# Equation of State and Freezeout in QCD with Staggered Quarks

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# Slowly convergent series

$$\mathcal{R} = 1 + 0.06 + 0.004 + 0. + \cdots$$

$$\mathcal{S} = 1 + 0.72 + 0.52 + 0.38 + \cdots$$

$$\mathcal{T} = 1 + 0.81 + 0.66 + 0.53 + \cdots$$

$$\mathcal{U} = 1 + 0.90 + 0.81 + 0.73 + \cdots$$

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$$1 + x^2 + x^4 + x^6 + \cdots$$

# Critical EOS and Padé Approximants

Our estimate of the QCD critical point:

$$\mu_B^E = (1.85 \pm 0.04)\, T^E, \qquad T^E = (0.94 \pm 0.01)\, T_c.$$

Close to a critical point a branch cut

$$\chi_B^2 \propto [\mu_B^2 - (\mu_B^E)^2]^{-\psi} + \text{non} - \text{singular}$$

Implies pole in the DLOG

$$m_1 \simeq \frac{\partial \log \chi_B^2}{\partial \mu_B} \quad \longrightarrow \quad \frac{2\psi\mu_B}{\mu_B^2 - (\mu_B^E)^2}.$$

Convert series expansion for $\chi_B^2$ to series for $m_1$ and then integrate to get equation of state. **ILGTI, 2013, 2016, 2017**

BONUS: $m_1$ directly measured in experiment, so use this to understand freezeout. **ILGTI 2010**

# Using a DLOG Padé

Error in series coefficients give rise to errors in $\psi$ and $\mu_B^E$. As a result finite probability of infinite value of $m_1$ at any $\mu$!

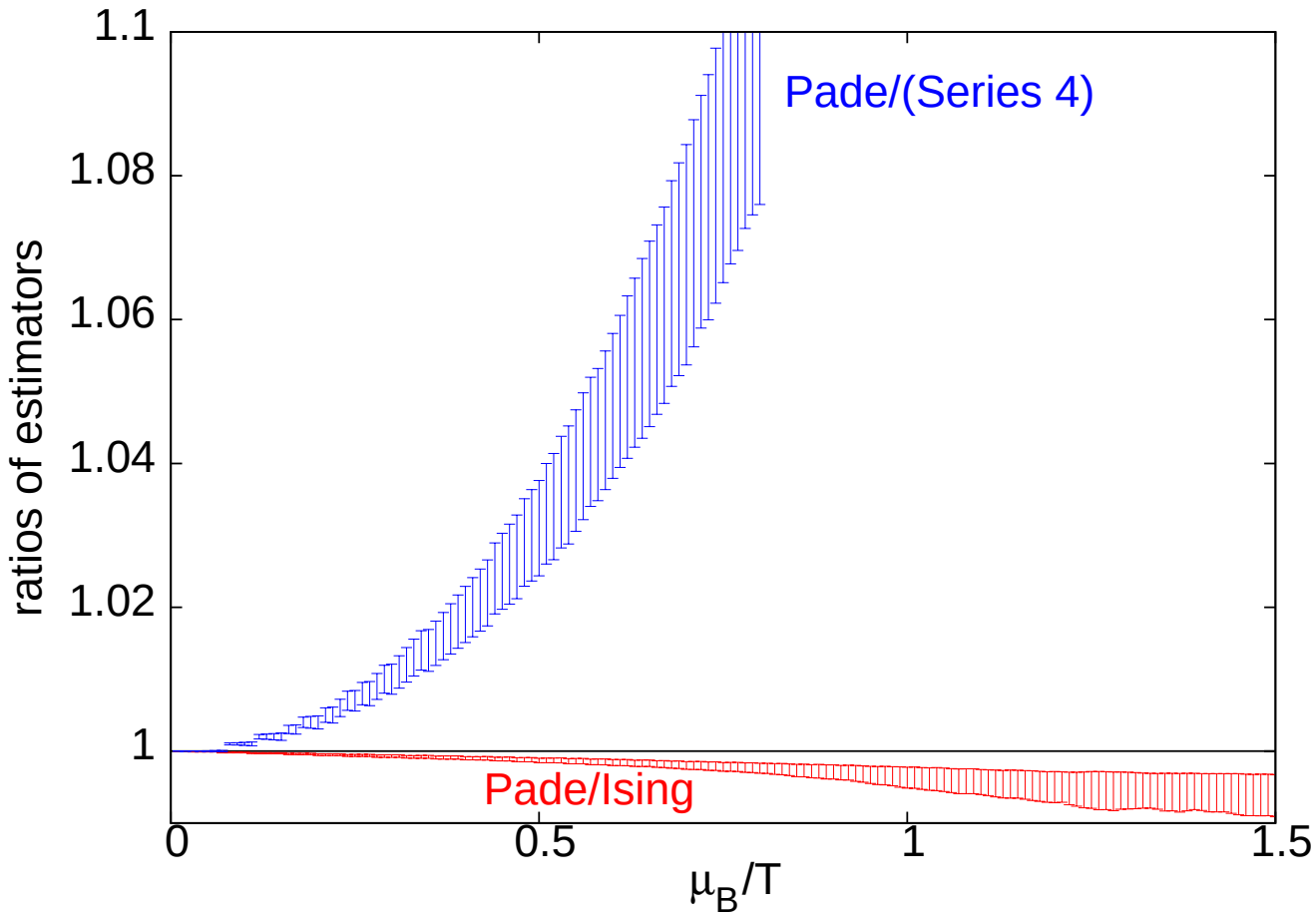
Regularization due to bootstrap with finite statistics. Goes to reasonable limit with infinite statistics, but with critical slowing down!

**SG, Karthik, Majumdar (2014)**

Is the Padé a good resummation? Use higher order terms in the series to check this. At present only one term can give a reasonable check. In future?

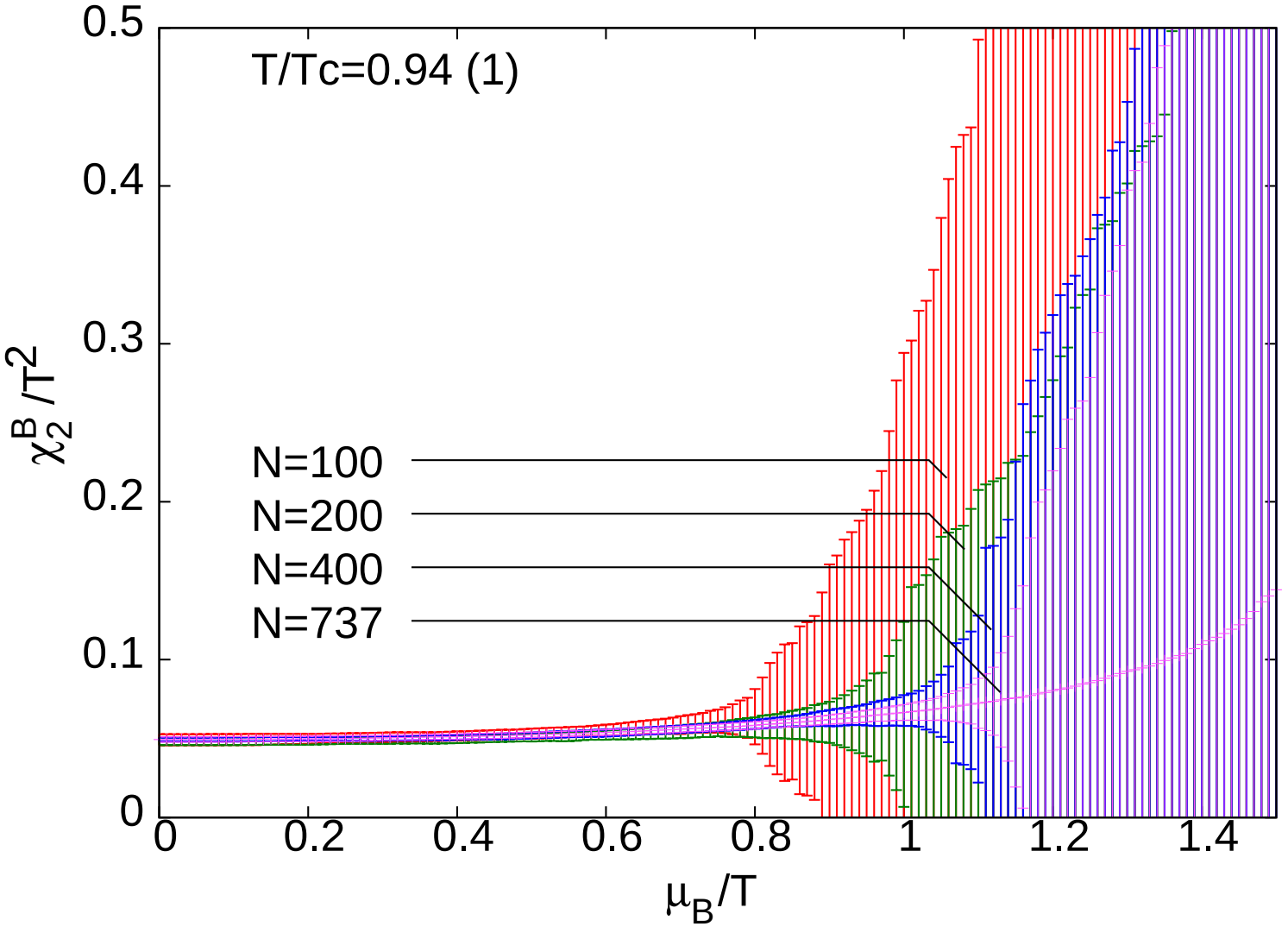
If we assume Ising critical exponent, then only one free parameter: $\mu_B^E$. Two terms serve as tests.

# Comparing extrapolations

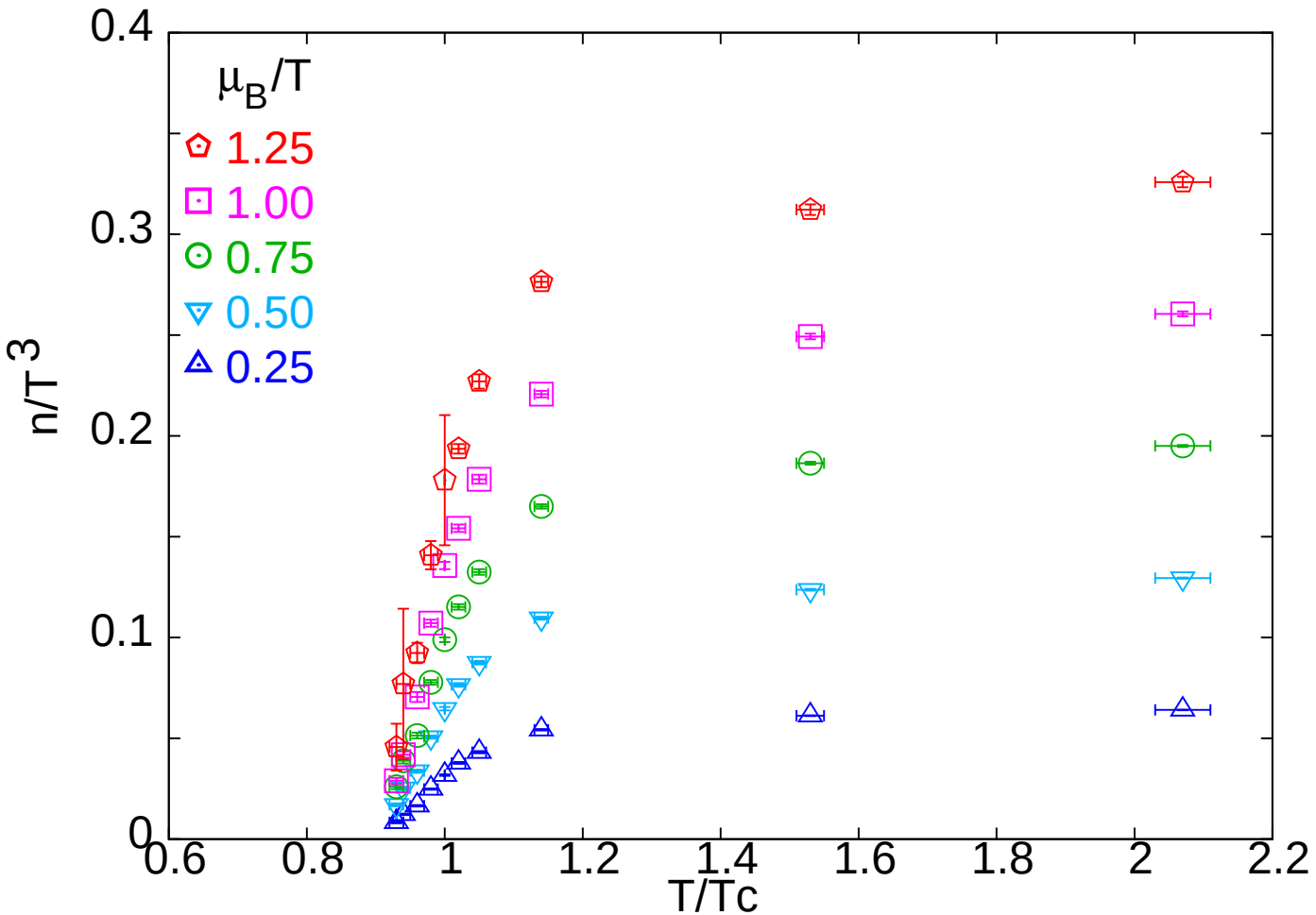



Measurements consistent with Ising exponents (also with mean field, at present). Not consistent with truncated series expansion.

# Extrapolation and critical slowing down



# Equation of state




$\Delta P = P(T, \mu_B) - P(T, 0)$, and $n = \partial \Delta P / \partial \mu_B$.

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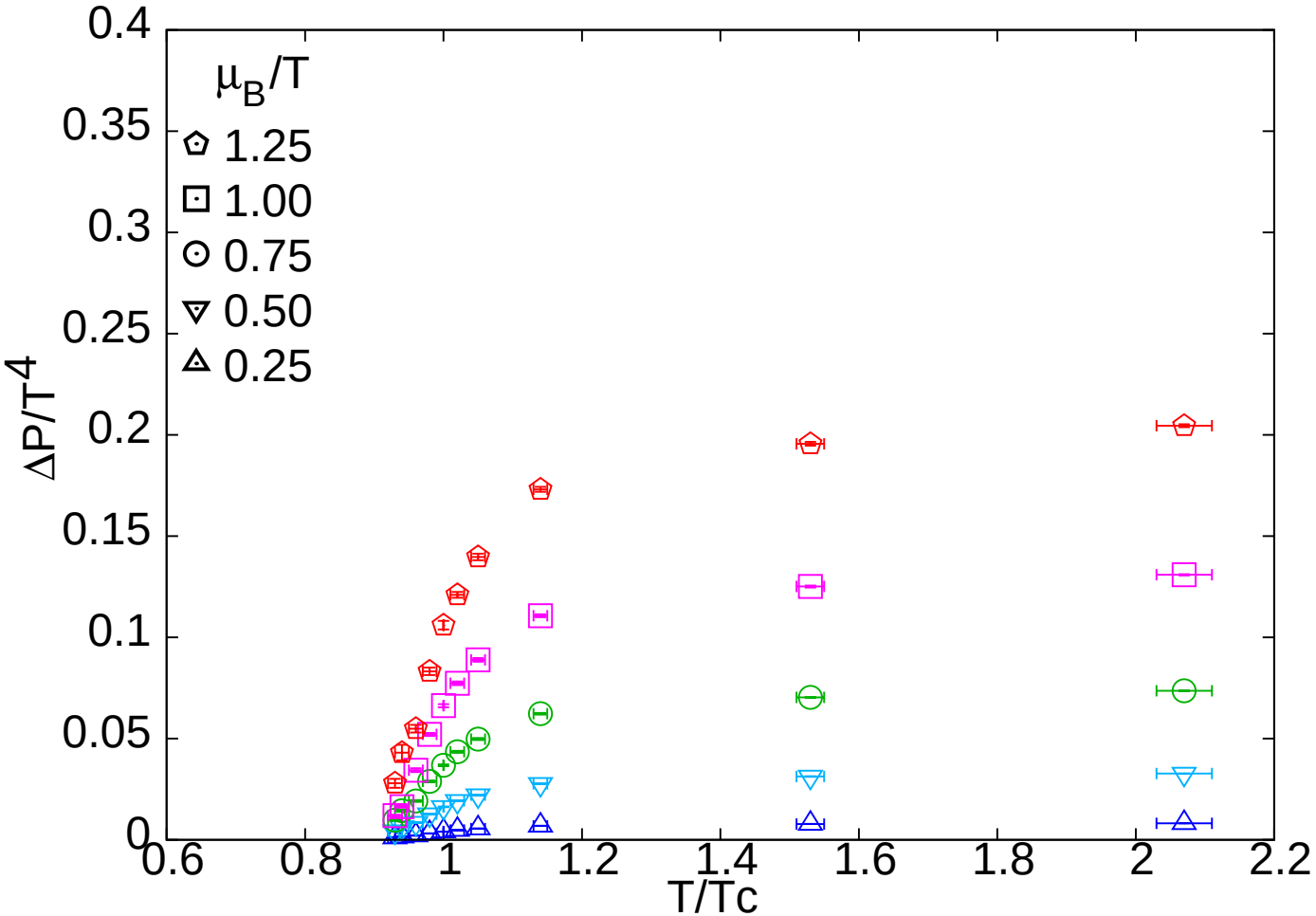


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# Freezeout

$$m_1 = \frac{\partial \log(\chi_B^2/T^2)}{\partial(\mu_B/T)} = \frac{\chi_B^3/T}{\chi_B^2/T^2}$$

Suggested as an experimental measurement in **SG (2009), Gavai and SG (2010)**. First data from RHIC in **STAR (2010)**.

Freezeout is a signal of non-equilibrium evolution. Why compare data to equilibrium statistics determined on lattice?

Compare different signals with lattice data and extract $T$ and $\mu_B$. Different signals will be slightly different. Degree of difference tells us about how close to equilibrium the system was.

Much later work including important measurements of $m_1$ by **Bazavov et al (2012)**, **Borsanyi et al (2013)**, and several others.

# Sources of error

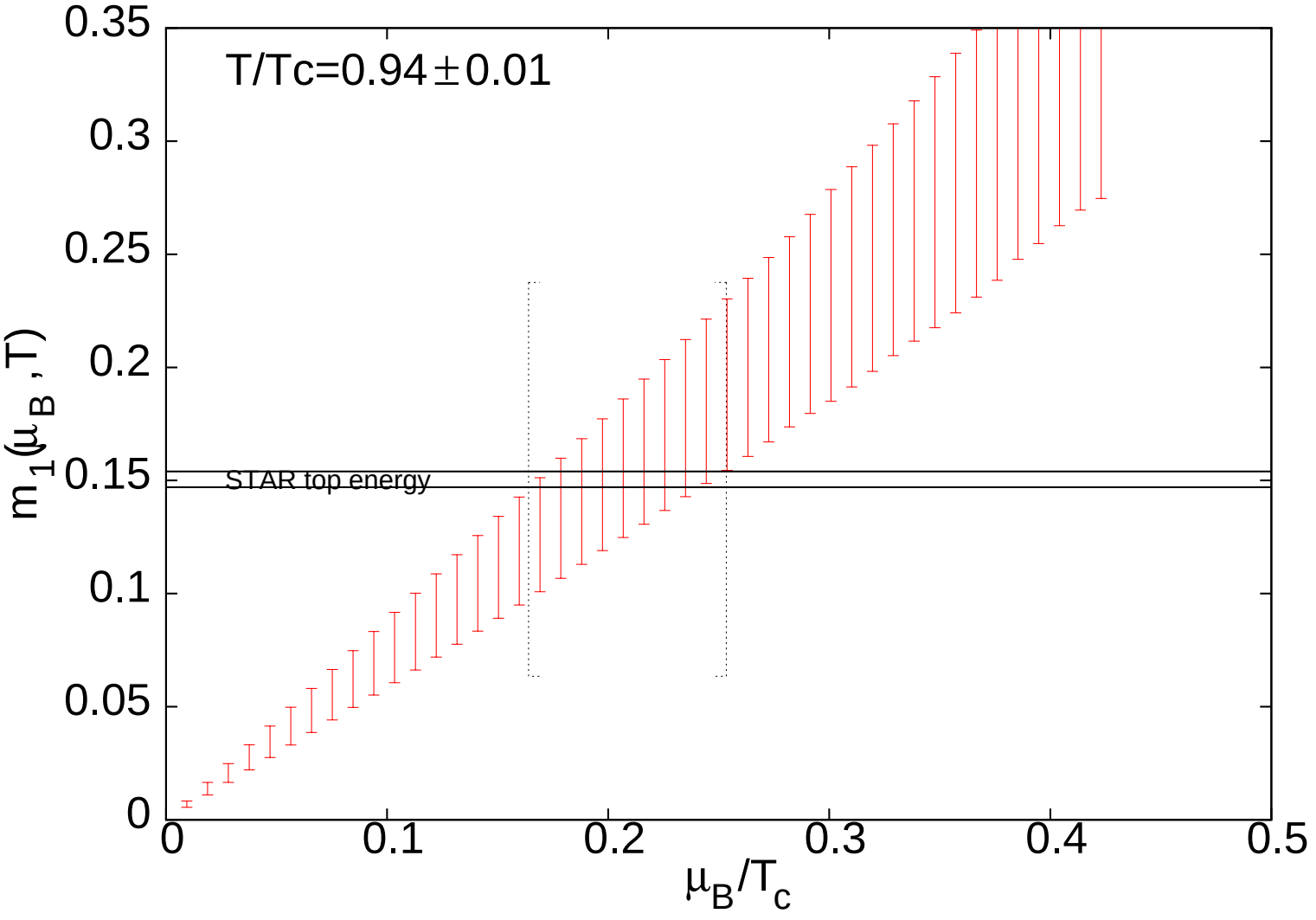
## Statistical and lattice systematics

Previously unexamined source of error: series truncation and critical slowing down. Since freezeout occurs close to the critical temperature, lattice computations may suffer from freezeout. Not visible in truncated series analysis.

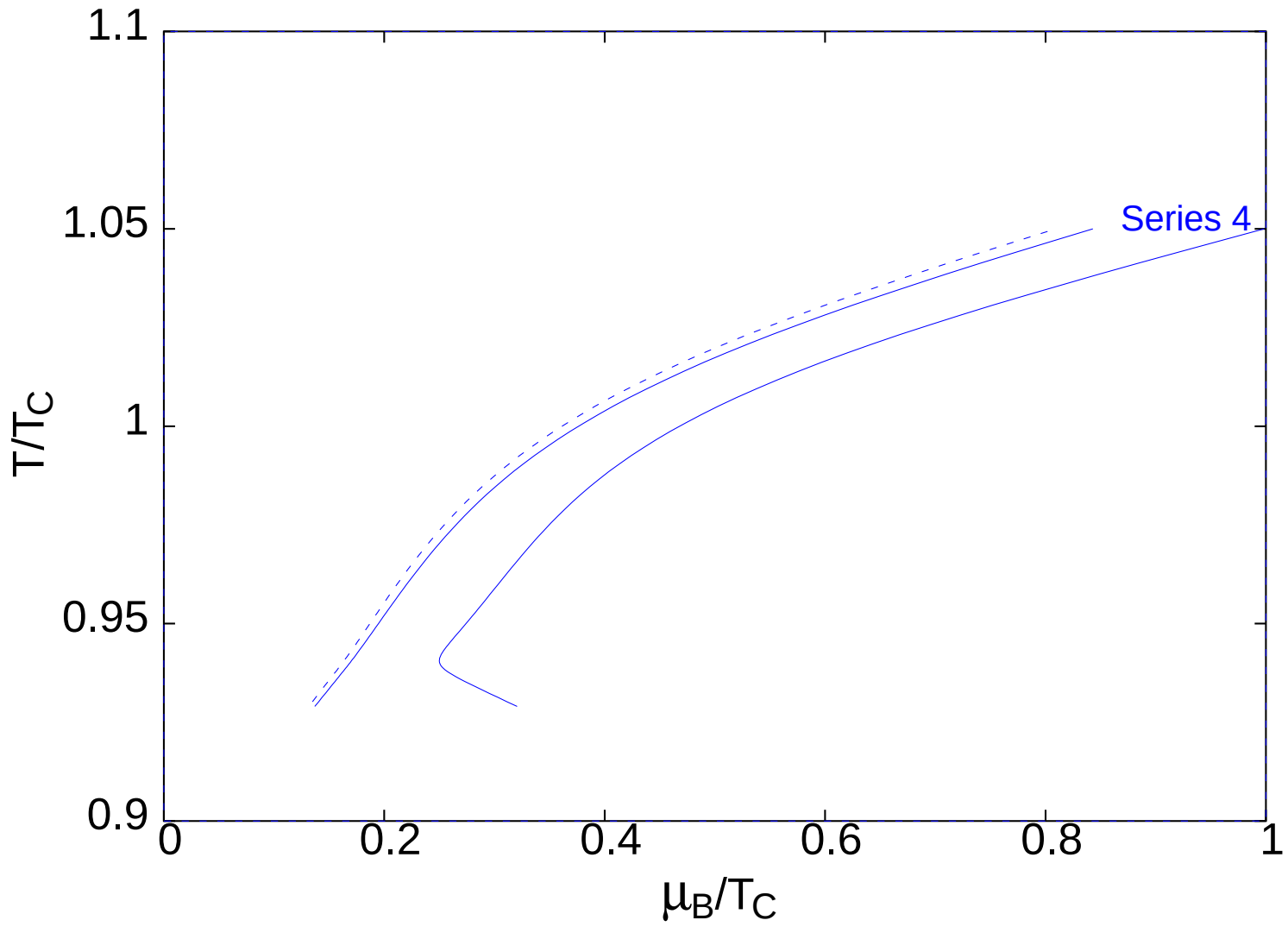
## Mathematical

At least two freezeout parameters to be determined: $T$ and $\mu$ at each collision energy. Cannot use one measurement to extract them from first principles. One lattice measurement gives an allowed region of parameters.

# Critical slowing down



# Freezeout parameters



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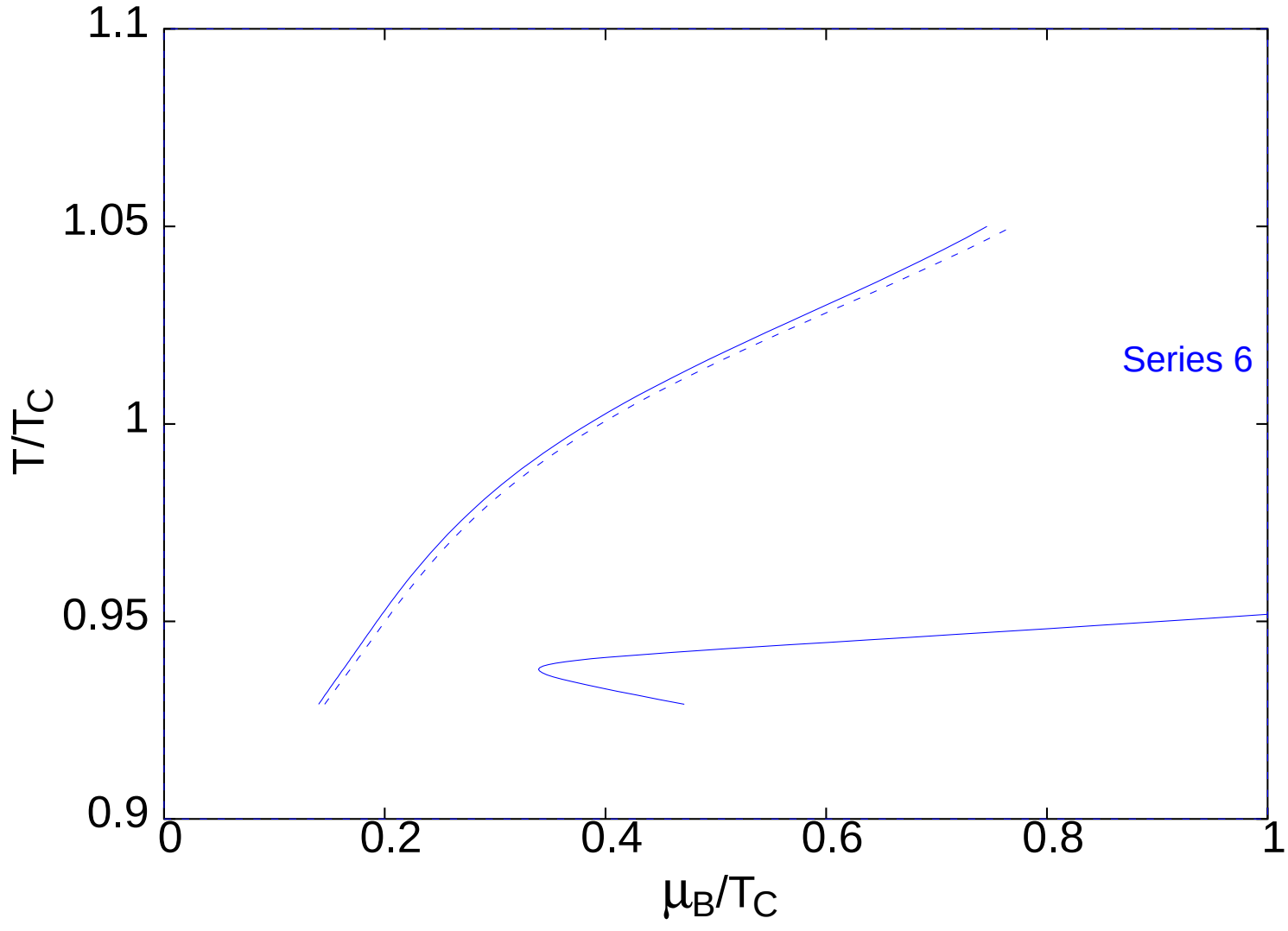

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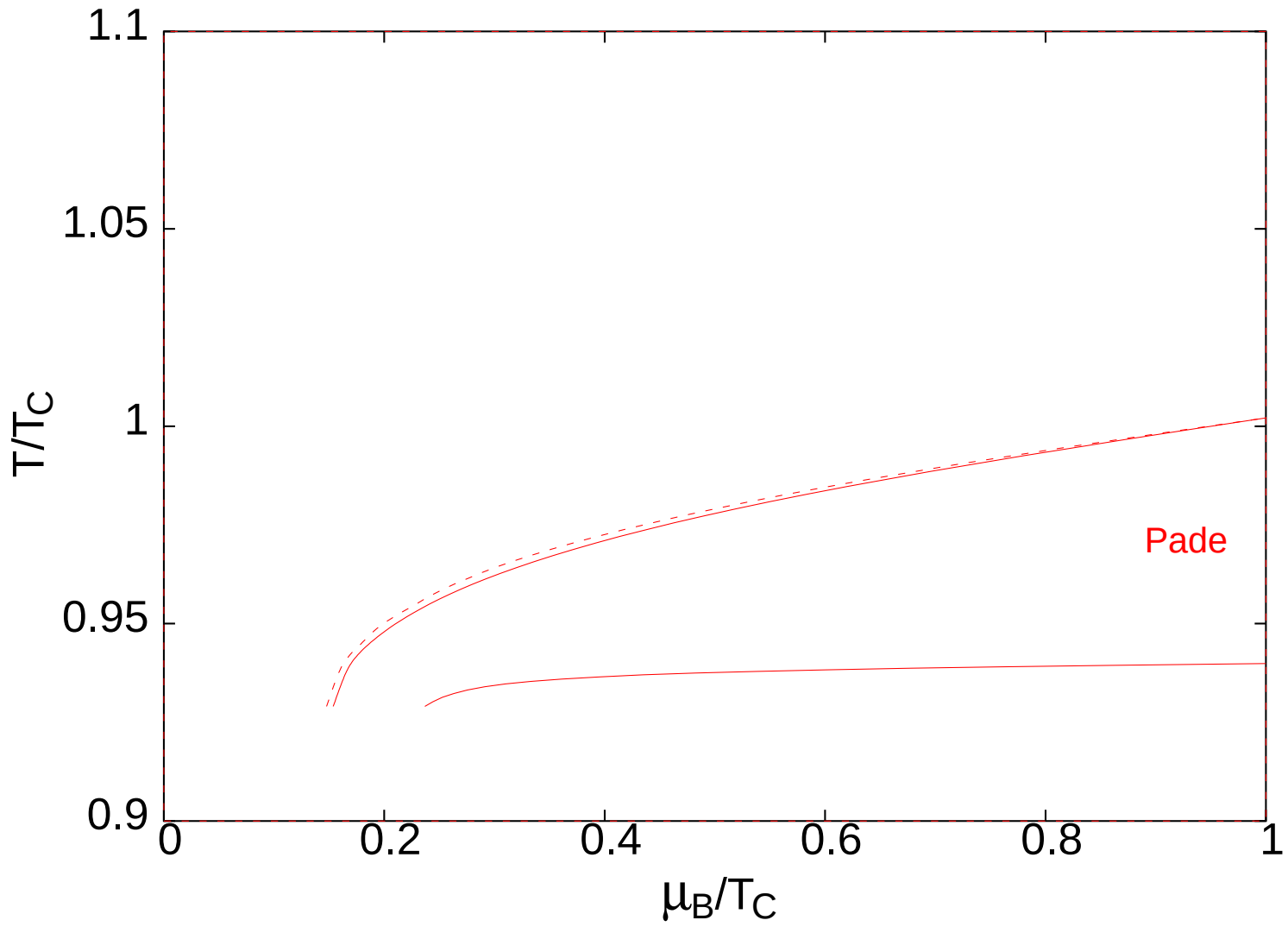

# Conclusions

## Equation of state

Critical slowing down visible in numerical evaluation of DLOG Padé ($m_1$). Reconstruction of singular part of free energy and its derivatives straightforward.

## Freezeout

The DLOG Padé $m_1$ is unlikely to be a baryometer. Resummed series shows that freezeout temperature is likely to be below $T_c$. Extraction of freezeout parameters still error bound.