Computation of parton distribution functions from the quasi-PDF approach at the physical point

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Parton distribution functions (PDFs) describe the inner dynamics of partons inside a hadron. The PDFs cannot be directly measured in experiments and have a non-perturbative nature. Naturally, lattice QCD methods would be the most suitable tools for a determination from first principles. We present prelimary results for unpolarized and polarized PDFs within the proton at the physical point ensemble with $N_f = 2$ using the method suggested by Ji [X. Ji, Parton Physics on a Euclidean Lattice, Phys.Rev.Lett. 110 (2013) 262002].

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Obtaining PDFs from lattice QCD

From operator analysis of nucleon-electron deep inelastic scattering, one finds that PDFs are given in terms of non-local light-cone correlations in the Minkowskian space-time

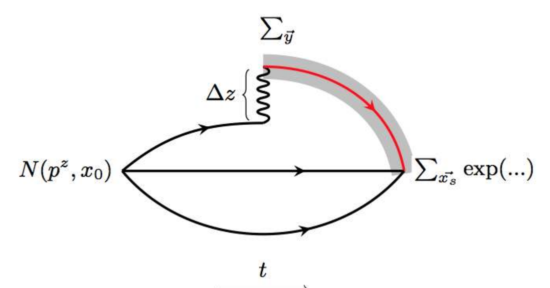
$$q(x) = \int_{-\infty}^{\infty} \frac{d\xi}{4\pi} e^{-ix\xi P} \langle P|\bar{\psi}(0)\lambda \cdot \gamma W(0, \xi\lambda)\psi(\xi\lambda)|P\rangle$$

In the Euclidean space, the light cone condition requires $(\xi\lambda)^2 = t^2 + \vec{x}^2 = 0$. It becomes then very hard to access these objects on the lattice due to a non-zero lattice spacing!

New proposal by Ji: compute quasi-distributions $\tilde{q}$

$$\tilde{q}(x,P) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izxP} \langle P|\bar{\psi}(0)\Gamma W(0,z)\psi(z)|P\rangle$$

where xP is the quark momentum in any spatial z-direction and the boosted momentum P of the nucleon is usually taken in the same direction as the Wilson line.

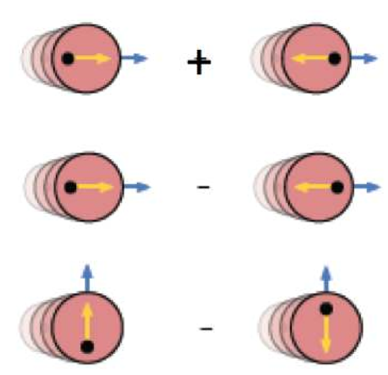


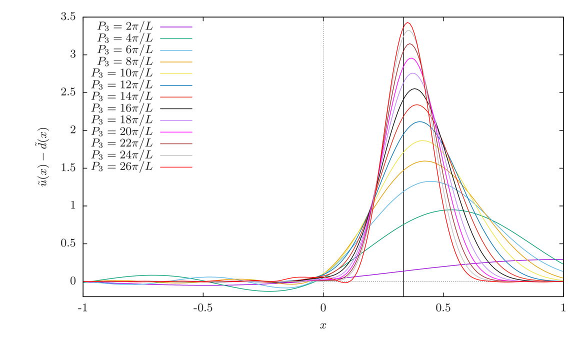
In the limit $P \to \infty$, we recover the light-cone PDFs. On the lattice we calculate $\tilde{q}$ at finite and sufficiently large P and then relate $\tilde{q}$ to the physical distribution via perturbation theory.

Lattice Setup

Specifically we study three types of PDFs:

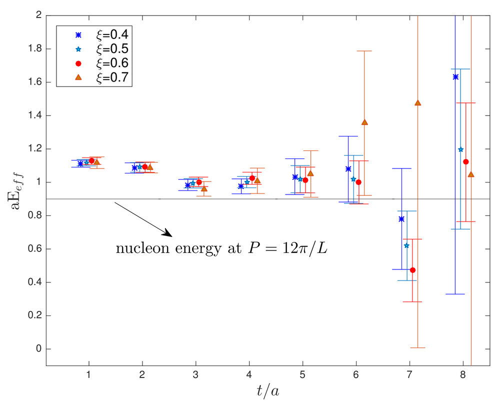
- Unpolarized: $\Gamma = \gamma_i$, $\tilde{q}(x) = \tilde{q}(x)_\downarrow + \tilde{q}(x)_\uparrow$
- Helicity: $\Gamma = \gamma_i\gamma_5$, $\Delta\tilde{q}(x) = \tilde{q}(x)_\downarrow - \tilde{q}(x)_\uparrow$
- Transversity: $\Gamma = \sigma_{ij}$, $\delta\tilde{q}(x) = \tilde{q}(x)_\perp - \tilde{q}(x)_\top$





Free quark momentum distribution
[C. Alexandrou et al. arXiv:1610.03689]

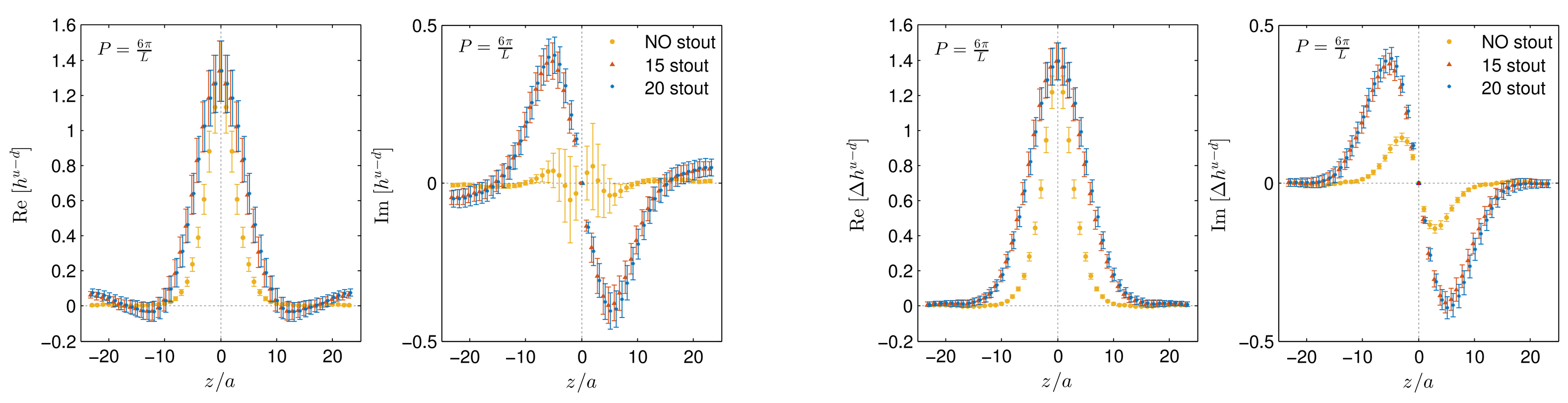
We aim at computations with as high momenta as possible. However, the signal-to-noise ratio decreases and new techniques are required. To improve the overlap with the boosted state we use the *momentum smearing* [G. Bali *et. al*].

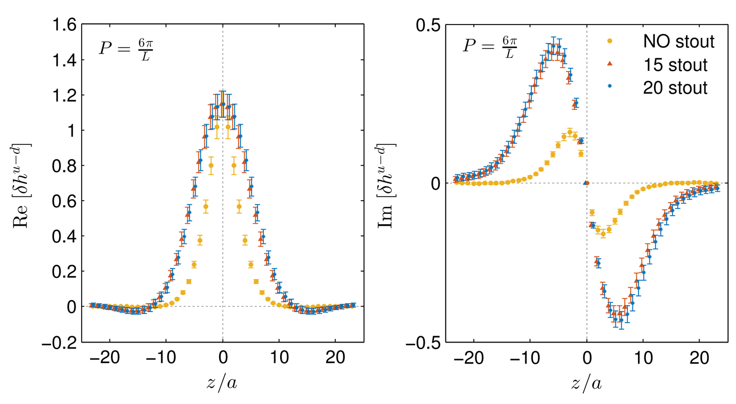


Simulations details:
the simulations are carried out using the ensemble $cA2.09.48$ at the physical quark masses ($M_\pi \simeq 135$ MeV), with $N_f = 2$, volume $V = 48^3 \times 96$ and lattice spacing $a \simeq 0.093$ fm.

Bare matrix elements

To estimate the influence of the renormalization we apply multiple steps of stout smearing to the gauge links in the operators. We present results for the bare unpolarized, helicity and transversity matrix elements computed at $P = 6\pi/L \simeq 0.8$ GeV, fixing the source-sink separation at $12a \simeq 1.1$ fm.

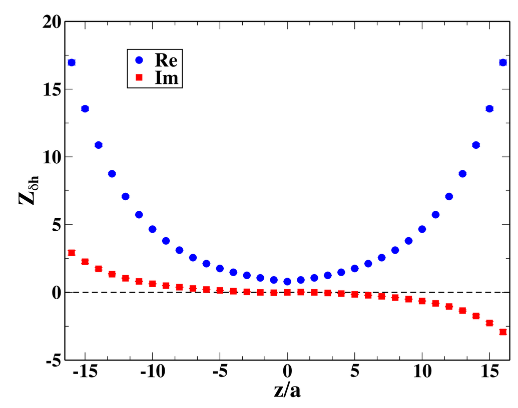




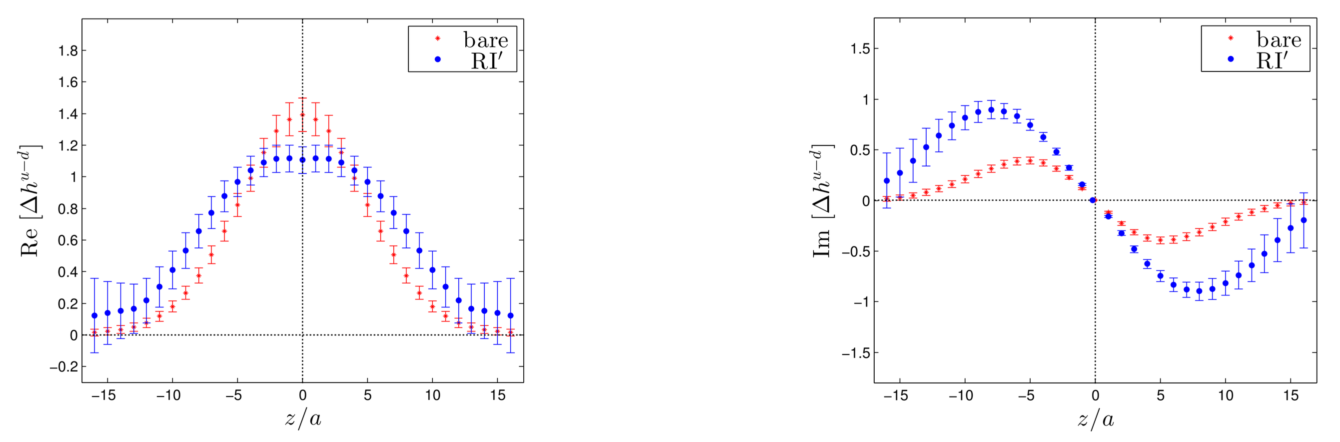
A valuable cross-check of the obtained results can be done by examining the form factors at $z = 0$, that at $Q^2 = 0$ can be identified with the local vector, axial and tensor currents. For this ensemble the local Z-factors at $z = 0$ are $Z_V \simeq 0.75$, $Z_A \simeq 0.79$, $Z_T \simeq 0.85$.

Renormalization - helicity

We apply the non-perturbative renormalization prescription in the RI′ scheme for the helicity quasi-PDF, as discussed by C. Alexandrou *et. al* [arXiv:1706.00265]. In the polarized case there is no mixing with other operators [see the work by M. Constantinou and H. Panagopoulos, arXiv:1705.11193].

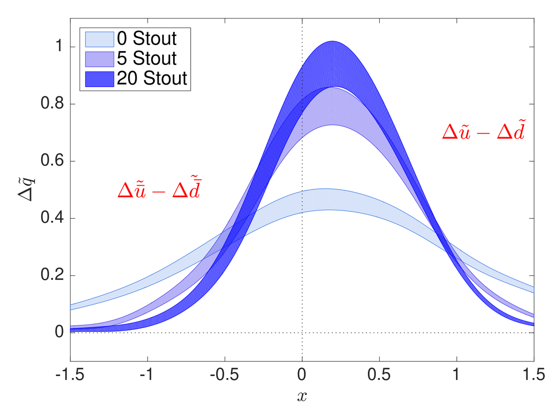


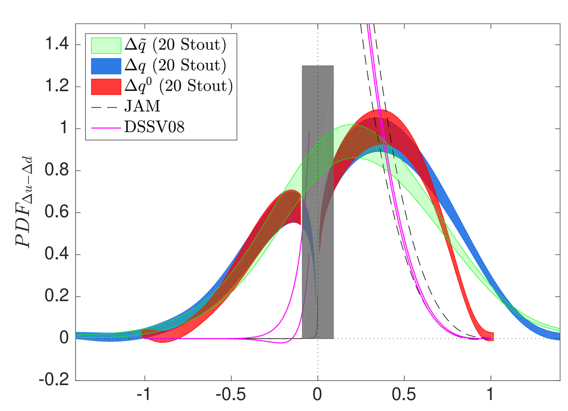
The Z-factors become larger with the length of the Wilson line, cause of the power divergence typical of non-local operators and this leads to an increase of the errors of the renormalized matrix elements in the region of large z.



Asymmetry $q\bar{q}$ - Bare helicity PDF

From the bare matrix elements we extract the bare *quasi*-PDFs $\tilde{q}(x)$, summing over all possible lengths z of the Wilson line. A strong asymmetry of the quasi-PDFs develops when going from 0 to 5 steps of stout smearing, showing that the renormalization plays an important role when looking at the quark distributions. Subsequently, from $\tilde{q}(x)$ we compute the physical distribution $q(x)$ by applying wave function and vertex functions corrections and then nucleon target mass corrections.





The bare helicity PDF does not show yet a qualitative agreement with the distributions extracted from phenomenological analyses. The discrepancy in the region of small x might be related to the presence of the infrared $1/L$ and ultra-violet $1/a$ cut-off regulators on a finite lattice. We also expect the range where the computation is reliable to increase for larger momenta.

Conclusions and Outlook

- We present preliminary results for PDFs at the physical pion mass, obtained with the pioneering method suggested by Ji.
- Enormous progress has been achieved in renormalizing the quasi-PDFs also at the physical point. Since the Dirac structure of the unpolarized distribution exhibits mixing, we focus at the moment on one of two simplest cases, helicity PDF, showing the Z-factors in RI′ scheme and the renormalized matrix elements. The *special* treatment needed for the unpolarized case will be addressed in the near future.
- We certainly need to compute parton distributions with higher momenta and this calculation is in working progress. However, the calculation is not trivial cause of an increasing of the *noise-to-signal* ratio when quark masses approach to the physical ones. The momentum smearing technique helps in this direction, as it was found a gain of a factor 200 in statistics in the previous work by C. Alexandrou *et. al* [arXiv:1610.03689].