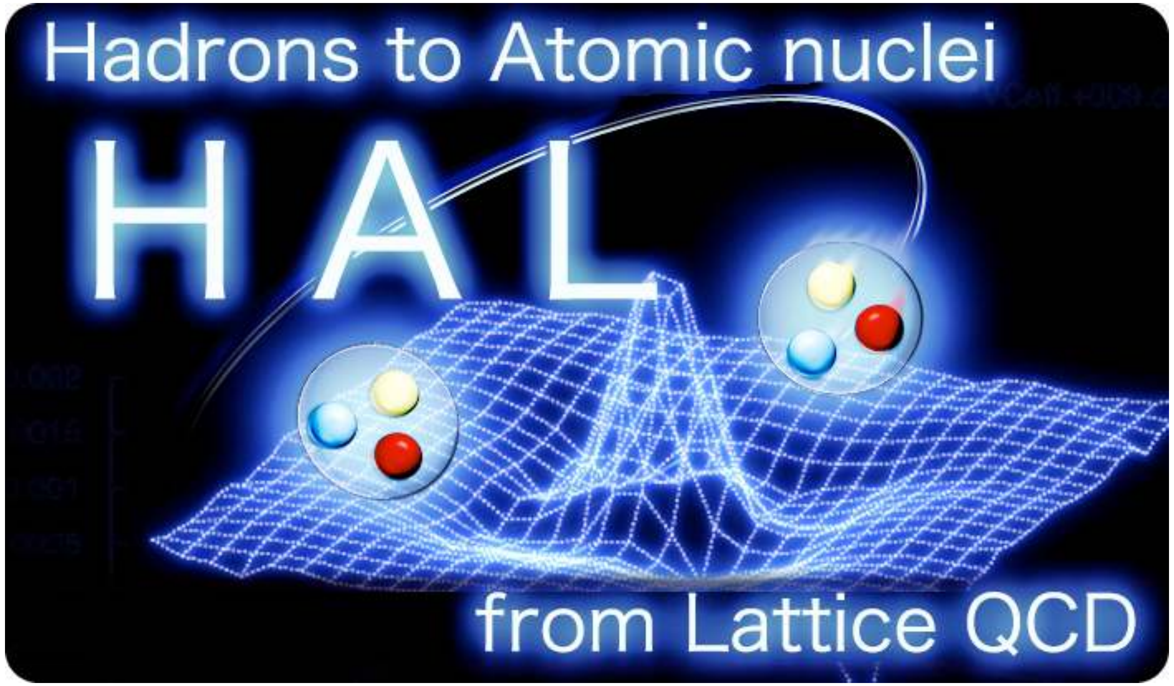
ρ resonance from the I=1 $\pi\pi$ potential in lattice QCD



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For HALQCD collaboration
@Lattice 2017, Granada, Spain
2017/06/19

Introduction

Lattice QCD uncovered a lot of important properties of hadron from the first principle calculation.

HALQDC collaboration contributed in the field like [S.Aoki, T.Hatsuda, N. Ishii, Prog. Theor.Phys., 123 (2010)]

[Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001]

- Negative parity channel
- Heavy quark hadron
- Potential at the physical point *(Wed. 15:20~, T. Doi, N. Ishii, K. Sasaki and H. Nemura's talk)*
- etc.

However, **all of them** are computed with *point-to-all* propagator. (∵ Number of inversions)

→ Some important channels are yet to be done.

(e.g. $\boldsymbol{\rho}$ **resonance**, σ resonance)

We incorporate **distillation smearing** and use all-to-all propagator in order to overcome this difficulty .

[Michael Peardon, John Bulava et al. Phys.Rev.D80:054506,2009]

Time dependent HAL method [Ishii et al.,PLB712(2012)437]

R-correlator

$$R(\mathbf{r}, t - t_0) = e^{2mt} \sum_{\mathbf{x}} \left\langle 0 | T \left\{ N(\mathbf{x}, t) N(\mathbf{x} + \mathbf{r}, t) \right\} \bar{\mathcal{J}}(t_0) | 0 \right\rangle$$
$$= \sum_n A_n \psi_{k_n}(\mathbf{r}) e^{-(E_n - 2m)(t - t_0)}$$

time-dependent Schrödinger-like equation

$$\left[\frac{1}{4m} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right] R(\mathbf{r}, t) = \int dr'^3 U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t)$$

From the velocity expansion, the potential is given by

$$V(\mathbf{r}) = \frac{1}{4m} \frac{(\partial/\partial t)^2 R(\mathbf{r}, t)}{R(\mathbf{r}, t)} - \frac{(\partial/\partial t) R(\mathbf{r}, t)}{R(\mathbf{r}, t)} - \frac{H_0 R(\mathbf{r}, t)}{R(\mathbf{r}, t)}$$

The operator dependence of HAL QCD potentials

Conventional HAL QCD method ➡ Wall src. and point sink are used.

$$R(\mathbf{r}, t - t_0) = e^{2mt} \sum_{\mathbf{x}} \langle 0 | T \underbrace{\{B(\mathbf{x}, t) B(\mathbf{x}+\mathbf{r}, t)\}}_{\text{Point sink}} \underbrace{\bar{\mathcal{J}}(t_0)}_{\text{Wall src}} | 0 \rangle$$

⬇ Distillation smearing

$$R(\mathbf{r}, t; \mathbf{P}, t_0) = e^{2m(t-t_0)} \sum_{\mathbf{x}} \sum_{\mathbf{y_1}, \mathbf{y_2}} e^{-i\mathbf{P}\cdot\mathbf{y_1}} e^{i\mathbf{P}\cdot\mathbf{y_2}} < 0 | T\{ \underbrace{M^s(\mathbf{x}, t) M^s(\mathbf{x}+\mathbf{r}, t)}_{\text{smeared sink}} \underbrace{M^s(\mathbf{y_1}, t_0) M^s(\mathbf{y_2}, t_0)}_{\text{smeared src}} \} | 0 >$$

➡ Smeared src. and smeared sink are used

Question : Systematic change due to operator construction is not known.

<u>How is the operator dependence of the HAL QCD potentials ?</u>

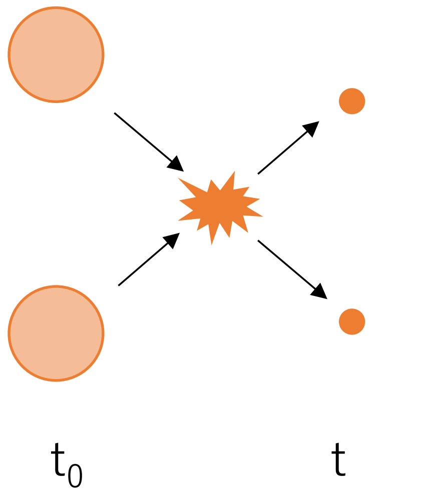
Check the operator dependence of pion-pion interaction potentials with 2 setups

Point sink – Smeared src

- 2-pt correlation $C_M^2(t,t_0) = \sum_{\mathbf{x},\mathbf{y}} \langle 0|\pi^+(\mathbf{x},t)\pi^{-s}(\mathbf{y},t_0)|0\rangle$

  $\pi^{-s}(\mathbf{x},t) = \bar{u}^s\gamma_5 d^s(\mathbf{x},t)$, $q^s(\mathbf{x},t) = S_{\mathbf{x},\mathbf{y}}(t)q(\mathbf{y},t)$

- 4-pt correlation $C_M^4(\mathbf{r},t;t_0) = \sum_{\mathbf{x}}\sum_{\mathbf{y_1},\mathbf{y_2}} \langle 0|\pi^+(\mathbf{x},t)\pi^+(\mathbf{x}+\mathbf{r},t)\pi^{-s}(\mathbf{y_1},t_0)\pi^{-s}(\mathbf{y_2},t_0)|0\rangle$



Smeared sink – Smeared src

- 2-pt correlation $C_M^2(t,t_0) = \sum_{\mathbf{x},\mathbf{y}} \langle 0|\pi^{+s}(\mathbf{x},t)\pi^{-s}(\mathbf{y},t_0)|0\rangle$

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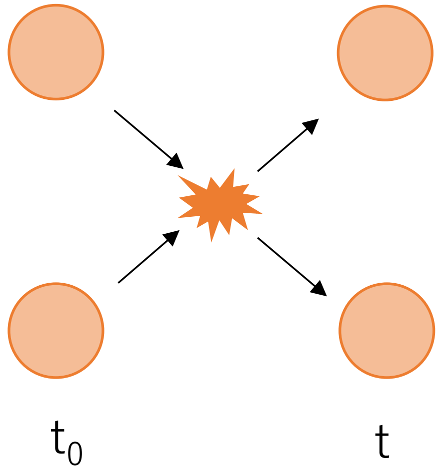
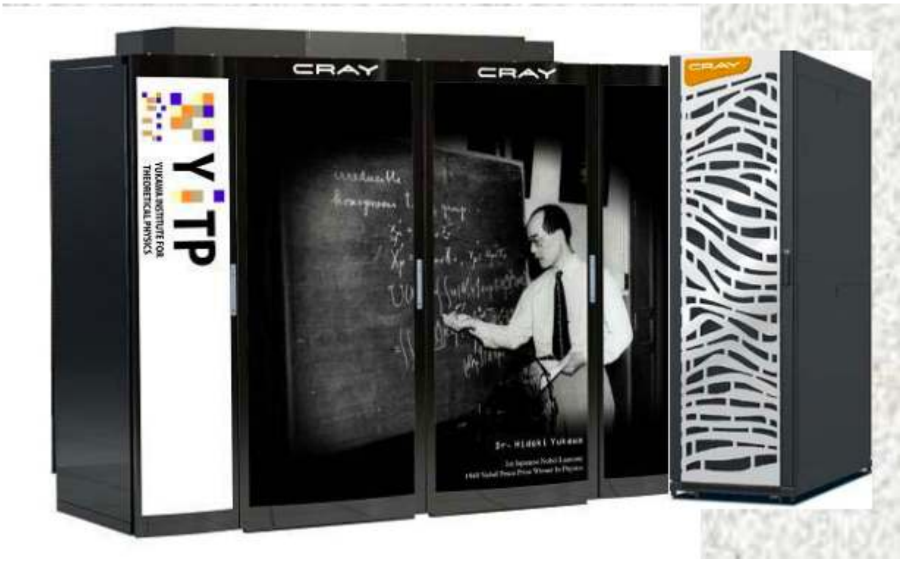


Numerical setup

- 2 + 1 flavor gauge configuration by CP-PACS & JLQCD
  [CP-PACS/JLQCD Collaboration : T.Ishikawa, et al., PRD 78 (2008) 011502(R)]
- Wilson clover fermion and Iwasaki gauge action
- a = 0.1214 fm , $16^3 \times 32$ lattice
- $m_\pi \simeq 870\mathrm{MeV}$
- 60conf × 32 time slice
- Calculated on Cray XC40 in YITP



Cray XC40 in YITP

Remark : the sum over source space improves statistics.

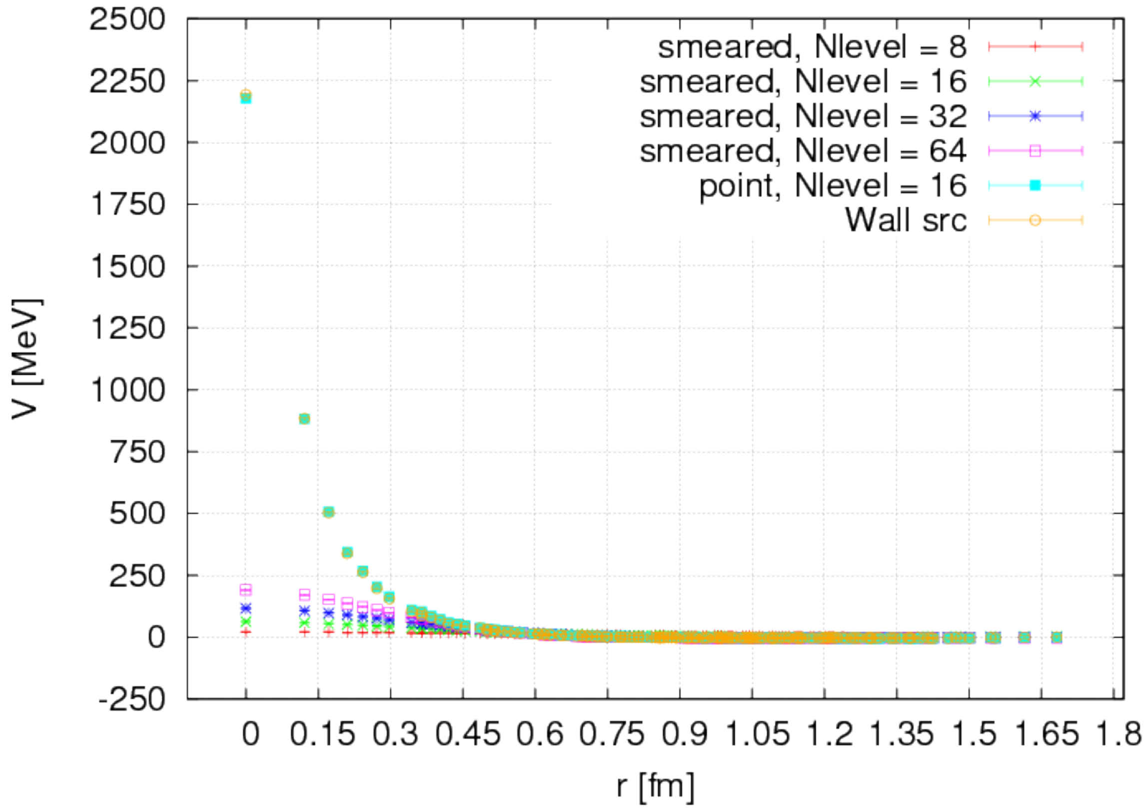
$$C^4_M(\mathbf{r},t;t_0) = \sum_{\mathbf{x}} \sum_{\mathbf{y_1},\mathbf{y_2}} \langle 0|\pi^+(\mathbf{x},t)\pi^+(\mathbf{x}+\mathbf{r},t)\pi^{-s}(\mathbf{y_1},t_0)\pi^{-s}(\mathbf{y_2},t_0)|0\rangle$$

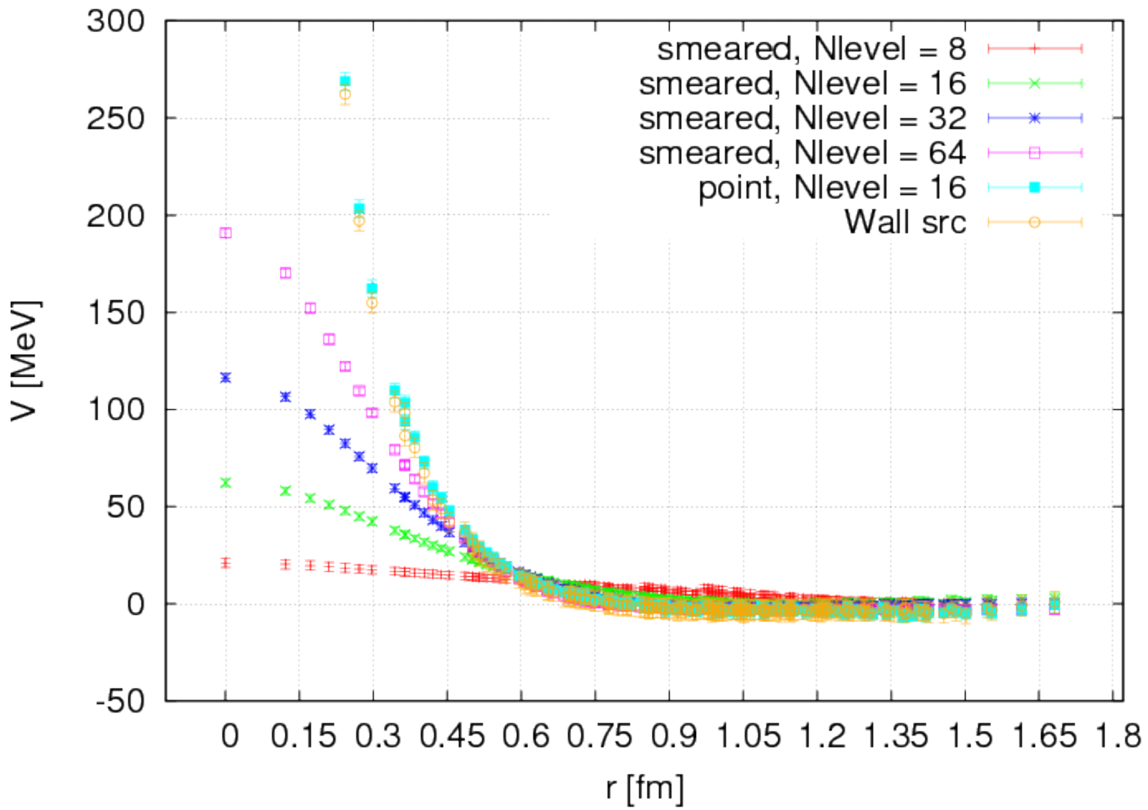
The operator dependence of potentials

Point sink-Smeared src. ➡ Quite similar to Point sink-Wall src.

Smeared sink-Smeared src. ➡ Repulsive core is weakened and strong dependence on the number of Laplacian eigenvalue appears.

Strong operator dependence in smeared sink case ➡ Is smeared sink useless in HAL QCD method ?





Next-to-leading order potential

We consider the potential to next-to-leading order. $\left[\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\mathbf{r},t) = \left(V_0 + \nabla^2 V_1\right) R(\mathbf{r},t)$

Now we assume V_0 and V_1 is the same regardless of operator combination.

$$V_0(r) + V_1(r)\frac{\sum_{g\in O_h} R_{stat.}(g^{-1}\mathbf{r},t)^* \nabla^2 R_{stat.}(g^{-1}\mathbf{r},t)}{\sum_{g\in O_h} R_{stat.}(g^{-1}\mathbf{r},t)^* R_{stat.}(g^{-1}\mathbf{r},t)} = V_{stat,tot.}(\mathbf{r},t)\ ,\quad R_{stat.}(\mathbf{r},t) \equiv \sum_{t_0} R(\mathbf{r},t+t_0;\mathbf{0},t_0)$$

$$V_0(r) + V_1(r)\frac{\sum_{g\in O_h} R_{A1}(g^{-1}\mathbf{r},t)^* \nabla^2 R_{A1}(g^{-1}\mathbf{r},t)}{\sum_{g\in O_h} R_{A1}(g^{-1}\mathbf{r},t)^* R_{A1.}(g^{-1}\mathbf{r},t)} = V_{A1,tot.}(\mathbf{r},t)\ ,\quad R_{A1}(\mathbf{r},t) \equiv \sum_{g\in O_h}\sum_{t_0} R(\mathbf{r},t+t_0;g\hat{\mathbf{e}}_x,t_0)$$

Then singular value decomposition (SVD) can be used in order to decompose

leading order and next-to-leading order potential.

$$\begin{pmatrix} 1 & \frac{\sum_{g\in O_h} R_{stat.}(g^{-1}\mathbf{r},t)^* \nabla^2 R_{stat.}(g^{-1}\mathbf{r},t)}{\sum_{g\in O_h} R_{stat.}(g^{-1}\mathbf{r},t)^* R_{stat.}(g^{-1}\mathbf{r},t)} \\ 1 & \frac{\sum_{g\in O_h} R_{A1}(g^{-1}\mathbf{r},t)^* \nabla^2 R_{A1}(g^{-1}\mathbf{r},t)}{\sum_{g\in O_h} R_{A1}(g^{-1}\mathbf{r},t)^* R_{A1}(g^{-1}\mathbf{r},t)} \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} V_0(r) \\ V_1(r) \end{pmatrix} = \begin{pmatrix} V_{stat,tot.}(\mathbf{r},t) \\ V_{A1,tot.}(\mathbf{r},t) \\ \vdots \end{pmatrix}$$

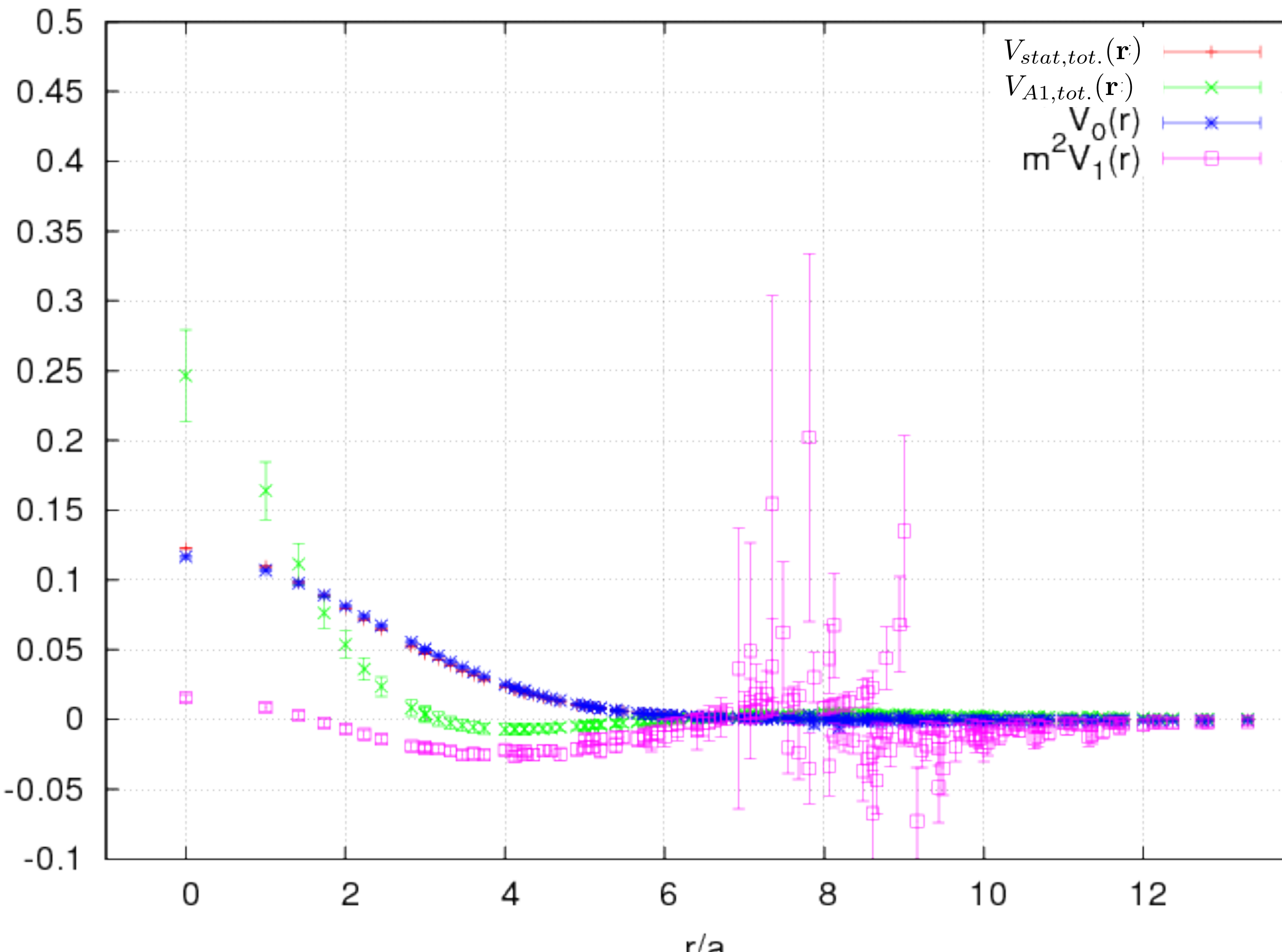
Here are the potentials given by the SVD of Rcorrelators.

Interestingly, the potential from $R_{stat.}$ and leading order potential is quite similar.

The leading order potential draws low energy region well.

But it's not enough to cover higher energy.

In this case, the behavior at $2\sqrt{s} - 2m_\pi \sim 600\mathrm{MeV}$ is largely modified by next-to-leading order.

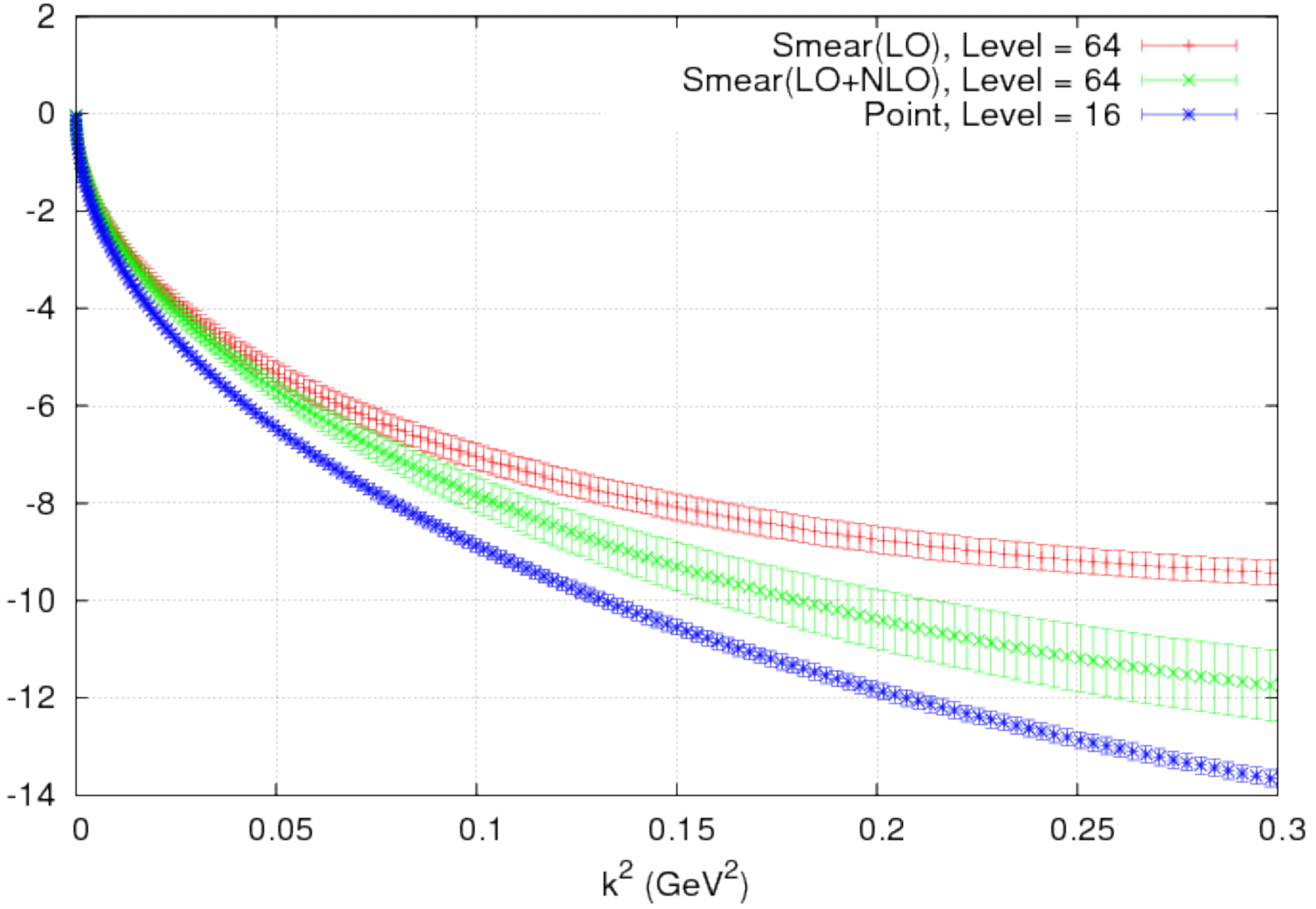


Phase shift from the LO+NLO potential

The comparison of phase shift from the LO, LO+NLO and point sink potential.

LO and LO+NLO potential have similar behavior at low energy.

However, as energy rises, phase shift given by the LO+NLO potential deviate from the one given by the LO potential and approach to the one by point sink.



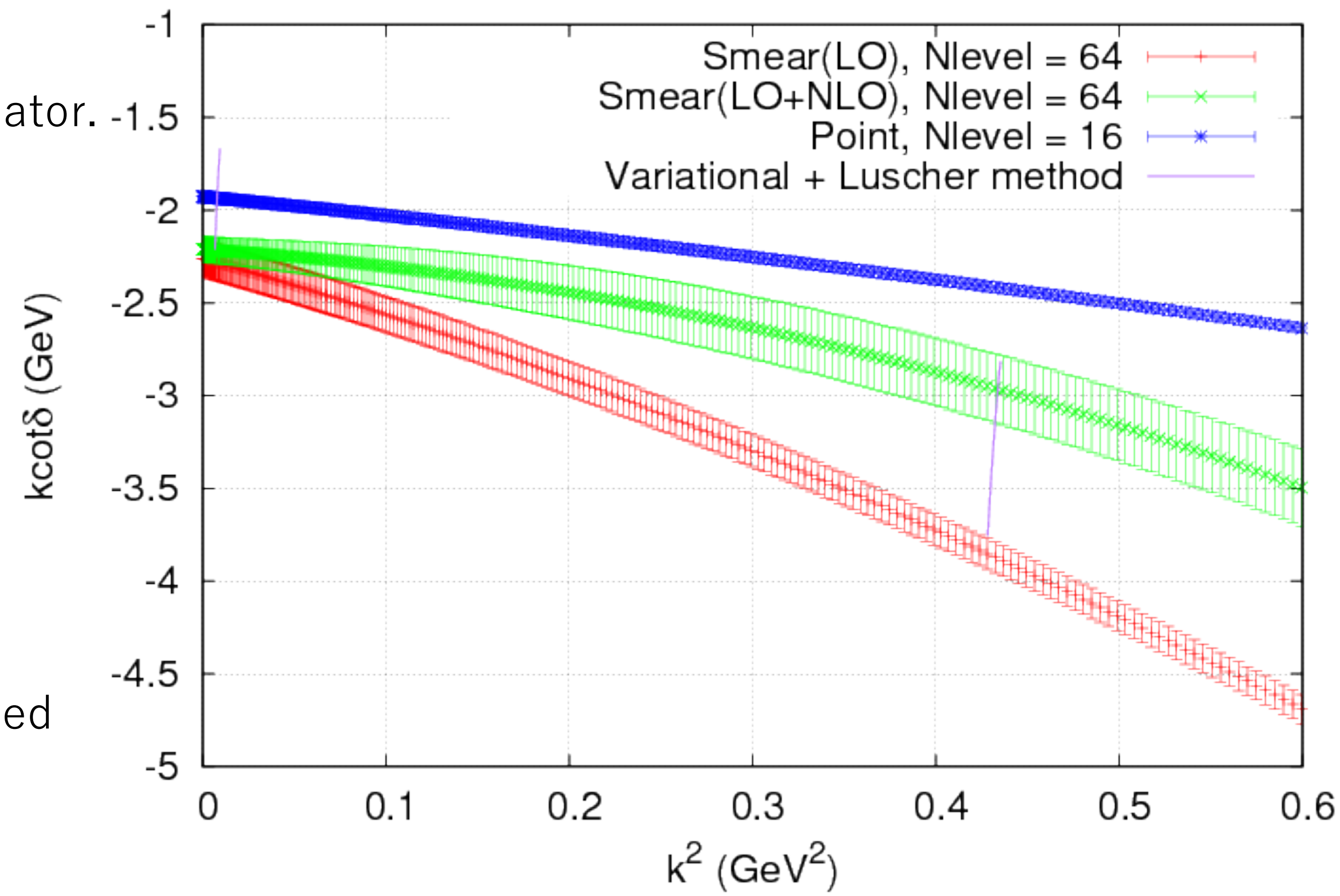
kcot δ from the LO+NLO potential

Phase shifts by LO+NLO is

Consistent with the one by Lüscher's finite size formula.

[M. Lüscher, Commun. Math. Phys. 105, 153 (1986); Nucl. Phys. B354, 531 (1991).]

- HAL QCD method can draws correct phase shifts within the same correlator.
- Potentials by smeared sink is Improved by considering higher order.



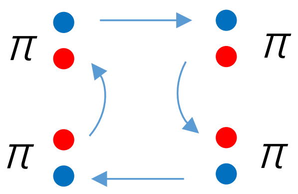
Conclustion for this part

- The potentials given by smeared sink have relatively large operator dependence.
- The deviation from point sink will be recovered by thinking higher order term.

$\mathrm{I} = 1$ $\pi\ \pi$ potential

I=1 channel

Compared with I = 2 channel, I = 1 $\pi\pi$ scattering is very difficult because of so-called box-like diagrams.



⇨ We have to calculate all-to-all correlators to get NBS wave functions.

⇨ It needs a lot of computational cost and time.

However, this channel deserves to calculate because ρ resonance is in this channel.

> Question : Can the potential correctly generate ρ resonance ?

The significant by-product of getting potentials is the capability to calculate S-matrix

in complex energy plane.

⇨ Direct search of pole is possible.

Numerical setup

- 2 + 1 flavor gauge configuration by CP-PACS collaboration

  [PACS-CS Collaboration: S. Aoki, K.-I. Ishikawa, N. Ishizuka, T. Izubuchi, D. Kadoh, K. Kanaya, Y.Kuramashi, Y. Namekawa, M. Okawa, Y. Taniguchi, A. Ukawa, N. Ukita, T. Yoshie Phys. Rev. D. 79 (2009) 034503]

- Wilson clover fermion and Iwasaki gauge action
- a = 0.0907 fm , $32^3 \times 64$ lattice
- m_π = 410MeV, m_ρ = 890 MeV

  ⇒ ρ meson will appear as a resonant state

- 83conf × 64 time slice
- Periodic boundary condition is used for all direction.

κ_{ud}	0.13754
κ_s	0.13640
π	0.18903(79)
	0.002
K	0.29190(67)
	0.002
η_{ss}	0.36870(71)
	0.000
ρ	0.4108(31)
	0.017
K^*	0.4665(23)
	0.007
ϕ	0.5156(21)
	0.002
N	0.5584(53)
	0.358
Λ	0.6208(36)
	0.089
Σ	0.6437(39)
	0.041
Ξ	0.6910(30)
	0.028
Δ	0.6956(66)
	0.102
Σ^*	0.7464(43)
	0.022
Ξ^*	0.7964(41)
	0.005
Ω	0.8456(37)
	0.009

The dimensionless mass spectrum in the ensamble

Potential

Potential computed with HAL QCD method with distillation smearing.

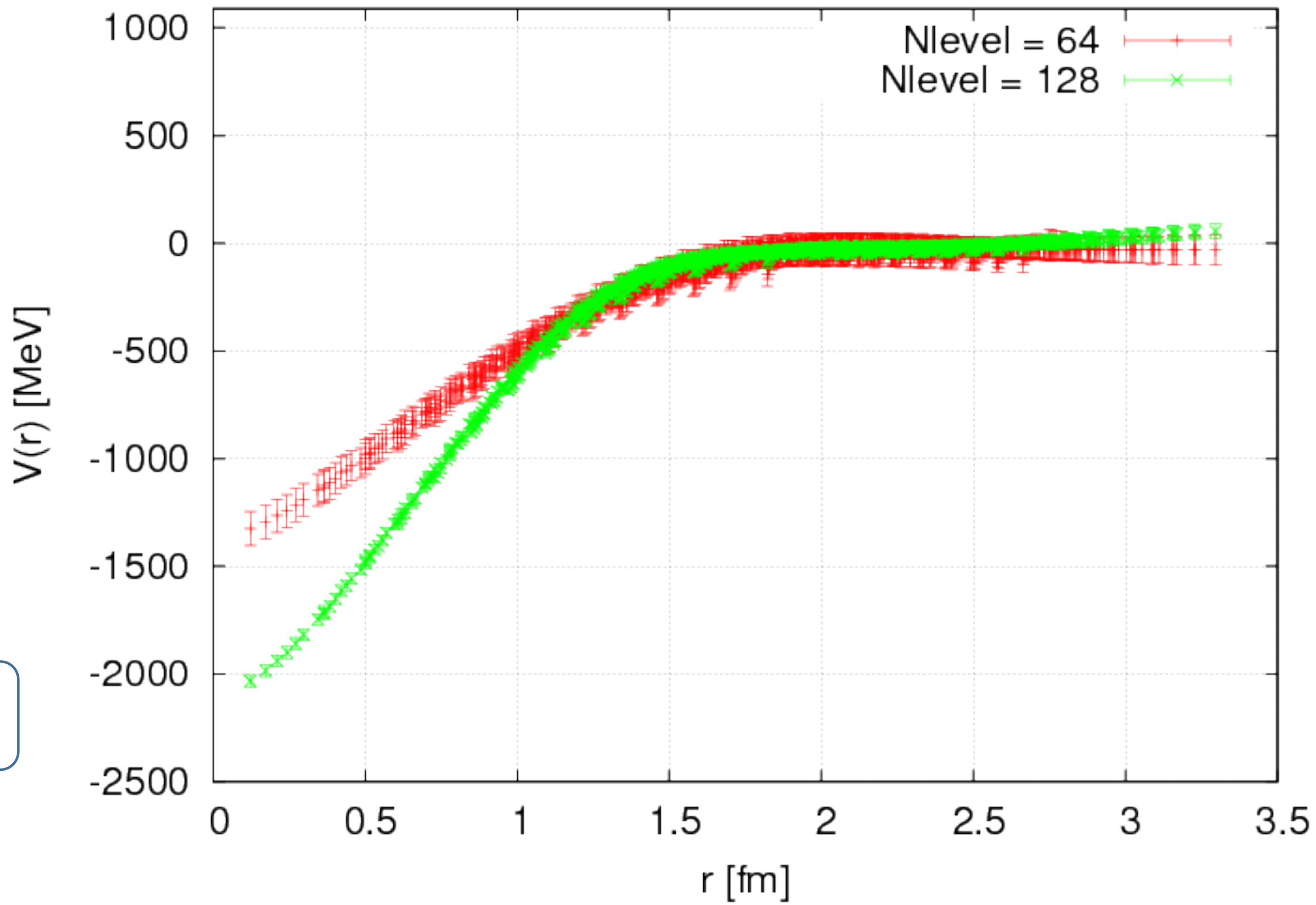
We tried 2 setups for the number of eigenvalue of gauge covariant Laplacian, 128 and 64.

(64 eigenvalue result is preliminary.)

➡ Short range behavior strongly depends on the number of eigenvalues.

This channel shares the same dependence with I = 2 $\pi\pi$ channel.

How is physical quantities like phase shift ?



Phase shift

The phase shift based on the potential in the previous slide.

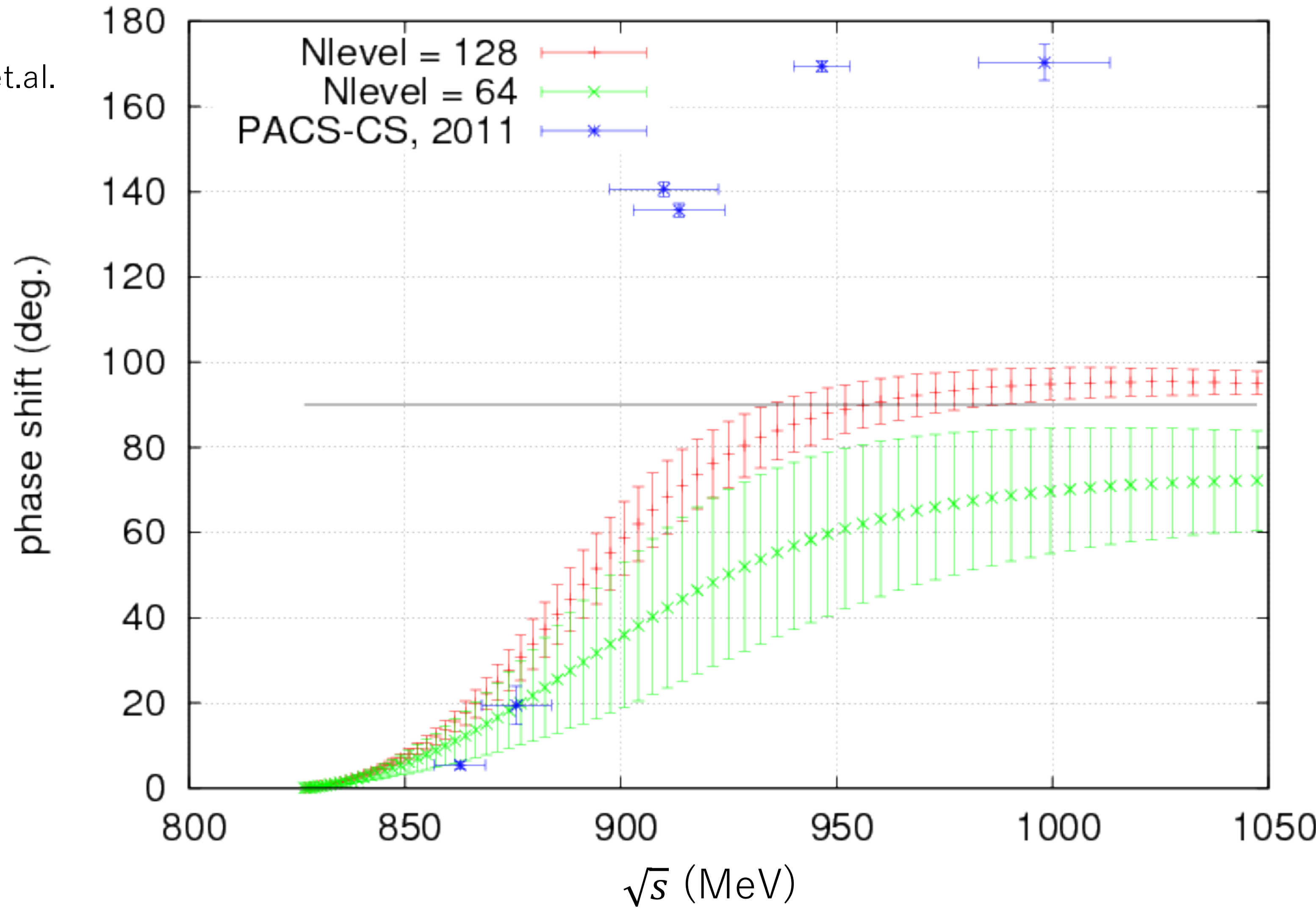
(Blue points are the results by PACS-CS collaboration.) [PACS-CS Collaboration, S.Aoki et.al. Phys.Rev.D84(2011)094505.]

The phase shift with 128 eigenvalues crosses 90 degree.

➡ Resonant behavior appears.

But, the energy phase shift crosses 90° deviates from PACS-CS result.

➡ Source and sink operator might be smeared too much.



Complex scaling method

Rotate coordinates with $\theta \in R$ simultaneously.

$$k \to k e^{-i\theta} \qquad r \to r e^{i\theta}$$

In this rotated coordinate, the Schrodinger-like equation NBS wave function follows is

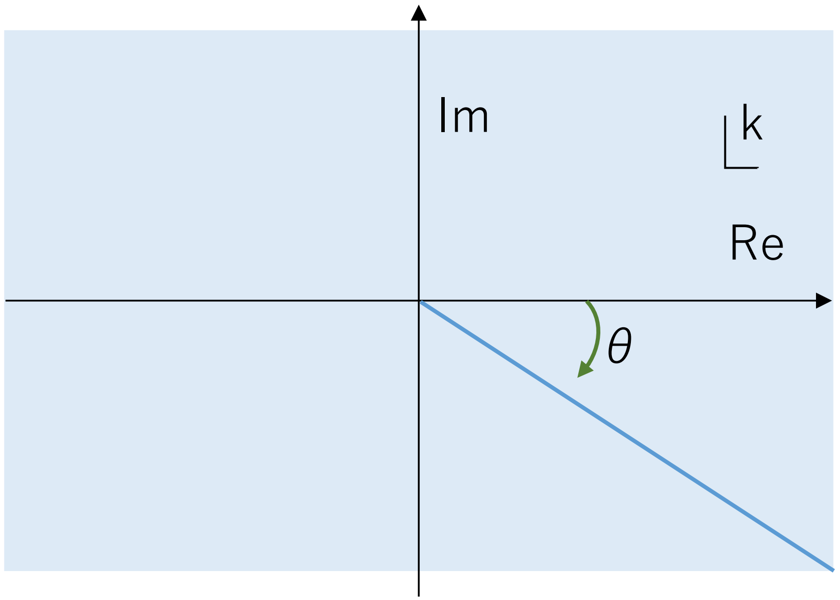
$$\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - e^{2i\theta} V(e^{i\theta} r) + k^2\right) \phi(r) = 0$$

At long distance, the solution is approximated by

$$\phi(r) \to \frac{i}{2} \left[\mathcal{J}_l(ke^{-i\theta}) h_l^-(kr) - \mathcal{J}_l^*(ke^{-i\theta}) h_l^+(kr)\right]$$

From this relation, S-matrix in complex plane is given by

$$\mathcal{S}_l(ke^{-i\theta}) = \frac{\mathcal{J}_l^*(ke^{-i\theta})}{\mathcal{J}_l(ke^{-i\theta})}$$



Pole position

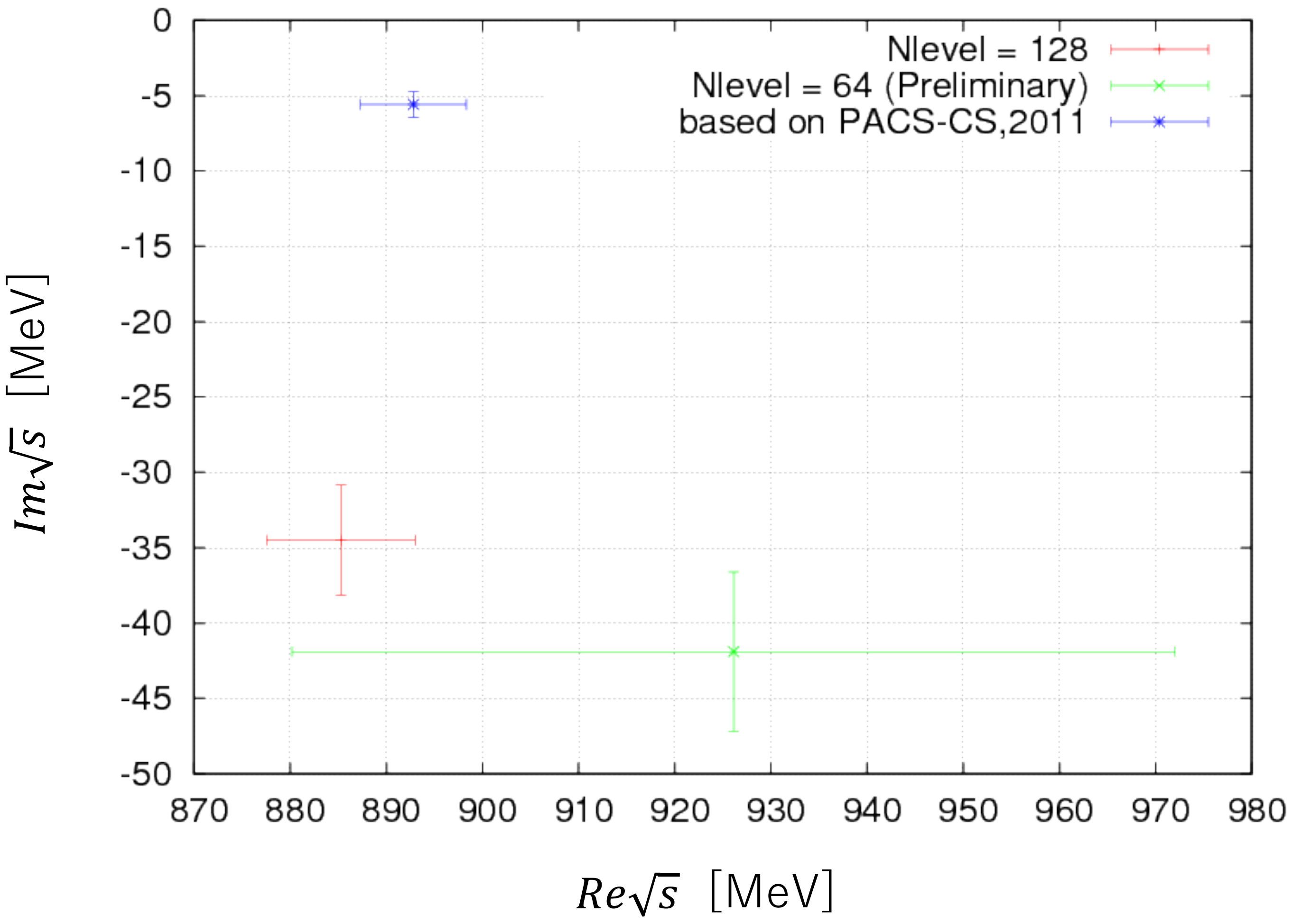
The imaginary part of ρ meson pole potision calculated by HAL QCD method have larger value compared with PACS-CS one.

➡ The reason for the deviation from Breit-Wigner formula.

Interesting point is the similarity of real part.

The real part of 128 eigenvalue and PCAS-CS is consistent within 1 σ error.

➡ Operator dependence of real part might not be so severe.



Summary

Operator dependence

- The potential with smeared sink have large operator dependence.

  ⇒ The height of repulsive core drastically chances.

- Even with the dependence, phase shift can be measured by considering higher order terms.

  ⇒ Phase shifts in high energy are improved by considering the NLO term.

I=1 $\pi\pi$ scattering

- The potential have the sign of ρ resonance.

  ⇒ Peak point is not consistent with Lüscher method.

  Some improvement might be necessary to get correct behavior in higher energy.

- Complex scaling method will be useful to search the resonance.

  ⇒ Research on resonance without fitting will be possible.