$ud\bar{b}\bar{b}$ tetraquark resonances with lattice QCD potentials and the Born-Oppenheimer approximation

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Outline

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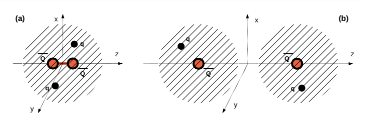
Introduction

Physics of $ud\bar{b}\bar{b}$ tetraquarks



(a) At small separations the static antiquarks $\bar{Q}\bar{Q}$ interact by perturbative one-gluon exchange.
(b) At large separations the light quarks qq screen the interaction and the four quarks form two rather weakly interacting B mesons.
The quantitative question: is the b quark heavy enough for binding?

- A main long standing problem in modern physics is to understand exotic hadrons.
- We investigate tetraquarks by combining lattice QCD potentials and the Born-Oppenheimer approximation with quantum mechanics techniques.
- We specialize in systems with two heavy antiquarks, where a $ud\bar{b}\bar{b}$ tetraquark bound state with quantum numbers $I(J^P) = 0(1^+)$ has recently been predicted with lattice QCD potentials.
- In this talk we search for resonances. We utilize the new emergent wave method, a technique from scattering theory.

Bicudo:2012qt, Brown:2012tm.

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   - Solving the differential equations for the emergent wave
   - Phase shifts and **S** and **T** matrix poles
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Introduction

The lattice QCD results for the potentials can be parametrized by a screened Coulomb potential,

$$V(r) = -\frac{\alpha}{r} e^{-r^2/d^2} \ . \qquad (1)$$

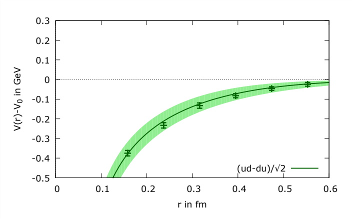
inspired by one-gluon exchange at small $\bar{Q}\bar{Q}$ separations r and a screening of the Coulomb potential by the two B mesons at large r.

Clearly, in lattice QCD potentials, the scalar ($I = 0, j = 0$) potential is the more attractive one.

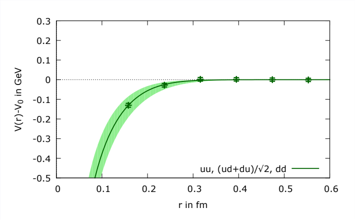
Wagner:2010ad, Wagner:2011ev, Bicudo:2015kna.

Fit with a screened Coulomb potential

scalar:



vector:



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Fit of the lattice QCD potential

I	j	α	d in fm
0	0	$0.34^{+0.03}_{-0.03}$	$0.45^{+0.12}_{-0.10}$
1	1	$0.29^{+0.05}_{-0.06}$	$0.16^{+0.05}_{-0.02}$

Table: Parameters α and d of the potential of Eq. (1) for two static antiquarks $\bar{Q}\bar{Q}$, in the presence of two light quarks qq with quantum numbers I and j.

Potentials $V(r)$ of two static antiquarks $\bar{Q}\bar{Q}$ in the presence of two light quarks qq have been computed using lattice QCD.

- There are both attractive and repulsive channels.
- Most promising with respect to the existence of tetraquark bound states or resonances are light quarks $q \in \{u, d\}$ together with $(I = 0, j = 0)$ or $(I = 1, j = 1)$,
- the corresponding potentials $V(r)$ are not only attractive, but also rather wide and deep

Bicudo:2015vta, Bicudo:2015kna, Bicudo:2016ooe.

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Emergent wave method

The first step in the emergent wave method is to split the wave function of the Schrödinger Eq. $\Big(H_0 + V(r) - E\Big)\psi = 0$, into two parts,

$$\Psi = \Psi_0 + X \, , \tag{2}$$

where Ψ_0 is the incident wave, a solution of the free Schrödinger equation, $H_0\Psi_0 = E\Psi_0$, and X is the emergent wave. We obtain

$$\Big(H_0 + V(r) - E\Big)X = -V(r)\Psi_0 \, . \tag{3}$$

- For any energy E we calculate the emergent wave X by providing the corresponding Ψ_0 and fixing the appropriate boundary conditions.
- From the asymptotic behaviour of the emergent wave X we then determine the phase shifts δ_l , the S matrix and the T matrix.
- Continuing to complex energies $E \in \mathbb{C}$ we find the poles of the S matrix and the T matrix in the complex plane.
- We identify a resonance with a pole of S in the second Riemann sheet at $m - i\Gamma/2$, where m is the mass and Γ is the resonance decay width.

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Emergent wave method

We consider an incident plane wave $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$, which can be expressed as a sum of spherical waves,

$$\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_l (2l+1) i^l j_l(kr) P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}) \; , \tag{4}$$

where j_l are spherical Bessel functions, P_l are Legendre polynomials and the relation between energy and momentum is $\hbar k = \sqrt{2\mu E}$. For a spherically symmetric potential $V(r)$ as in Eq. (1) and an incident wave $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$ the emergent wave X can also be expanded in terms of Legendre polynomials P_l,

$$X = \sum_l (2l+1) i^l \frac{\chi_l(r)}{kr} P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}) \; . \tag{5}$$

Inserting Eq. (4) and Eq. (5) into Eq. (3) leads to a set of ordinary differential equations for χ_l,

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} + V(r) - E \right) \chi_l(r) = -V(r) kr j_l(kr) \; . \tag{6}$$

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Emergent wave method

The potentials $V(r)$, Eq. (1), are exponentially screened, i.e. $V(r) \approx 0$ for $r \geq R$, where $R \gg d$. For large separations $r \geq R$ the emergent wave is, hence, a superposition of outgoing spherical waves, i.e.

$$\frac{\chi_l(r)}{kr} = i\, t_l h_l^{(1)}(kr), \tag{7}$$

where $h_l^{(1)}$ are the spherical Hankel functions of first kind.
Our aim is now to compute the complex prefactors t_l, which will eventually lead to the phase shifts. To this end we solve the ordinary differential equation (6). The corresponding boundary conditions are the following:

- At $r = 0$: $\chi_l(r) \propto r^{l+1}$.
- For $r \geq R$: Eq. (7).

The boundary condition for $r \geq R$ fixes t_l as a function of E.
We solve it numerically, with two different numerical techniques approaches: (1) a fine uniform discretization of the interval $[0, R]$, which reduces the differential equation to a large set of linear equations, which can be solved rather efficiently, since the corresponding matrix is tridiagonal; (2) a standard 4-th order Runge-Kutta shooting method.

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Emergent wave method

The quantity t_l is a T matrix eigenvalue. From t_l we can calculate the phase shift δ_l and also read off the corresponding S matrix eigenvalue s_l, [1]

$$s_l \equiv 1 + 2it_l = e^{2i\delta_l} \ . \tag{8}$$

Moreover, note that both the S matrix and the T matrix are analytical in the complex plane. They are well-defined for complex energies $E \in \mathbb{C}$.

- Thus, our numerical method can as well be applied to solve the differential Eq. (6) for complex $E \in \mathbb{C}$.
- We find the S and T matrix poles by scanning the complex plane $(\mathrm{Re}(E), \mathrm{Im}(E))$ and applying Newton's method to find the roots of $1/t_l(E)$. The poles of the S and the T matrix correspond to complex energies of resonances.
- Note the resonance poles must be in the second Riemann sheet with a negative imaginary part both for the energy E and the momentum k.

[1] At large distances $r \geq R$, the radial wavefunction is $kr[j_l(kr) + i\, t_l h_l^{(1)}(kr)] = (kr/2)[h_l^{(2)}(kr) + e^{2i\delta_l} h_l^{(1)}(kr)]$.

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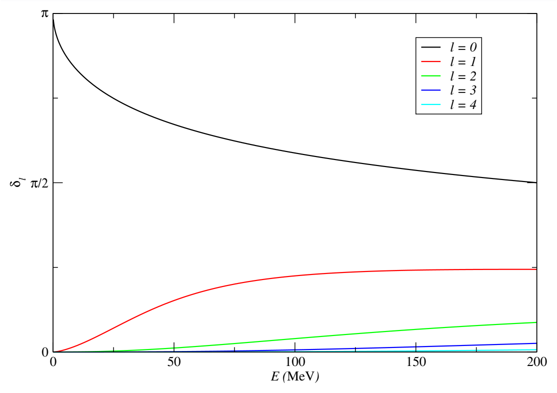
Results for the phase shifts and resonances

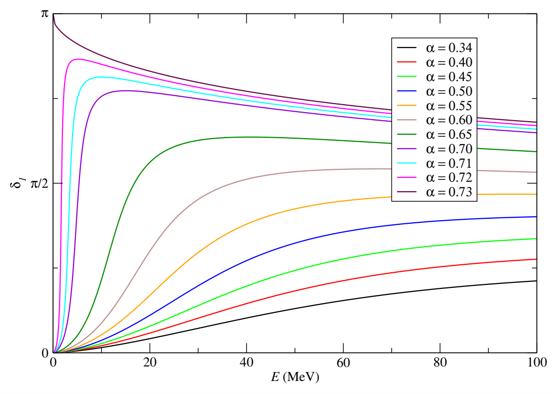
Phase shifts



Phase shift δ_l as a function of the energy E for different angular momenta $l = 0, 1, 2, 3, 4$ for the $(I = 0, j = 0)$ potential ($\alpha = 0.34$, $d = 0.45\,\mathrm{fm}$).

Results for the phase shifts and resonances

δ_1 for different α parameters



Phase shift δ_1 as a function of the energy E for different parameters α for the $(I = 0, j = 0)$ potential $(d = 0.45\,\text{fm})$.

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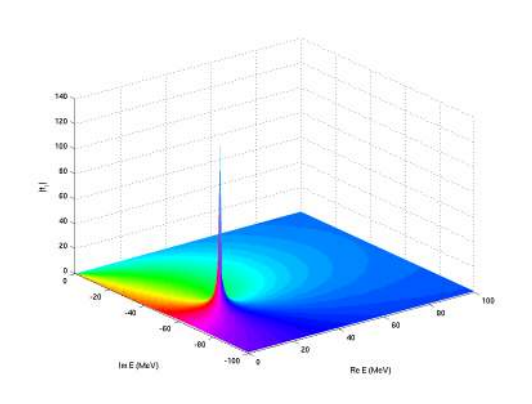
Results for the phase shifts and resonances

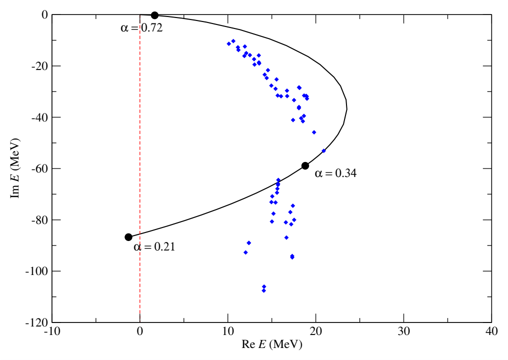
Pole in the complex plane of $E \in \mathbb{C}$



T matrix eigenvalue t_1 as a function of the complex energy E . The vertical axis shows the norm $|t_1|$, the colours represent the phase $\arg(t_l)$.

Results for the phase shifts and resonances

Pole trajectory in the complex space of $E \in \mathbb{C}$ as a function of α



Trajectory of the pole of the eigenvalue t_1 of the T matrix in the complex plane $(\mathrm{Re}(E), \mathrm{Im}(E))$, corresponding to a variation of parameter α. We also illustrate with a cloud of diamond points the systematic error Bicudo:2015vta .

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Summary and outlook

- For more details, please see the recent e-Print: arXiv:1704.02383, by, Pedro Bicudo, Marco Cardoso (CFTP, IST, Lisbon Univ.) , Antje Peters, Martin Pflaumer, Marc Wagner (Frankfurt Univ.).
- Searching for $ud\bar{b}\bar{b}$ resonances, we utilized lattice QCD potentials computed for two static antiquarks in the presence of two light quarks, the Born-Oppenheimer approximation and the emergent wave method.
- First we computed scattering phase shifts of a BB meson pair.
- Then we performed the analytic continuation of the S matrix and the T matrix to the second Riemann sheet and have searched for poles $\in \mathbb{C}$.
- From these results we have predicted a novel resonance, with quantum numbers $I(J^P) = 0(1^-)$. Performing a careful statistical and systematic error analysis has led to a resonance mass $m = 10\,576^{+4}_{-4}\,\mathrm{MeV}$ and a decay width $\Gamma = 112^{+90}_{-103}\,\mathrm{MeV}$.
- As and outlook we plan to address the experimentally observed quarkonia tetraquarks, (including $b\bar{b}$ or $c\bar{c}$), with our method.

Bicudo:2017szl

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