Nucleon structure from 2+1 flavor lattice QCD near the physical point

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for PACS Collaboration

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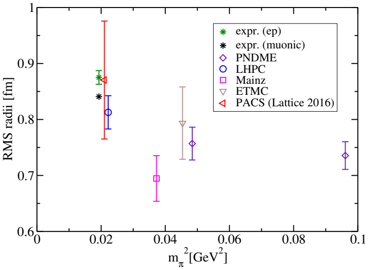
June 19, 2017

contents

Introduction

We access the information on the nucleon structure through nucleon form factors (FFs) that can be computed from first principles with lattice QCD.



size of the proton

- Lattice results of the electric charge radius (RMS) have been often underestimated.
- Note that chiral effective field theory predicts that the charge radius has the logarithmic divergence in the chiral limit.

Our preliminary result (Lattice 2016)* computed with almost physical pion mass on very large volume shows agreement with experimental results.

*Y. Kuramashi *et al.*, PoS Lattice2016 158 (2017).

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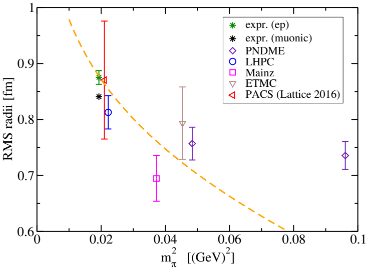

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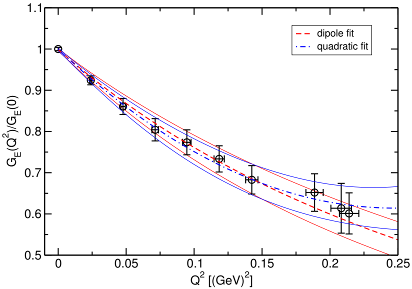
Method

How can we determine the proton radius?

$$G(t = Q^2) = 1 - \frac{1}{6}\langle r^2 \rangle t + \mathcal{O}(t^2) \quad (1)$$



- We can estimate the root mean squared (RMS) radius from the slope at zero momentum transfer.
- Low Q^2 data point is crucial for the determination of the RMS radius.

→ Large spacial size is required.

$$Q^2 = \left(\frac{2\pi}{aL}\right)^2 \times \text{integer}$$

The slope determination is highly sensitive to how we fit the Q^2-dependence of the form factor.

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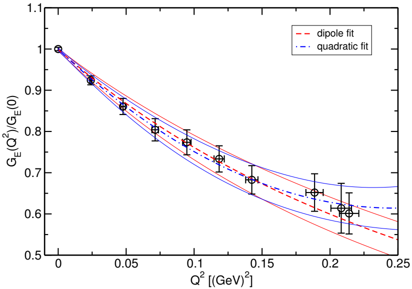

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polynomial? $G(t) = \sum a_k t^k$ or dipole? $G(t) = \frac{1}{(1+\frac{t}{m})^2}$

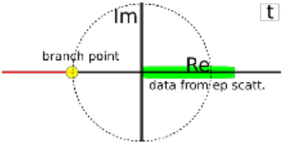
model dependence

Method

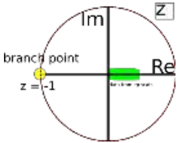
z-expansion method[†] is a model-independent approach.

$$G(t = Q^2) = \sum c_k z^k \qquad z = \frac{\sqrt{t_{\rm cut} + t} - \sqrt{t_{\rm cut}}}{\sqrt{t_{\rm cut} + t} + \sqrt{t_{\rm cut}}} \qquad t_{\rm cut} = 4m_\pi^2 \qquad (2)$$

We can avoid non-analytic region due to thresholds of two or more particles.



→conformal →
→mapping →



Analytic domain is mapped into the open unit disk ($|z|^2 < 1$).
So the z-expansion series $\sum_0^\infty c_k z^k$ should converge and reduce model-dependence.

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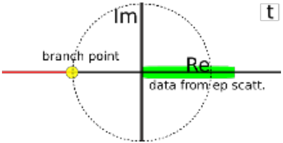

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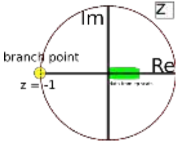

Analytic domain is mapped into the open unit disk ($|z|^2 < 1$).
So the z-expansion series $\sum_0^\infty c_k z^k$ should converge and reduce model-dependence.
We perform a rigorous analysis of the Q^2-dependence of nucleon form factors including very low Q^2 data point, by using z-expansion method.

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Simulation Details

Near Physical Point $2+1$ flavor Lattice QCD Simulation using K computer (PACS Collaboration)[‡]

HPCI Strategic Program Field 5
"The origin of matter and the universe"

(Nos. hp120281,hp130023,hp140209,hp140155,hp150135,hp160125,hp170022)

- Iwasaki gauge action
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- $L^3 \times T = 96^3 \times 96 (La \sim 8\mathrm{fm})$
- cutoff $a^{-1} \sim 2.3\mathrm{GeV}$
- pion mass $m_\pi \sim 0.145\mathrm{GeV}$
- 200 configurations (updated)

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We calculated isovector nucleon form factors in the vector (V), axial-vector (A) and also pseudo-scalar (P) channels.

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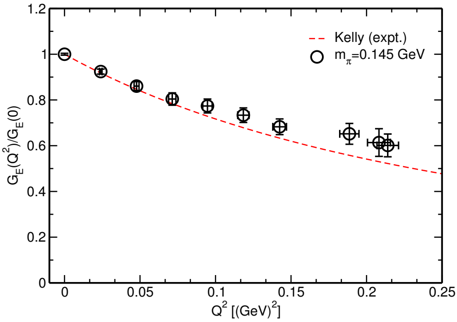
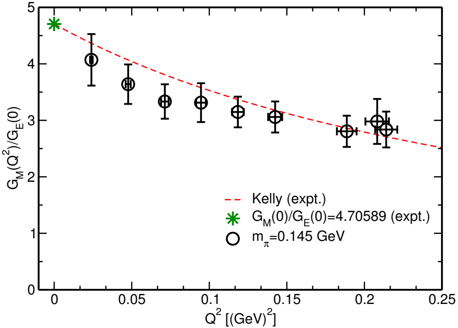
Numerical Results (Electric & Magnetic FFs)

Updated from LATTICE 2016

146 →200 configurations



isovector G_E



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Both G_E and G_M are consistent with experiments

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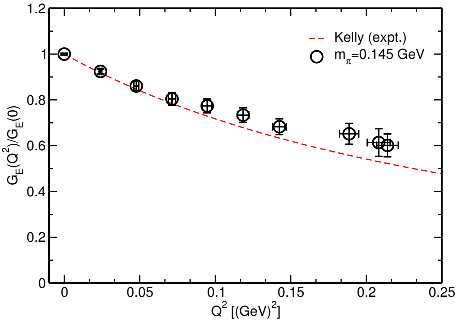

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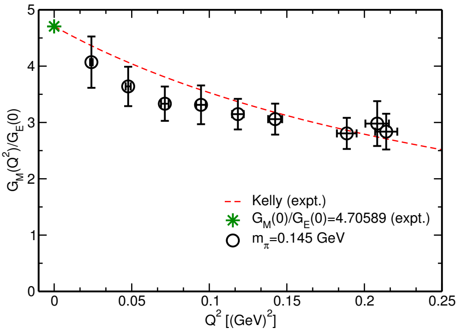

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Both G_E and G_M are consistent with experiments

We analyze electric & magnetic form factors using z-expansion method.

Numerical Results (Electric & Magnetic FFs)

$$G(t = Q^2) = \sum_{k=0}^{k_{\max}} c_k z^k \qquad z = \frac{\sqrt{t_{\rm cut} + t} - \sqrt{t_{\rm cut}}}{\sqrt{t_{\rm cut} + t} + \sqrt{t_{\rm cut}}} \qquad t_{\rm cut} = 4m_\pi^2 \tag{3}$$

To obtain coefficient c_k, we have to solve "minimize" problem :

$$\min |\mathrm{A}\mathbf{c} - \mathbf{b}| \qquad \mathrm{A}_{ij} = \frac{z(Q_i^2)^j}{dG(Q_i^2)} \qquad \mathbf{b}_i = \frac{G(Q_i^2)}{dG(Q_i^2)} \tag{4}$$

We can solve this problem using SVD and a pseudoinverse matrix.

$$\mathrm{A} = \mathrm{U} \times \mathrm{diag}(\mathrm{s}_1, \cdots) \times \mathrm{V}^\dagger \tag{5}$$

$$\mathrm{A}^+ = \mathrm{V} \times \mathrm{diag}(1/\mathrm{s}_1, \cdots) \times \mathrm{U}^\dagger \tag{6}$$

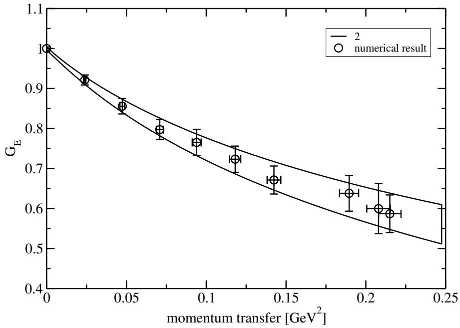
$$\mathbf{c} = \mathrm{A}^+\mathbf{b} = \sum_i^N \frac{1}{s_i}\mathbf{v}_{(i)}\mathbf{u}_{(i)}^\dagger\mathbf{b} \tag{7}$$

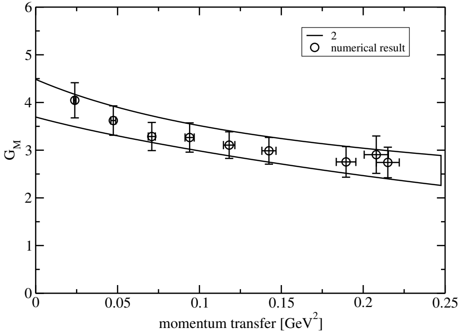
We may truncate the summation of Eq.(7) up to N before the singular values s_i become too small in order to prevent over-fitting.

Numerical Results (Electric & Magnetic FFs)

We show N-dependence.

$$\mathbf{c} = \mathrm{A}^{+}\mathbf{b} = \sum_{i}^{N} \frac{1}{s_i} \mathbf{v}_{(i)} \mathbf{u}_{(i)}^{\dagger} \mathbf{b} \qquad G(z) = c_k z^k$$





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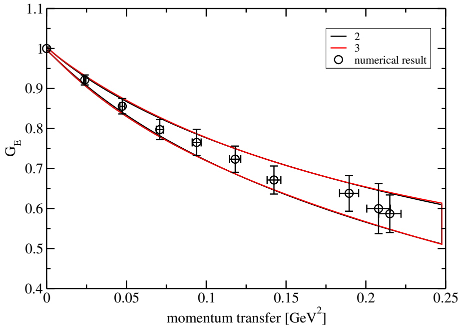

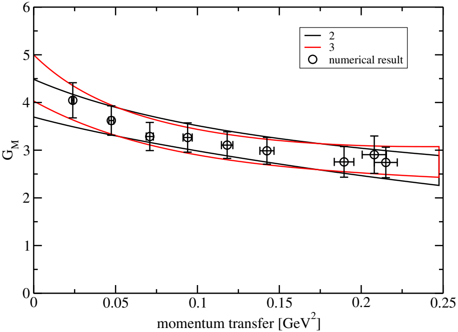

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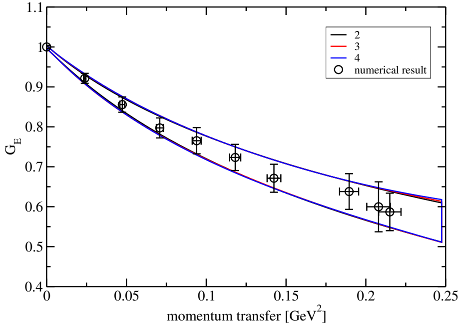

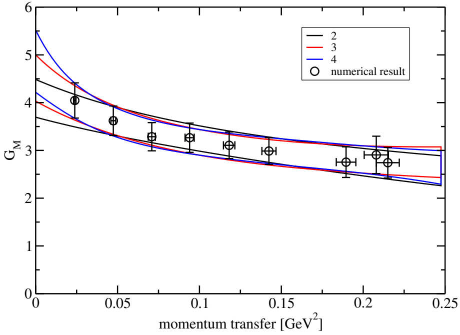

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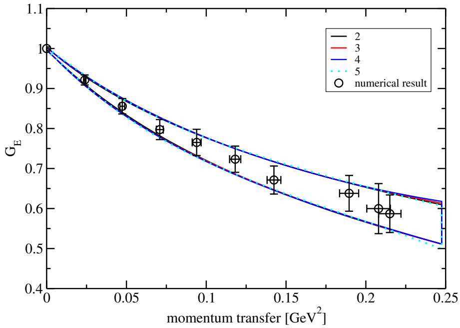

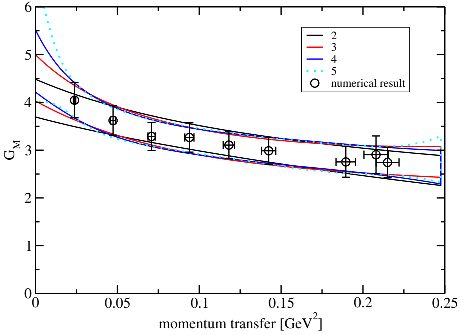

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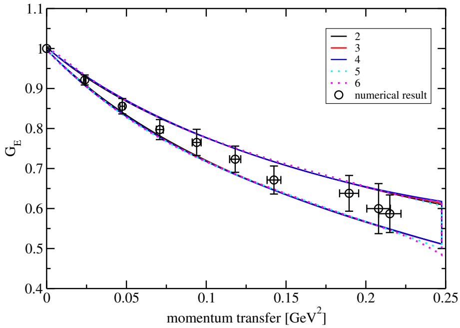

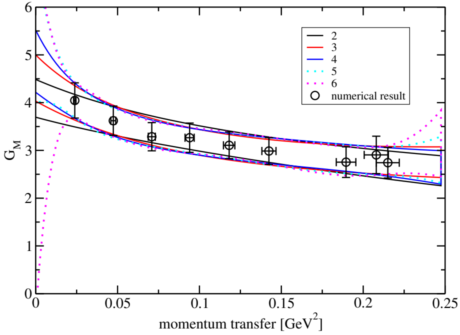

Numerical Results (Electric & Magnetic FFs)

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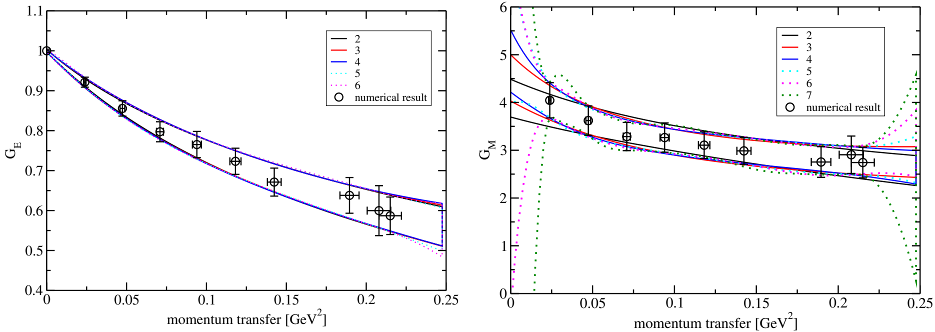
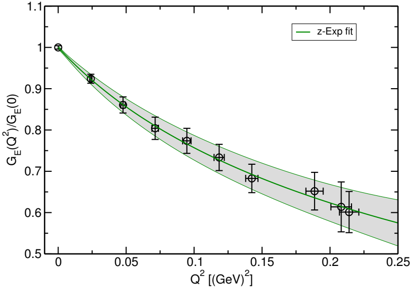
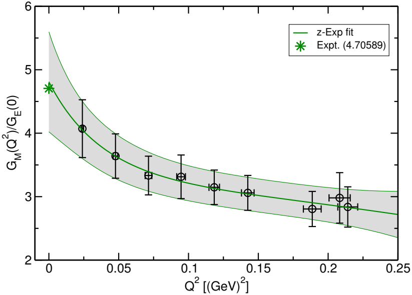


We observe that stable fit results are obtained up to $N = 4$.

$$s_4 \gg s_i \qquad \mathit{for}\ i > 4 \qquad \mathit{then\ set}\ \mathbf{u}_{(i)} = 0.$$

Numerical Results (Electric & Magnetic FFs)





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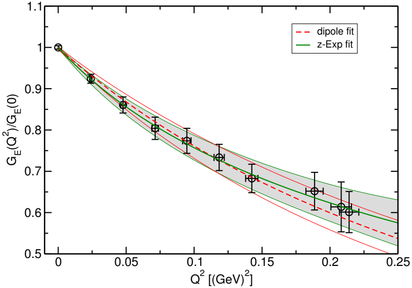

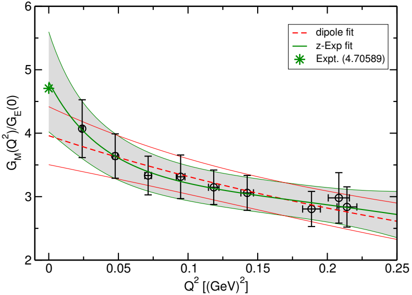

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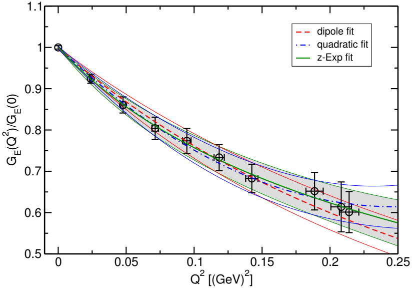

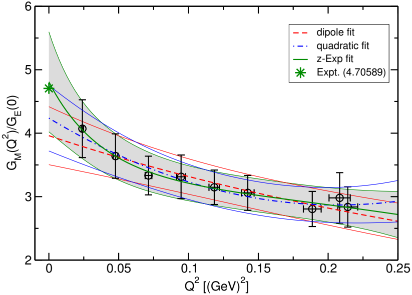

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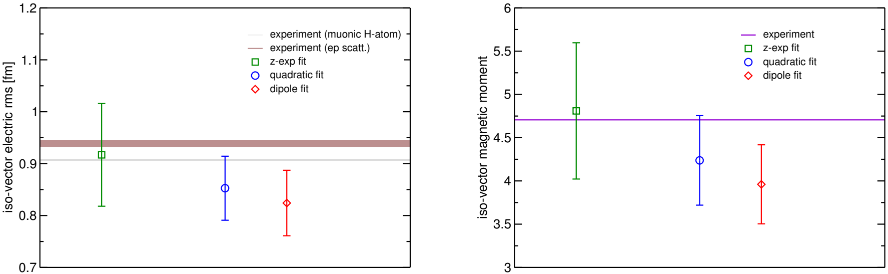
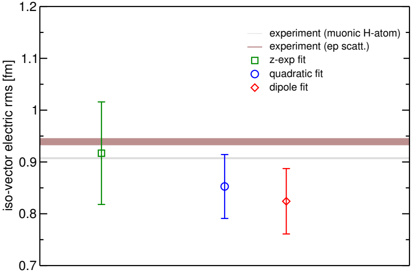


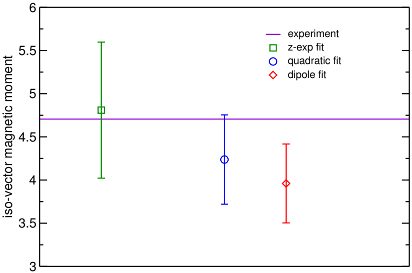
We extract the isovector ($\mathcal{V}$) radius and magnetic moment, which are defined as

$$\langle r_E^2 \rangle_{\mathcal{V}} = \langle r_E^2 \rangle_p - \langle r_E^2 \rangle_n, \quad (8)$$

$$\mu_{\mathcal{V}} = \mu_p - \mu_n. \quad (9)$$

Numerical Results (Electric & Magnetic FFs)





we obtain

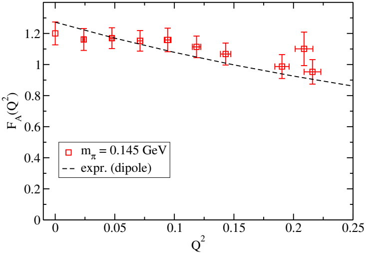
$$\sqrt{\langle r_E^2 \rangle_V} = 0.917 \pm 0.099 [\mathrm{fm}] \tag{8}$$

$$\mu_V = 4.81 \pm 0.79 \tag{9}$$

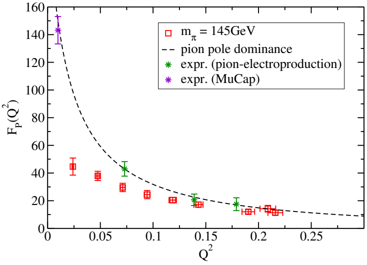
Numerical Results (Axialvector & Pseudoscalar FFs)

We also show a preliminary analysis on FFs in axial channel.

$$\langle N|A^a_\mu(x)|N\rangle = \overline{u}\left(\gamma_\mu\gamma_5 F_A(Q^2) + iq_\mu\gamma_5\frac{F_P(Q^2)}{2M_N}\right)\tau^a u e^{iq\cdot x} \tag{10}$$



F_A



F_P

Although F_A is barely consistent with experiments, F_P is underestimated in comparison with both experiments and the pion-pole dominance model.

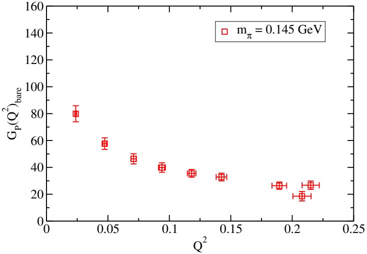
$$F_P(Q^2) = \frac{4M_N^2}{Q^2 + m_\pi^2}F_A(Q^2) \qquad \text{PPD ansatz} \tag{11}$$

Numerical Results (Axialvector & Pseudoscalar FFs)

Although axial-vector current is not conserved when quark mass is non-zero, the current still satisfies the following Ward-Takahashi identity (AWTI)

$$\partial^\mu A^a_\mu(x) = 2mP^a(x), \tag{12}$$

where m is quark mass.



G_P

Goldberger-Treiman (GT) relation is derived from the nucleon matrix element of the currents on both sides of Eq.(12)

$$2M_N F_A(Q^2) - \frac{Q^2}{2M_N} F_P(Q^2) = 2mG_P(Q^2), \tag{13}$$

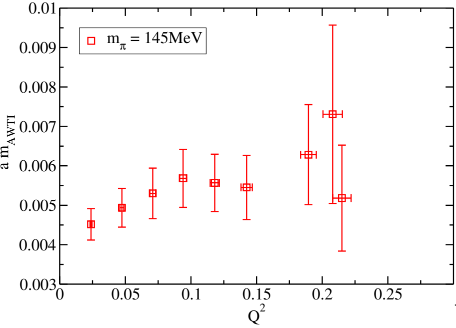
where $G_P(Q^2)$ is pseudoscalar form factor.

$$\langle N|P^a(x)|N\rangle = \overline{u}\left(\gamma_5 G_P(Q^2)\right)\tau^a u e^{iq\cdot x} \tag{14}$$

Numerical Results (Axialvector & Pseudoscalar FFs)

GT relation gives us a chance to determine a bare quark mass.

$$m = \frac{2M_N F_A(Q^2) - \frac{Q^2}{2M_N} F_P(Q^2)}{2G_P(Q^2)}. \tag{15}$$



quark mass from Goldberger Treiman (GT) relation

- Ratio m has no apparent Q^2 dependence.
- am is about 0.005 that corresponds to about 10 MeV.

→ Recall that am=0.001577(10) is obtained from the PCAC relation[a].

[a] PACS Collaboration, arXiv:1511.08549 and arXiv:1511.09222

Summary

We have studied the various nucleon form factors calculated in 2+1 flavor QCD near the physical point (m_π =145MeV) on a large spacial volume $(8.1\mathrm{fm})^3$.

- We examine both electric and magnetic form factor shapes with a model-independent analysis based on the z-expansion method.

  As a result, we obtain

  - RMS radius ($\sqrt{\langle r_E^2\rangle_\mathcal{V}}$ = 0.911(99)[fm]) from isovector G_E
  - magnetic moment ($\mu_\mathcal{V}$ = 4.81(79)) from isovector G_M

  both which are consistent with experimental values.

- And we also analyzed axial and pseudoscalar form factors.

  - F_A is barely consistent with experments ($g_A = 1.16(8)$)
  - Three form factors, F_A, F_P and G_P satisfy the GT relation, although F_P form factor is underestimated in comparison with experiments in low Q^2 region.