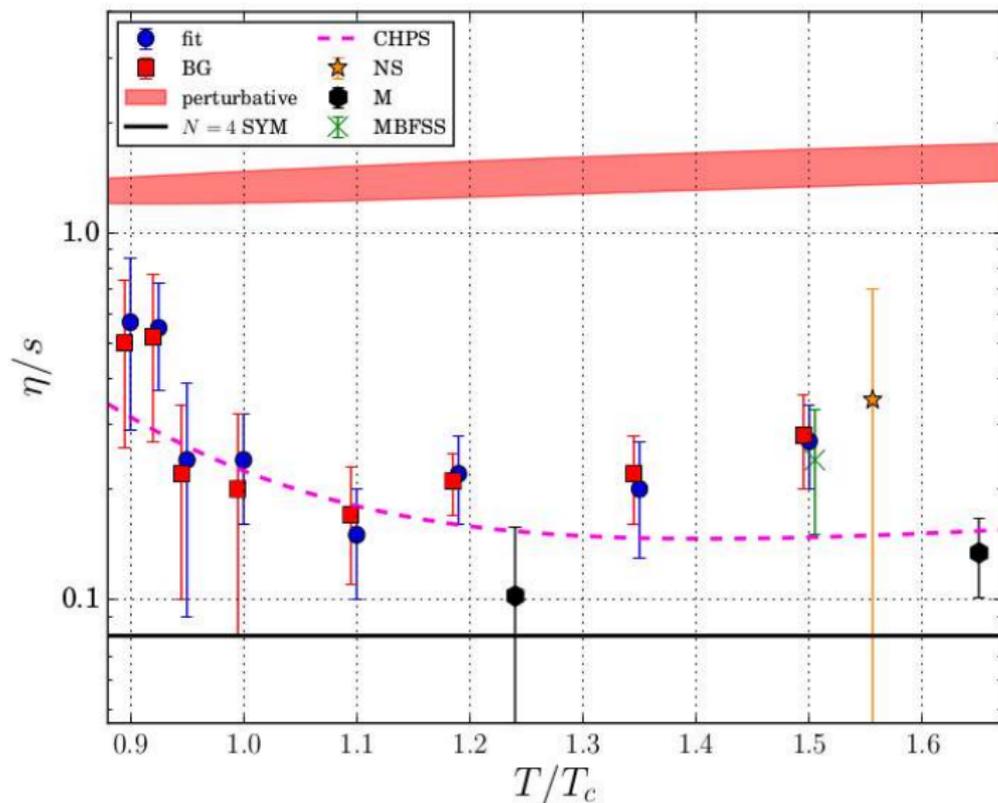
## Outline:

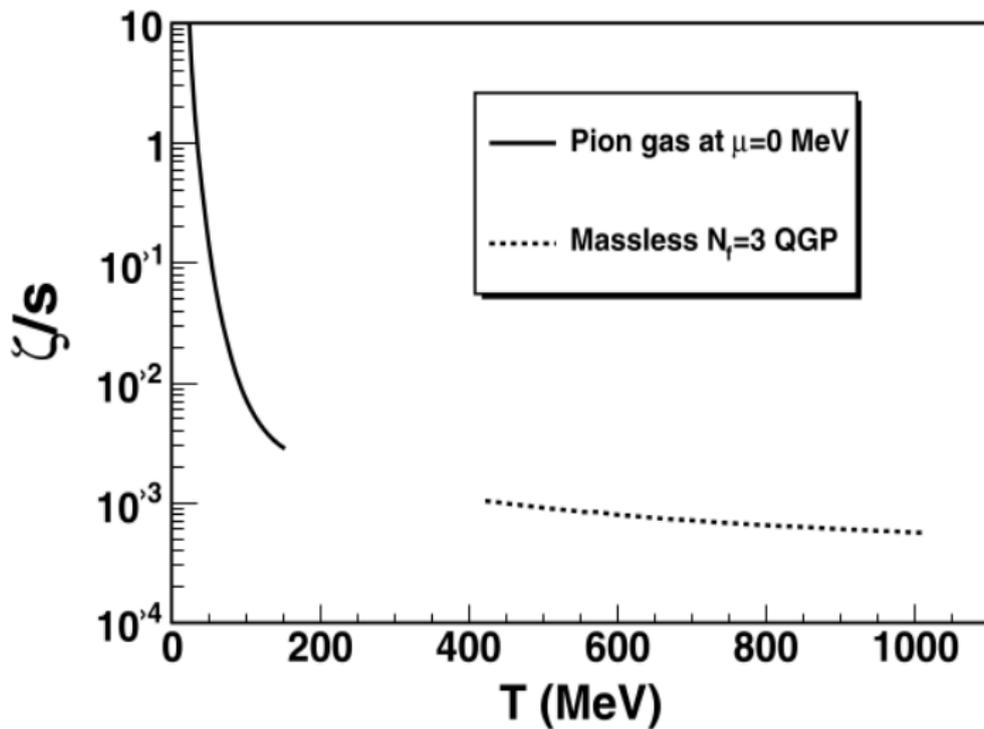
- Introduction
- Details of the calculation
- Bulk viscosity
  - Middle point method
  - Backus-Gilbert method
- Conclusion

## Relativistic Hydrodynamics

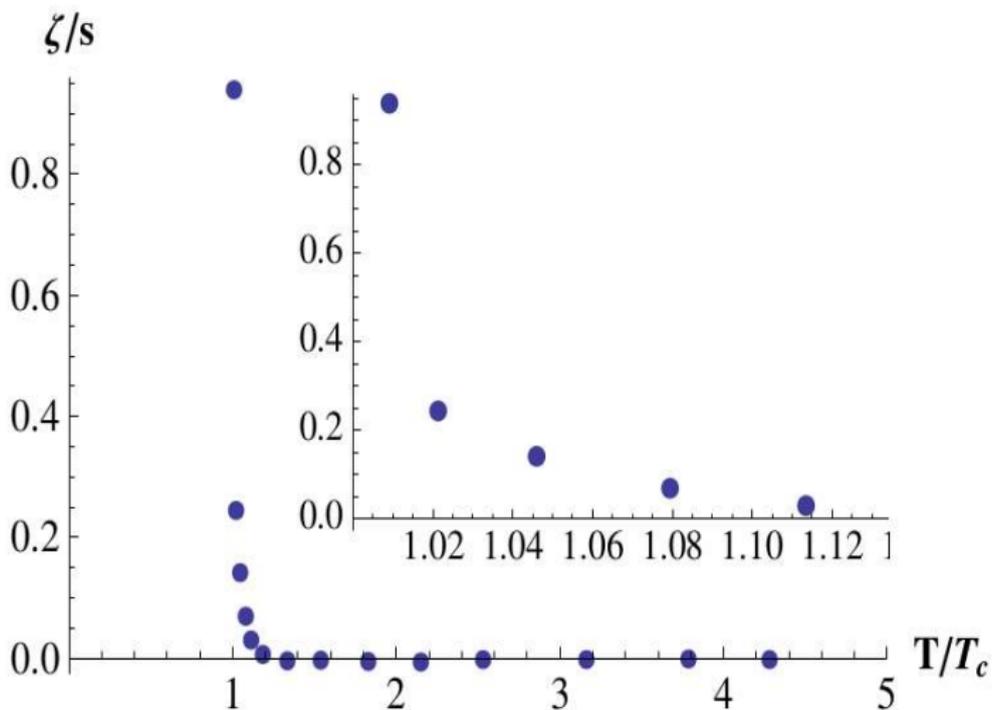
- $T^{\mu\nu} = (e + p)u^\mu u^\nu + pg^{\mu\nu} + (\eta\nabla^\langle\mu u^{\nu\rangle} + \zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \dots$   
 $\nabla^\alpha = \Delta^{\alpha\nu}\partial_\nu, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$   
 $\nabla^\langle\mu u^{\nu\rangle} = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$
- EOM  $\partial_\mu T^{\mu\nu} = 0$
- Non-relativistic limit ( $u^\mu = (1, \vec{v})$ )
  - *Continuity equation:*  $\partial_t \rho + \rho(\vec{\partial}\vec{v}) + \vec{v}\vec{\partial}\rho = 0$
  - *Navier–Stokes equation:*  $\frac{\partial v^i}{\partial t} + v^k \frac{\partial v^i}{\partial x^k} = -\frac{1}{\rho} \frac{\partial p}{\partial x^i} - \frac{1}{\rho} \frac{\partial \Pi^{ki}}{\partial x^k}$
  - *Viscous stress tensor:*  $\Pi^{ik} = -\eta\left(\frac{\partial v^i}{\partial x^k} + \frac{\partial v^k}{\partial x^i} - \frac{2}{3}\delta^{ik}\frac{\partial v^l}{\partial x^l}\right) - \zeta\delta^{ik}\frac{\partial v^l}{\partial x^l}$
- $\eta$ –shear viscosity,  $\zeta$ –bulk viscosity

# Shear viscosity (JHEP 1704 (2017) 101)





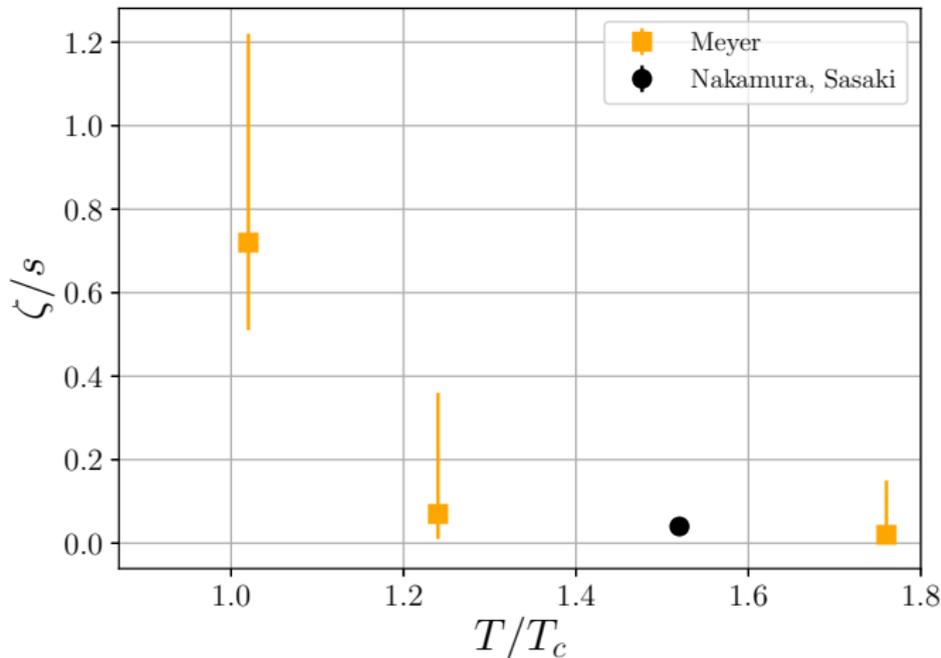
- CHPT: A. Dobado, F.J. Llanes-Estrada, J.M. Torres-Rincon, Physics Letters B 702 (2011) 43
- Perturbative QCD: P. Arnold, C. Dogan, G. Moore, Physical Review D 74, 085021 (2006)



● Low energy theorems of QCD: D. Kharzeev, K. Tuchin, JHEP 0809 (2008) 093

## Previous works (SU(3) gluodynamics):

- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev.Lett. 100 (2008) 162001



# Lattice calculation of bulk viscosity

The first step:

Measurement of the correlation function:

$$C_E(t) = \langle T_\mu^\mu(t) T_\nu^\nu(0) \rangle$$

The second step:

Calculation of the spectral function  $\rho(\omega)$ :

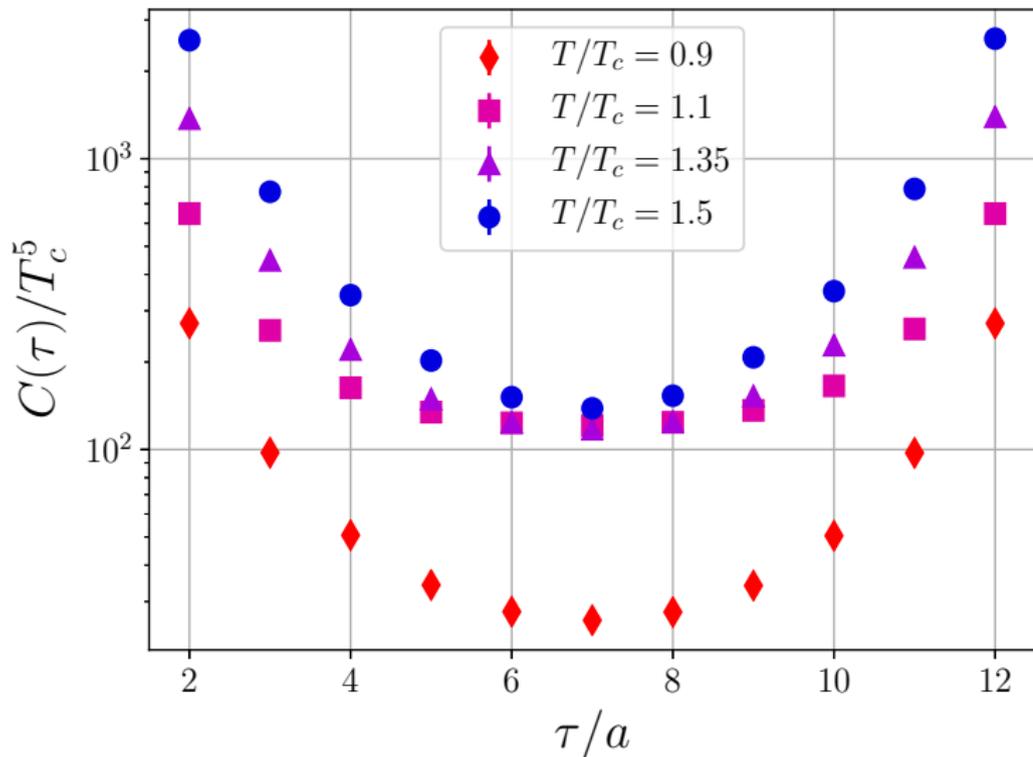
$$C_E(t) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$
$$\zeta = \frac{\pi}{9} \lim_{\omega \rightarrow 0} \frac{\rho(\omega)}{\omega}$$

## Details of the calculation

- SU(3) gluodynamics
- Two-level algorithm
- Lattice size  $32^3 \times 16$
- Temperatures  $T/T_c = 0.9, 0.925, 0.95, 1.0, 1.1, 1.2, 1.35, 1.5$
- Accuracy  $\sim 3\%$  at  $t = \frac{1}{2T}$
- Clover discretization for the  $\hat{F}_{\mu\nu}$
- Renormalization of EMT: F. Karsch, Nucl.Phys. B205 (1982) 285-300
- ...

**Our results are preliminary !**

# Correlation functions

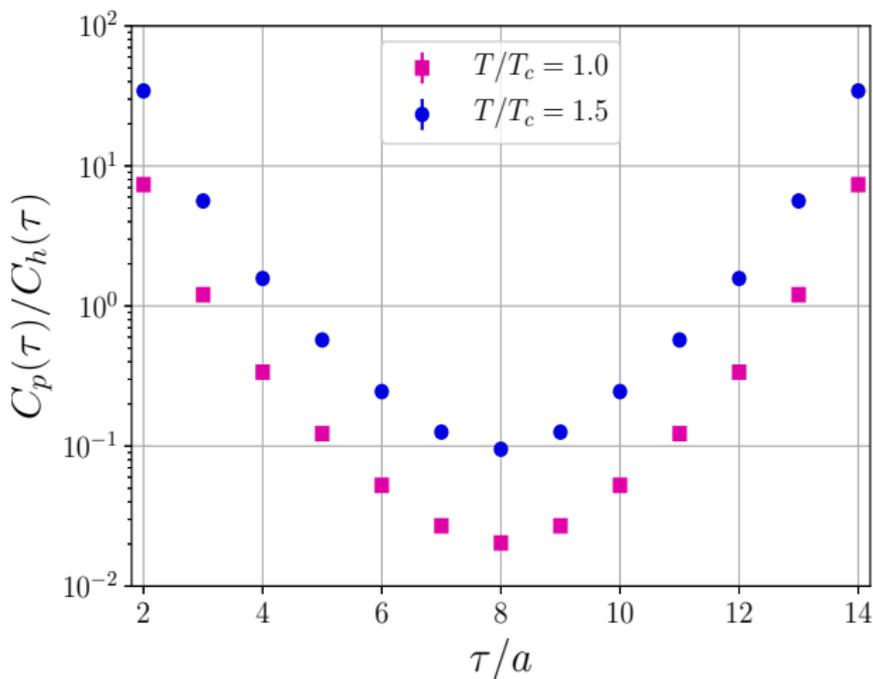


## Spectral function

$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{\text{ch}\left(\frac{\omega}{2T} - \omega t\right)}{\text{sh}\left(\frac{\omega}{2T}\right)}$$

Properties of the spectral function:

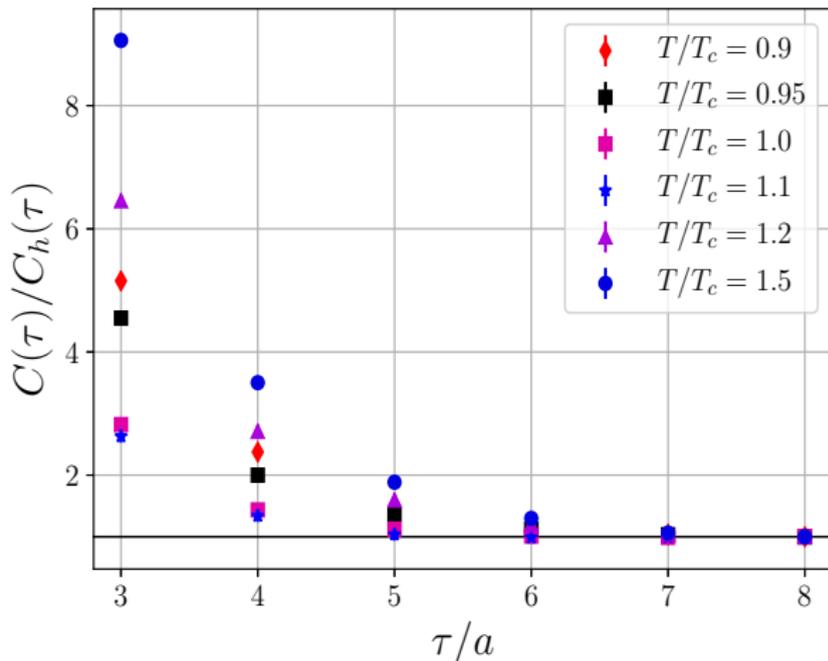
- $\rho(\omega) \geq 0$ ,  $\rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom:  $\rho(\omega)|_{\omega \rightarrow \infty}^{NLO} = d_A \left( \frac{11\alpha_s}{(4\pi)^2} \right)^2 \omega^4$   
compare with shear channel  $\sim d_A \frac{1}{10(4\pi)^2} \omega^4$
- Hydrodynamics:  $\rho(\omega)|_{\omega \rightarrow 0} = \frac{9}{\pi} \zeta \omega$

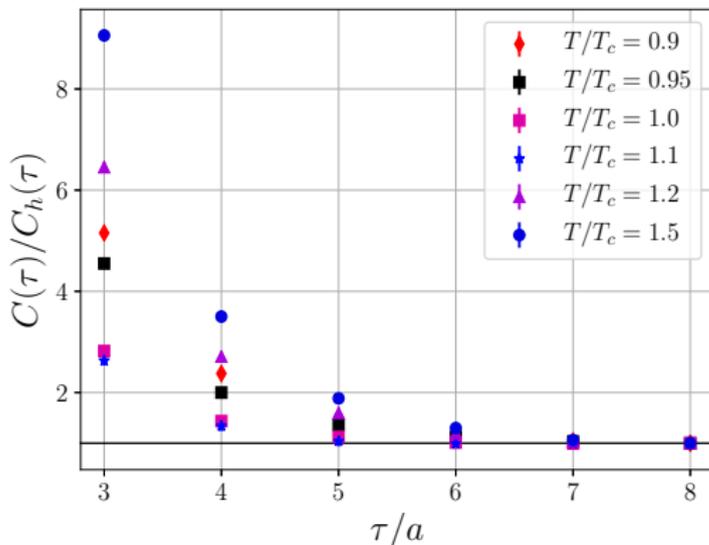


- In the region  $\tau/a \sim \frac{\beta}{2}$  hydrodynamics is dominant
- In the region  $\tau/a \sim \text{few}$  perturbative contribution is dominant

## Hydrodynamical approximation

$$C_h(\tau) = \int_0^\infty d\omega \rho_h(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega\tau\right)}{sh\left(\frac{\omega}{2T}\right)}, \quad \rho_h(\omega) = \frac{9}{\pi} \zeta \omega \theta(\omega_0 - \omega)$$





### Middle point estimation of bulk viscosity

- In the vicinity of the phase transition hydrodynamics very works well!
- $C_h\left(\frac{\beta}{2}\right) = \frac{9}{\pi} \zeta \int_0^{\omega_0} d\omega \frac{\omega}{sh\left(\frac{\omega}{2T}\right)}$
- $\omega_0$  is varied within the interval 2 – 3 GeV

## Backus-Gilbert method for the spectral function

- Problem: find  $f(\omega)$  from the integral equation

$$C(x_i) = \int_0^\infty d\omega f(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{\text{ch}\left(\frac{\omega}{2T} - \omega x_i\right)}{\text{sh}\left(\frac{\omega}{2T}\right)}$$

- Define an estimator  $\tilde{f}(\bar{\omega})$  ( $\delta(\bar{\omega}, \omega)$  - resolution function):

$$\tilde{f}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega}, \omega) f(\omega)$$

- Let us expand  $\delta(\bar{\omega}, \omega)$  as

$$\delta(\bar{\omega}, \omega) = \sum_i b_i(\bar{\omega}) K(x_i, \omega) \quad \tilde{f}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

- Goal: minimize the width of the resolution function

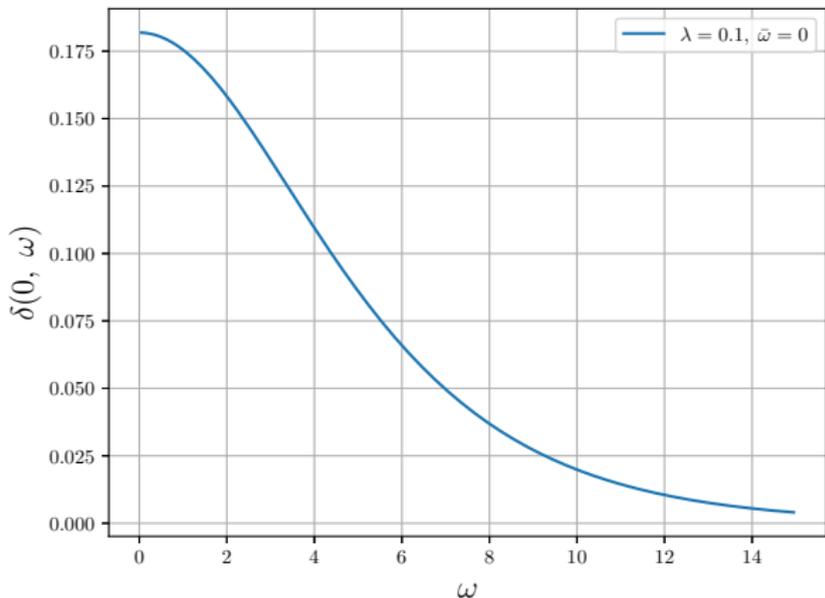
$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$

$$W_{ij} = \int d\omega K(x_i, \omega) (\omega - \bar{\omega})^2 K(x_j, \omega), \quad R_i = \int d\omega K(x_i, \omega)$$

- Regularization by the covariance matrix  $S_{ij}$ :

$$W_{ij} \rightarrow \lambda W_{ij} + (1 - \lambda) S_{ij}, \quad 0 < \lambda < 1$$

Resolution function  $\delta(0, \omega)$  ( $T/T_c = 1.5$ ,  $\lambda = 0.1$ )



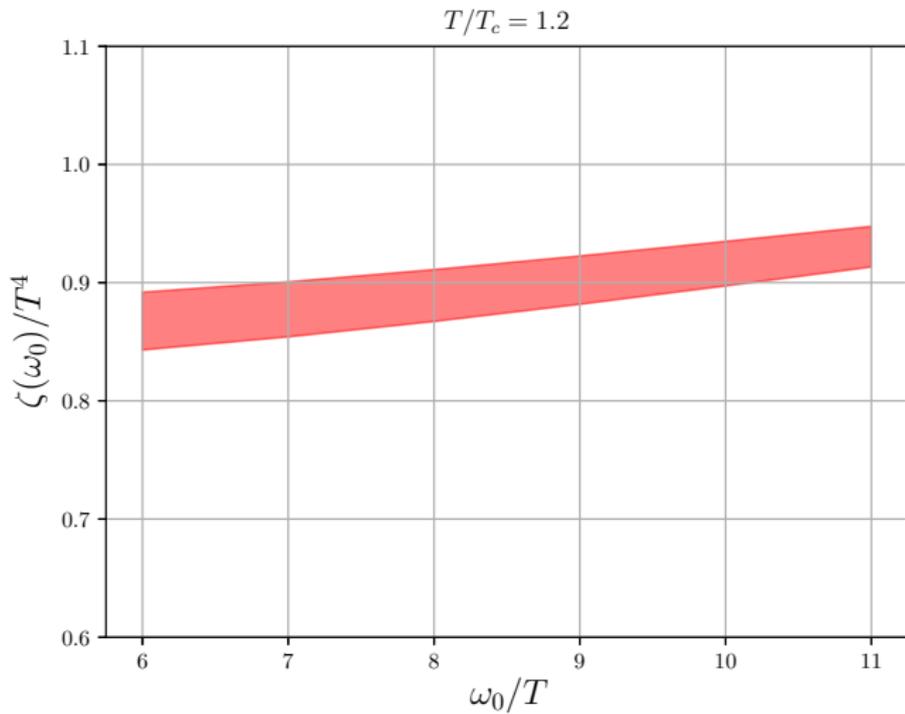
- Width of the resolution function  $\omega/T \sim 5$

## Removal of the ultraviolet contribution

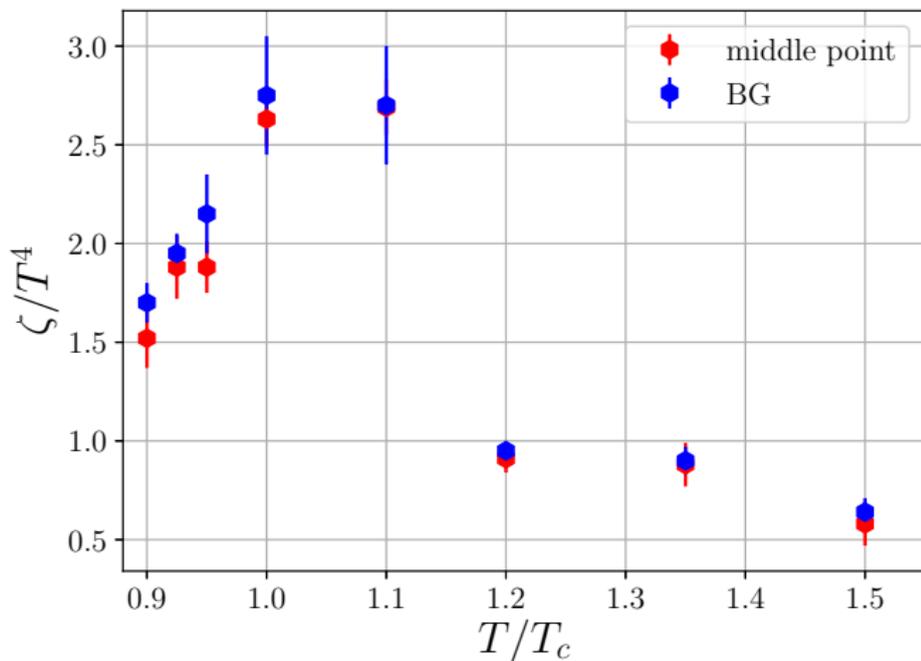
- Take ultraviolet contribution in the form:

$$\rho_{ultr} = A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

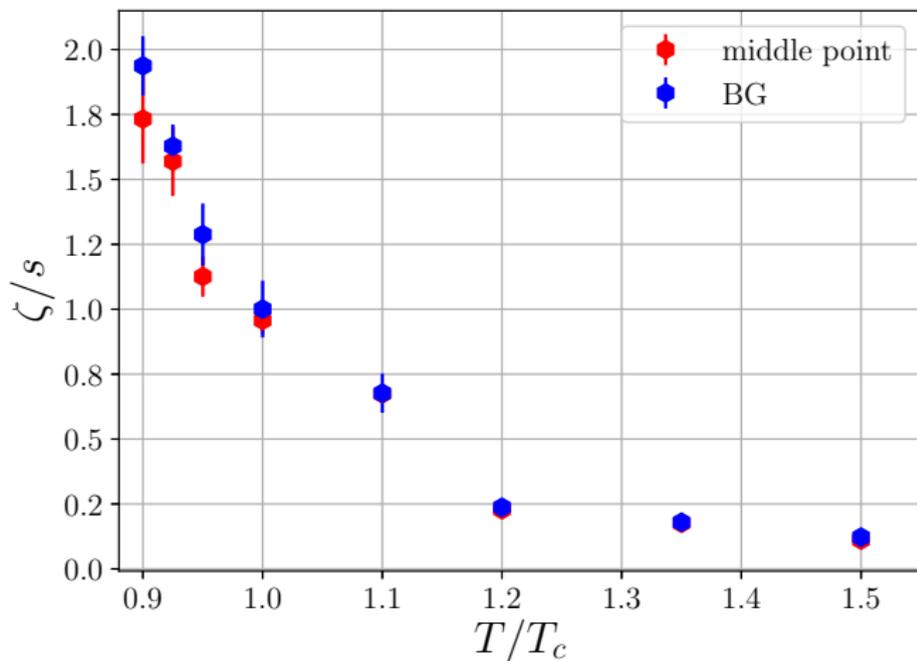
- Determine the value of the constant A from the  $C(\tau/a = 2)$
- Subtract ultraviolet contribution and obtain  $\zeta/T^4$  as a function of  $\omega_0$



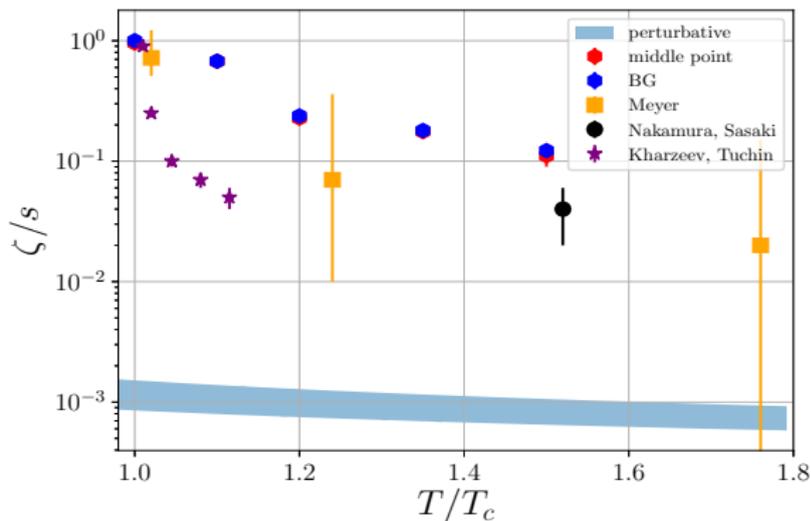
## Our results (preliminary!)



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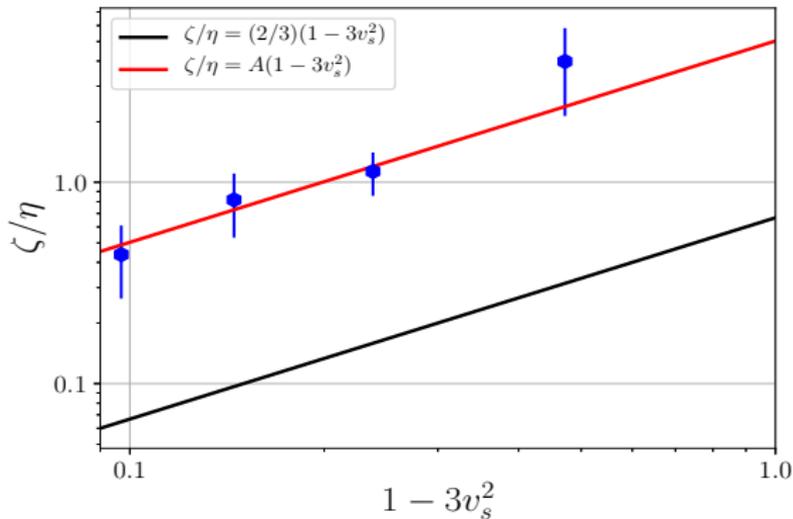


## Comparison with other approaches

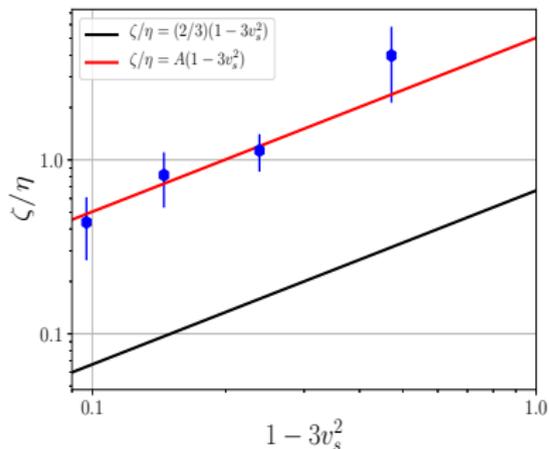
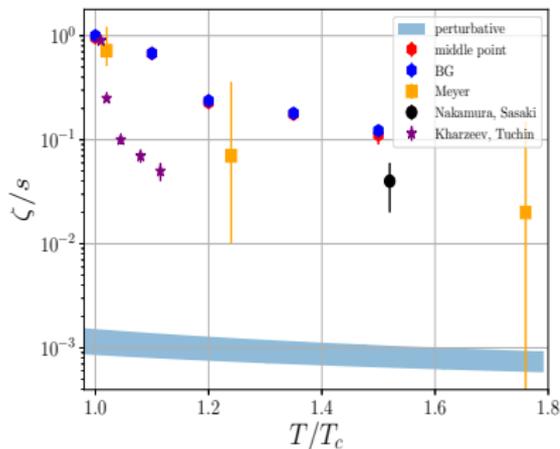


- Agreement with other lattice studies
- Large deviation from perturbative results

## Weakly or strongly coupled QGP?



- Weakly coupled system  $\zeta/\eta \sim (1 - 3v_s^2)^2$
- Strongly coupled system  $\zeta/\eta \sim (1 - 3v_s^2)$
- $\zeta/\eta \geq \frac{2}{3}(1 - 3v_s^2)$  (A. Buchel, Physics Letters B663, 286 (2008))



## Conclusion:

- We calculated  $\zeta/s$  for set of temperatures  $T/T_c \in (0.9, 1.5)$
- Agreement with previous lattice results
- Large deviation from perturbative calculation
- QGP reveals the properties of strongly coupled system