

# Degeneracy of vector-channel spatial correlators in high T. QCD

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L. Glozman, S. Hashimoto, C.B. Lang, S. Prelovsek

21 June 2017, Lattice 2017, Granada



## Related work

JLQCD collab. (G.Cossu *et al.*) hep-lat/1510.07395

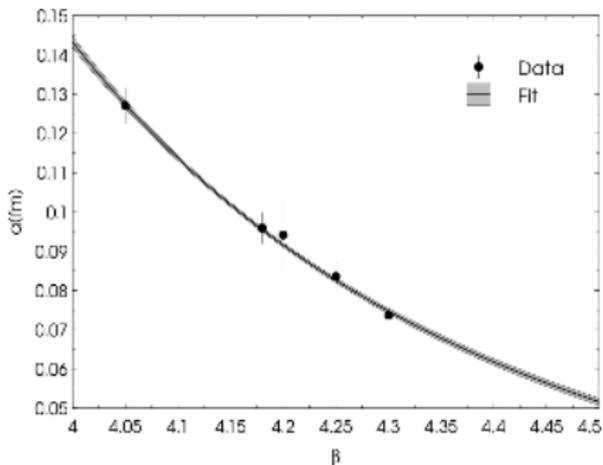
JLQCD collab. (A.Tomiya *et al.*) hep-lat/1612.01908

- ✓ K. Suzuki (Monday, Nonzero-T)  
*Axial  $U(1)$  symmetry at high temperature in 2-flavor lattice QCD*
- ✓ Y. Aoki (Monday, Nonzero-T)  
*Topological Susceptibility in  $N_f = 2$  QCD at Finite Temperature*
- ⇒ H. Fukaya ( **Thursday, plenary** )  
*Is axial  $U(1)$  anomalous at high temperature?*

***What happens to spatial correlators of spin one operators?***

# Lattice setup

- $n_f = 2$  Möbius DW fermions, Symanzik gauge action
- $32^3 \times 8$  lattices,  $T_c = 175\text{MeV}$
- local isovectors  $\mathcal{O}_\Gamma(x) = \bar{q}(x)\Gamma q(x)$
- measuring spatial correlations in  $z$ -direction



$\beta$	$m_{ud}$	$T$ [MeV]	
4.10	0.001	220	$1.3T_c$
4.18	0.001	260	$1.5T_c$
4.30	0.001	330	$1.9T_c$
4.37	0.005	380	$2.2T_c$

# The Dirac algebra

<i>standard terminology</i>			
$\mathbb{1}$	scalar		1
$\gamma_\mu$	vector		4
$\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$	tensor		6
$\gamma_\mu \gamma_5$	pseudo-vector		4
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The tensor elements can be combined to 2 more Vectors:

Tensor **T** and Axial Tensor **X**.

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Fix direction of propagation (*z-direction*):

$$C_{\Gamma}(n_z) = \sum_{n_x, n_y, n_t} \langle \mathcal{O}_{\Gamma}(n_x, n_y, n_z, n_t) \mathcal{O}_{\Gamma}(0, 0, 0, 0)^{\dagger} \rangle$$

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$$\mathbf{T} = \begin{pmatrix} \gamma_1 \gamma_3 & = & T_x \\ \gamma_2 \gamma_3 & = & T_y \\ \gamma_4 \gamma_3 & = & T_t \end{pmatrix} \quad \mathbf{X} = \begin{pmatrix} \gamma_1 \gamma_3 \gamma_5 & = & \gamma_2 \gamma_4 & = & X_x \\ \gamma_2 \gamma_3 \gamma_5 & = & \gamma_4 \gamma_1 & = & X_y \\ \gamma_4 \gamma_3 \gamma_5 & = & \gamma_1 \gamma_2 & = & X_t \end{pmatrix}$$

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## What to expect from $\mathcal{L}_{QCD}$ and $\chi$ S?

	<i>Pseudoscalar</i>		<i>Scalar</i>
<b>PS</b>	$(1, 0^{-+})$ $\bar{q}(\vec{\tau} \otimes \gamma_5)q$	<b>S</b>	$(1, 0^{++})$ $\bar{q}(\vec{\tau} \otimes \mathbb{1}_D)q$

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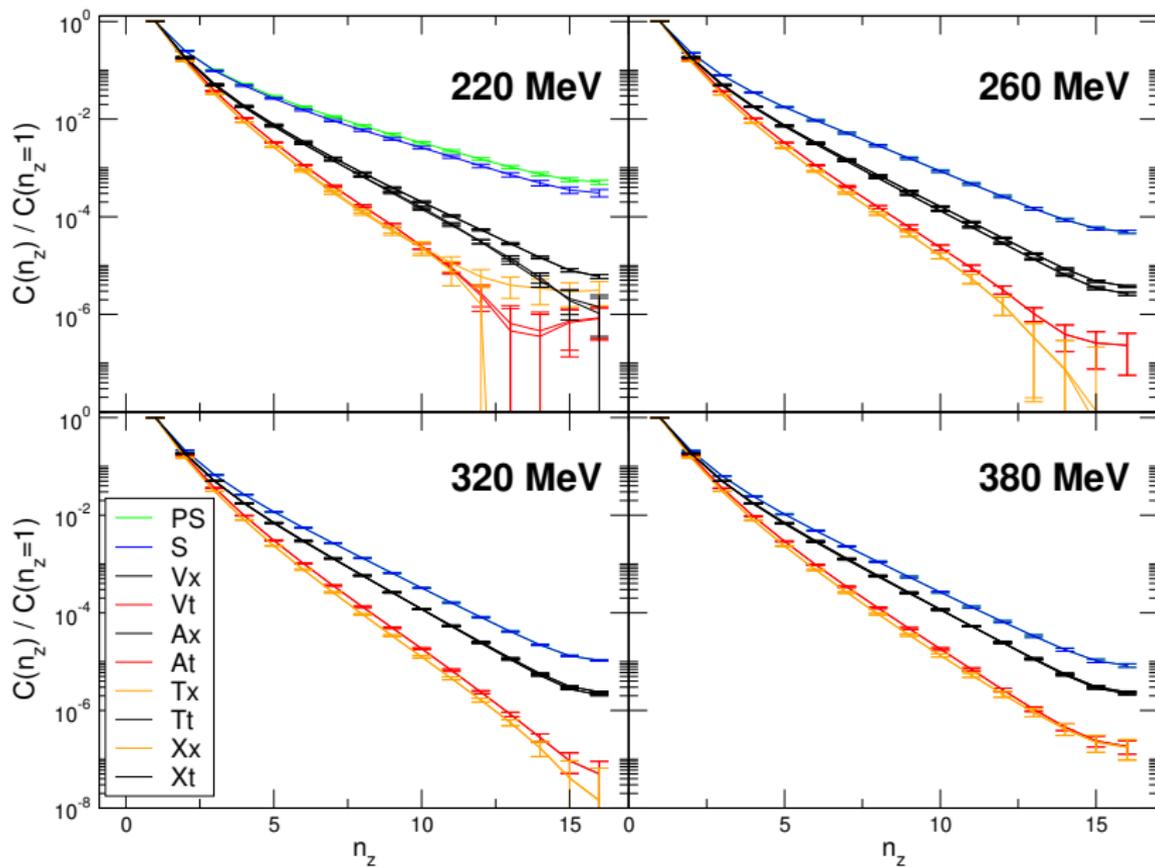
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	<i>Vector</i>		<i>Axial Vector</i>
<b>V</b>	$(1, 1^{--})$ $\bar{q}(\vec{\tau} \otimes \gamma_k)q$	<b>A</b>	$(1, 1^{+-})$ $\bar{q}(\vec{\tau} \otimes \gamma_5 \gamma_k)q$
	<i>Tensor Vector</i>		<i>Axial Tensor V.</i>
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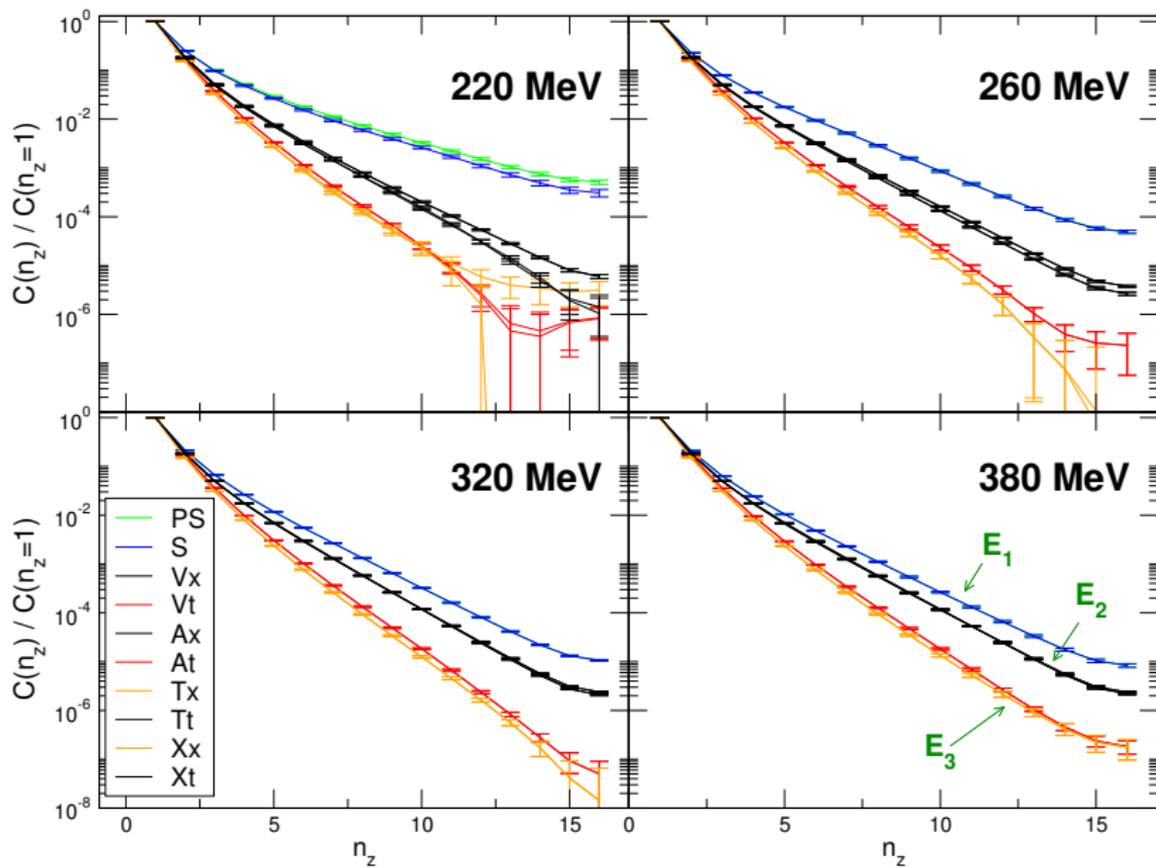
<b>PS</b>	<i>Pseudoscalar</i> $(1, 0^{-+})$ $\bar{q}(\vec{\tau} \otimes \gamma_5)q$	$\longleftrightarrow$ $U(1)_A$	<b>S</b>	<i>Scalar</i> $(1, 0^{++})$ $\bar{q}(\vec{\tau} \otimes \mathbb{1}_D)q$
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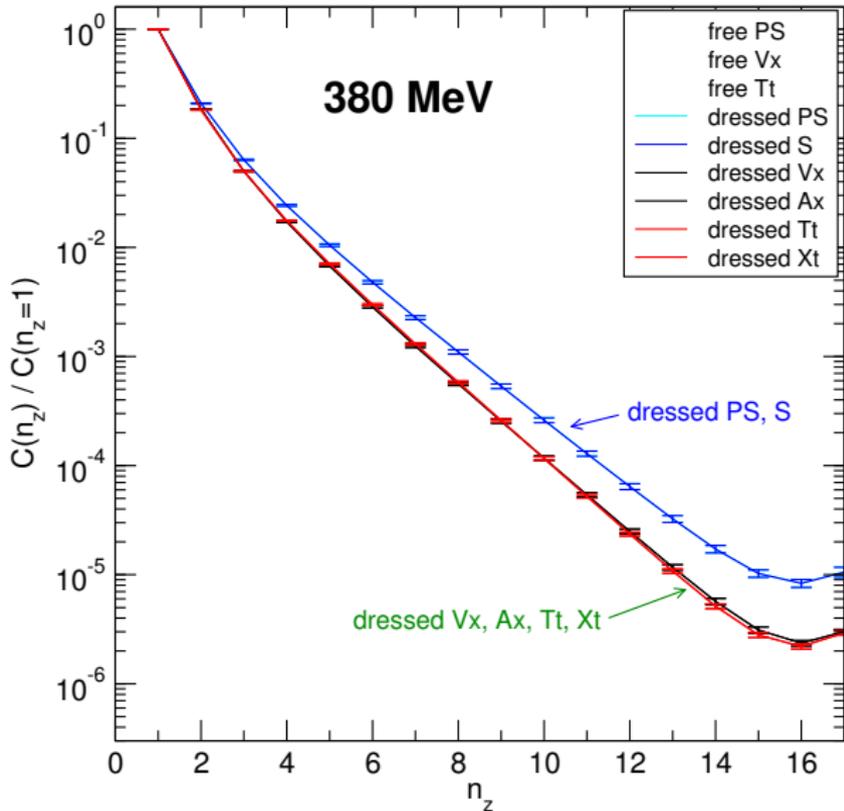


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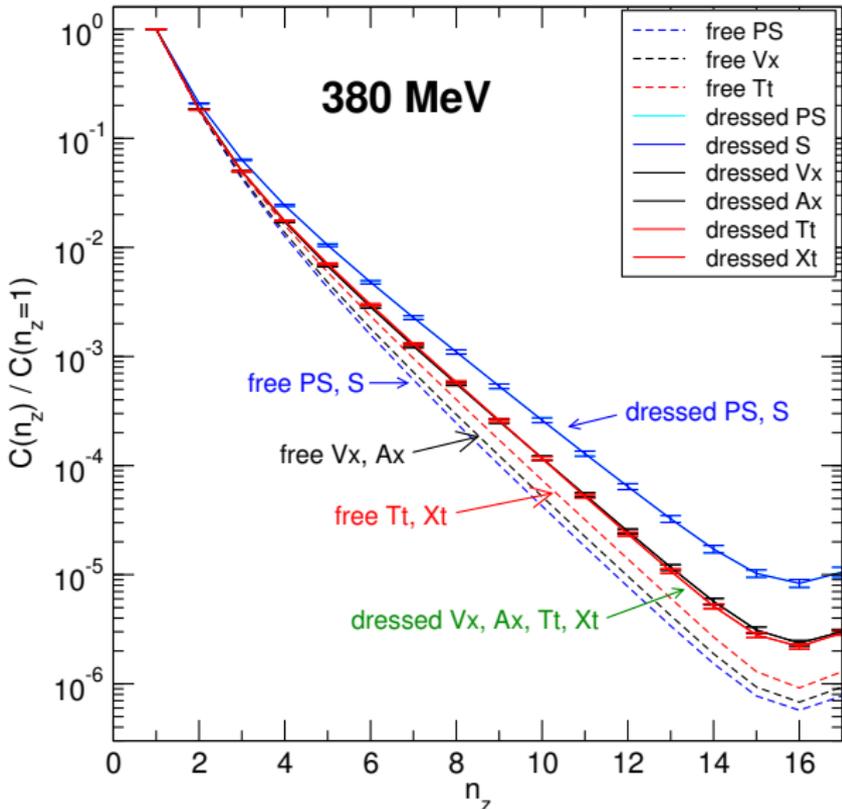


## $E_1$ and $E_2$ multiplets

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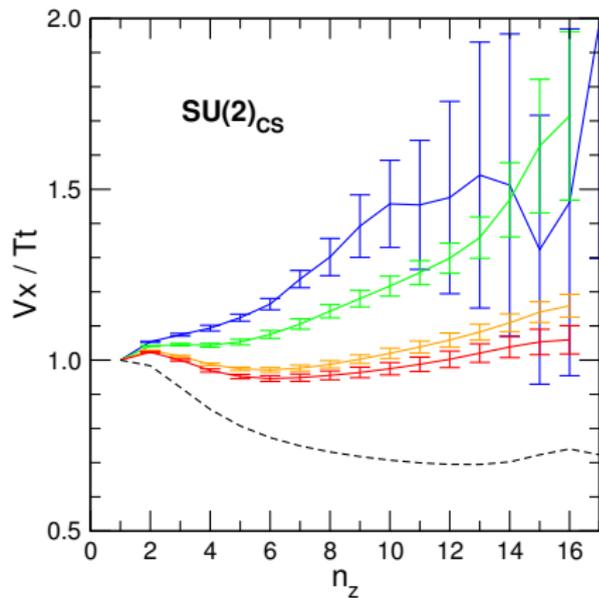
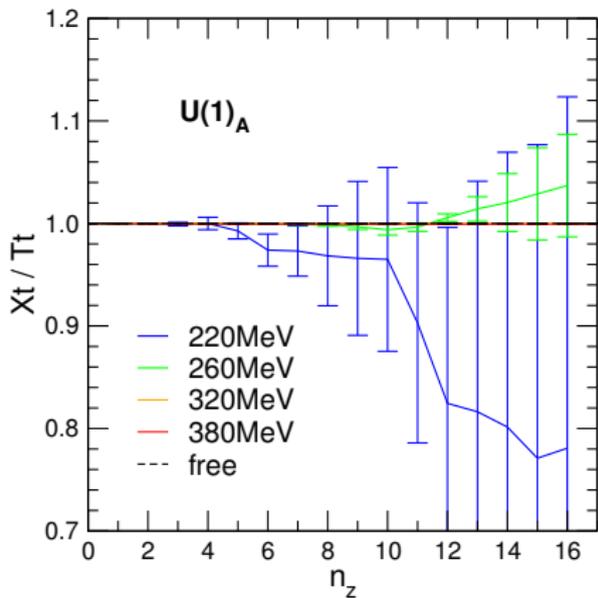


'free'  $U(x)_\mu = 1$   
correlators differ algebraically!

Asymptotic freedom:  
sum of free quarks  
 $C(n_z) \sim e^{-2\omega_0 n_z}$

W.Florkowski &  
B.L.Friman Z.Phys.  
A347 (1994) 271-276

# $U(1)_A$ and $SU(2)_{CS}$ detailed ratios



# $SU(2)_{CS}$ and $SU(4)$ symmetries

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$$A_1 : \quad \{\gamma^1, -i\gamma^5\gamma^1, \gamma^5\}$$

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- ◇ This connects following operators:

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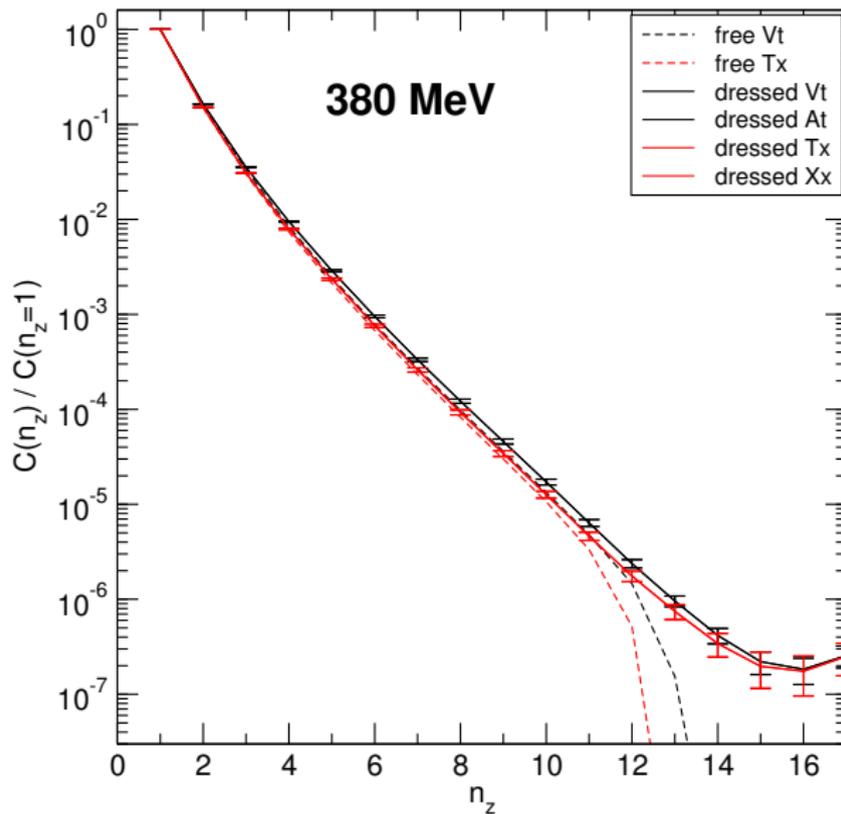
$$\left. \begin{array}{l} V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x \\ V_y \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_y \end{array} \right\} E_2$$

$$\left. \begin{array}{l} V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t \\ V_t \leftrightarrow T_y \leftrightarrow X_y \leftrightarrow A_t \end{array} \right\} E_3$$

This is exactly  
what we observe!

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Finite size effects are obvious for non-interacting  $U(x)_\mu = \mathbb{1}$  correlators!

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thank you!  
ありがとう  
ございます

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