Lattice study of finite volume effect in HVP for muon g-2

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Target precision in lattice QCD

$\mathrm{Err}[a_\mu^{BNL}] = 6.3 \times 10^{-10}$

Will be factor 5 improvement in FNAL, JPARC new experiment

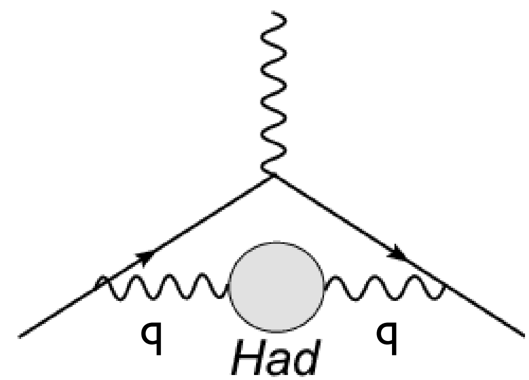
- Leading order of hadronic contribution (HLO)

  Integral of vacuum polarization from $q \in [0, \infty]$

  Target precision < 1% ~ $O(\mathrm{Err}[a_\mu^{BNL}])$

  Dispersion theory (N_f=5) using R-ratio (e+e-) :
  $a_\mu^{HLO} = 688.6(4.3) \times 10^{-10} \Rightarrow 0.6$ % precision

  Jegerlehner, 1511.04473



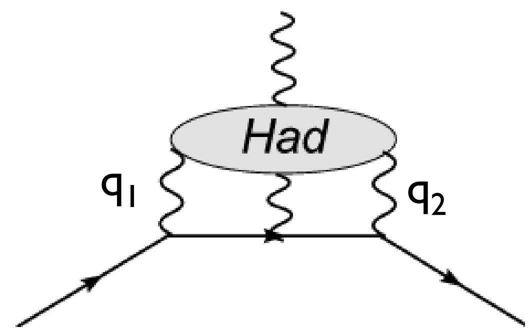
- Next-to-leading order (HNLO)

  Integral of hadronic light-by-light diagram from $q_1 \in [0, \infty]$, $q_2 \in [0, \infty]$

  Target precision ~ 10% ~ $O(\mathrm{Err}[a_\mu^{BNL}])$

  Model: $a_\mu^{HLO} = 10.6(0.3) \times 10^{-10} \Rightarrow$ ~3 %

  Prades et al., 0901.0306



g-2 with Q integral

- Euclidean momentum integral

Lautrup et al., Phys. Rep. 3 (1972), Blum, PRL91(2003)

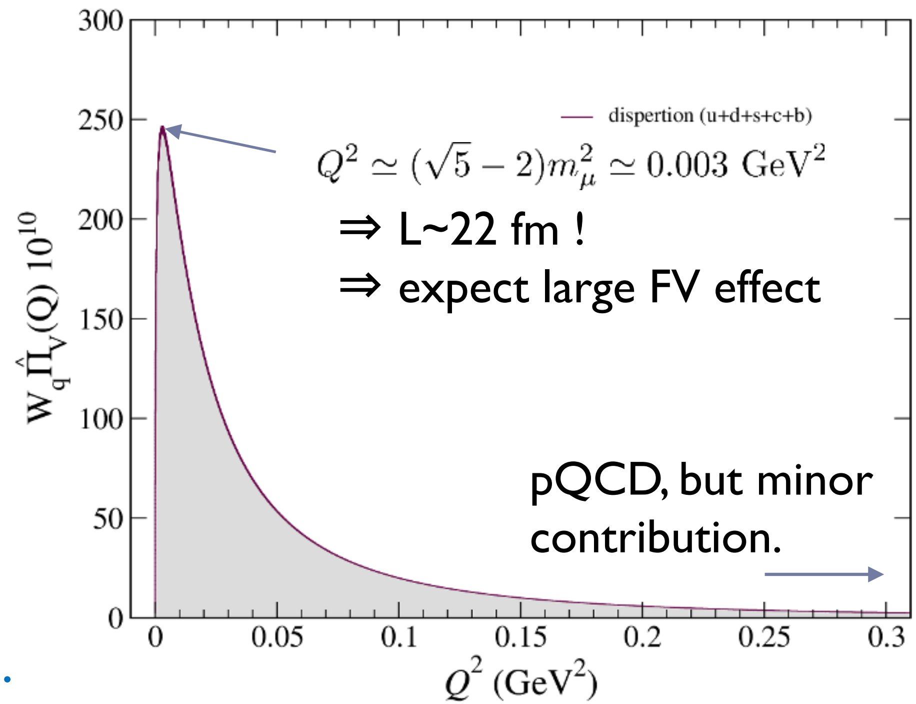
$$a_\mu^{\rm HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty ds K_E(s)\hat{\Pi}(s), \quad \hat{\Pi}(s) = 4\pi^2\left(\Pi(s) - \Pi(0)\right)$$

$$K_E(s) = \frac{1}{m_\mu^2} s Z(\hat{s})^3 \frac{1-\hat{s}Z(\hat{s})}{1+\hat{s}Z^2(\hat{s})}, \quad Z(s) = -\frac{\hat{s}-\sqrt{\hat{s}^2+4\hat{s}}}{2\hat{s}}, \quad \hat{s} = \frac{s}{m_\mu^2}$$

VPF tensor

$$\begin{aligned} \Pi_{\mu\nu} &= \int e^{iQx}\langle V_\mu(x)V_\nu(0)\rangle \\ &= (-Q_\mu Q_\nu + Q^2\delta_{\mu\nu})\Pi(Q) \\ V_\mu(x) &= \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d + \cdots \end{aligned}$$

- Pade function ($Q^2 < m_\rho^2$)
- Renormalization constant $\Pi(0)$ given from extrapolation.
- Q^2 integral from 0 -- ∞, but pQCD gives asymptotic function.



1. Introduction

g-2 with t integral

- Temporal integral

Bernecker, Meyer, EPL A47(2011)

$$a_\mu^{\rm HLO} = \int_0^\infty W_t(t)G(t), \quad G(t) = \int d^3x \langle V_i(x)V_i(0)\rangle$$

$$W_t(t) = 4\alpha^2 m_\mu t^3 \hat{K}(t)$$

$$\hat{K}(t) = \frac{2}{m_\mu t^3}\int_0^\infty \frac{d\omega}{\omega} K_E(\omega^2)\left[\omega^2 t^2 - 4\sin^2(\omega t/2)\right]$$

Pros

- On the lattice, <VV>(t) without momentum.
- Integral (summation) without extrapolation/interpolation.

Cons

- Temporal integral from 0 – ∞, we need to know asymptotic function
- Temporal boundary effect, backward propagation
- Discrete sum.

Possible to involve the large uncertainty due to FV effect and lattice artifact.

2. HVP on the lattice

Studies of finite volume

- ChPT

  Aubin et al., PRD93(2016)

  - Lowest-order SChPT gives VPF tensor: $\Pi_{\mu\nu}(q)$
  - 10% -- 15% discrepancy between $a_\mu^{HLO}[A_1]$ and $a_\mu^{HLO}[A_1^{44}]$

    consistent with lattice calculation (L=3.8 fm, 0.22 GeV pion, $m_\pi L=4.2$)

- Gounaris-Sakurai model

  Wittig (2016,2017), Mainz 1705.01775

  - By using time-like pion form factor, g-2 can be described in infinite volume.
  - 3% FV effect in L=4 fm, 0.19 GeV pion, $m_\pi L=4$

- Anisotropic study

  Lehner (2016)

  - Coordinate space integral along temporal or spatial direction.
  - Discrepancy is a_μ^{HLO} [spatial] – a_μ^{HLO} [temporal] ~ 3%.

- **Direct lattice study (PACS)**
  - Comparison between two volumes in physical pion at fixed *a*
  - L > 5 fm, $m_\pi L \gtrsim 3.8$
  - Compare the different boundary

PACS 96^4 and 64^4 at a=0.08 fm

PACS group recently generates two gauge ensembles:

- Nf=2+1 O(a) improved clover fermion + Stout smearing
- a=0.083 fm, and two lattice sizes 64^4 and 96^4
- (almost) physical pion,

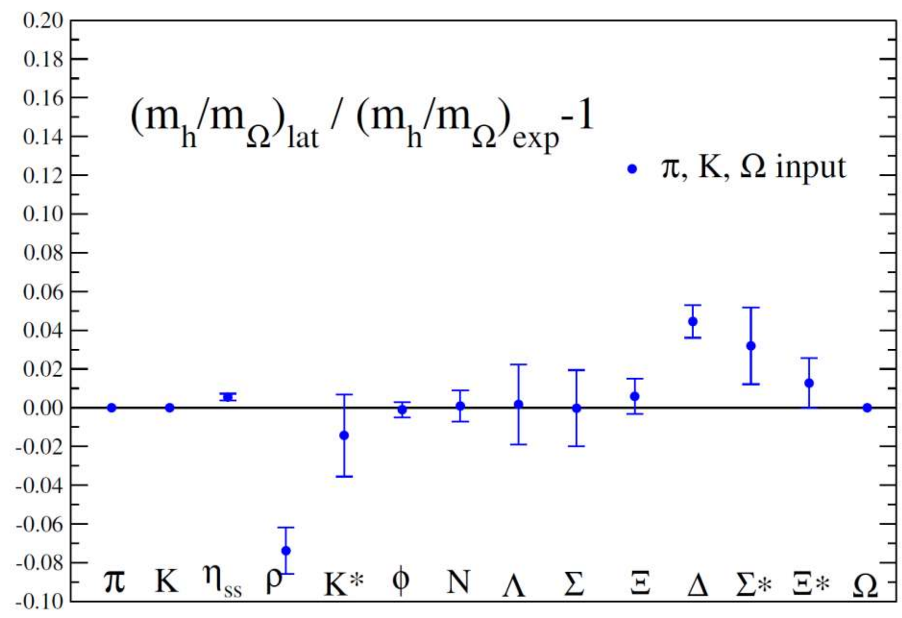
L=5.4 fm, 0.140 GeV ($m_\pi L$=3.8),

with K_{ud}=0.126117, K_s=0.124790

L=8.1 fm, 0.145 GeV ($m_\pi L$=6.0)

with K_{ud}=0.126117, K_s=0.124902

~5 MeV difference in pion mass



PACS, 1511.09222

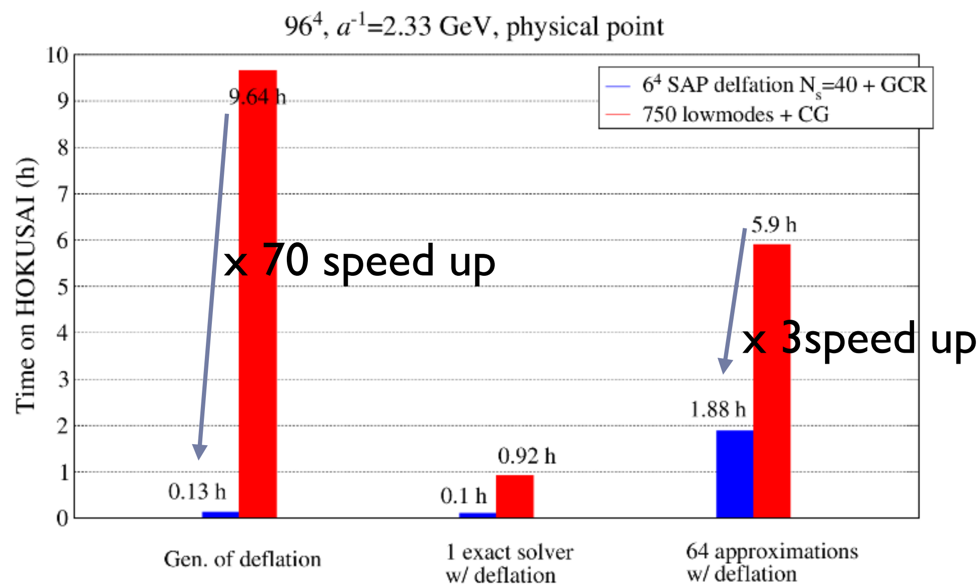
Computation with AMA

- Optimized AMA with SAP + deflation
  - Domain-decomposition, 6^4 domain size is chosen.
  - Deflation field, $N_s = 40$, 5 SAP cycles in single precision.
  - Deflated SAP + GCR for exact and approximation
    - Exact: ~30 GCR iteration (outer double precision loop)
    - Approximation: 5 fixed GCR iteration, $|r| \sim O(10^{-5})$

Blum et al., PRD88(2013), PRD91(2015), Mainz, NPB914 (2017)

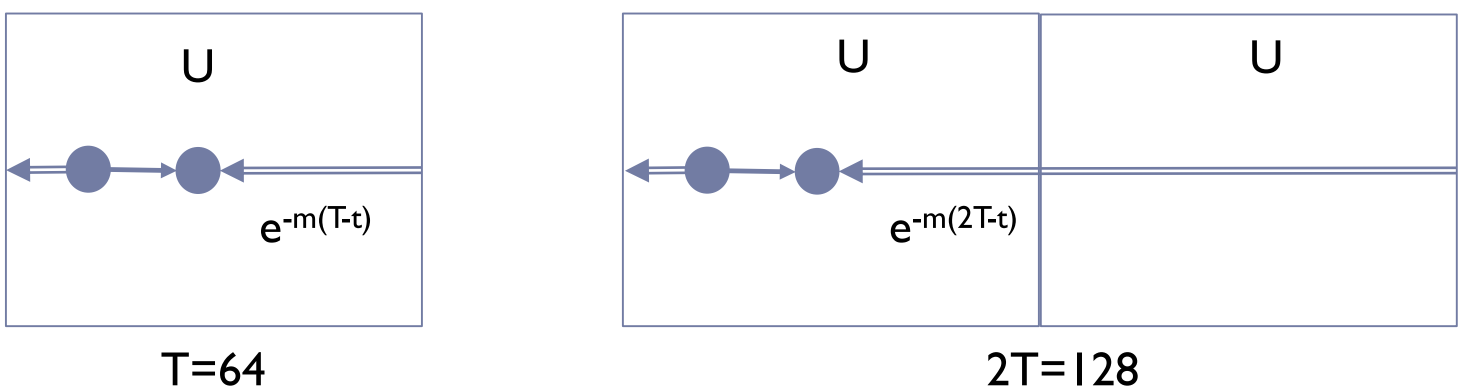
Luscher, JHEP07 (2008)

- ➢ Small cost for a generation of deflation field.
  ⇒ no need **huge** storage (or memory) to store eigenvector
- ➢ Totally 3x faster than lowmode deflated CG



Study of backward state propagation

- Extension of temporal length
  - To study backward state effect, we extend temporal length.
  - Using duplicated gauge configurations for 64^4 lattice
  - Suppress the backward state effectively (consistently using periodic anti-periodic fermion)
  - Important check of finite t effect in t integral

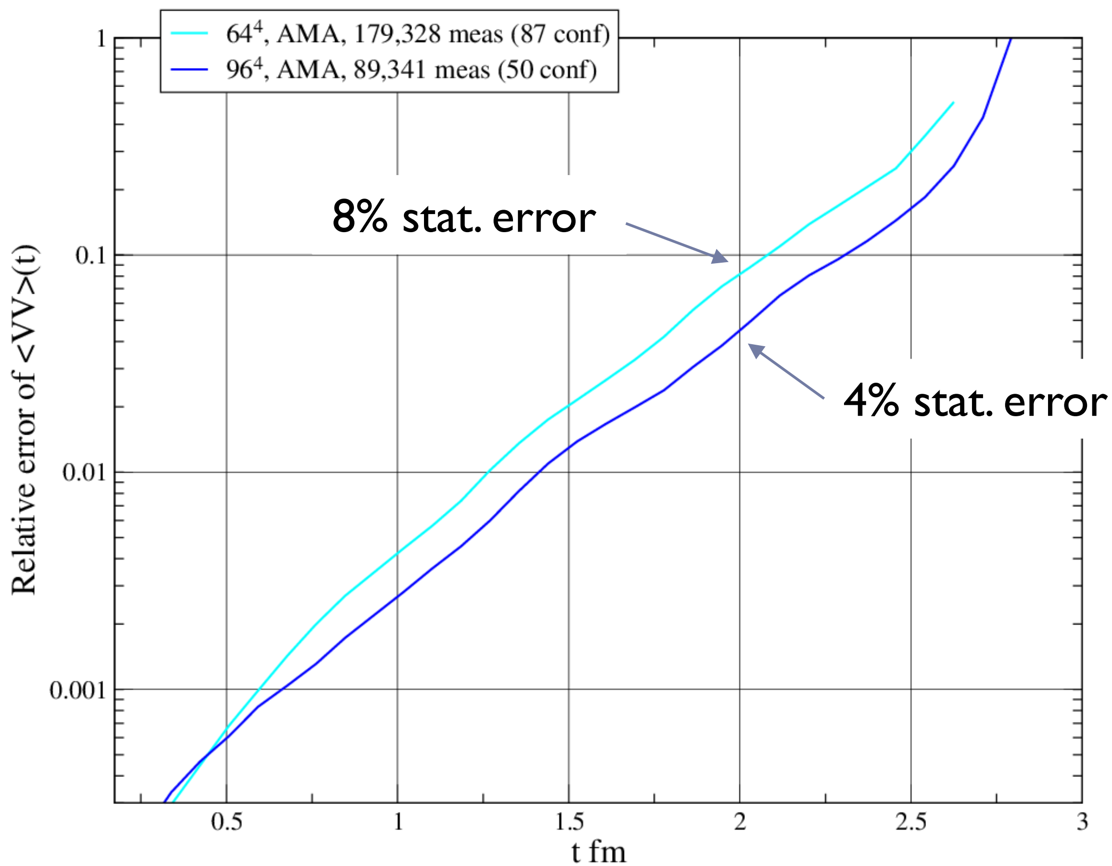


4. Preliminary result

High statistics in PACS configurations

96^4 : 50 configs., 89,341 meas (light), 3,382 meas (strange)

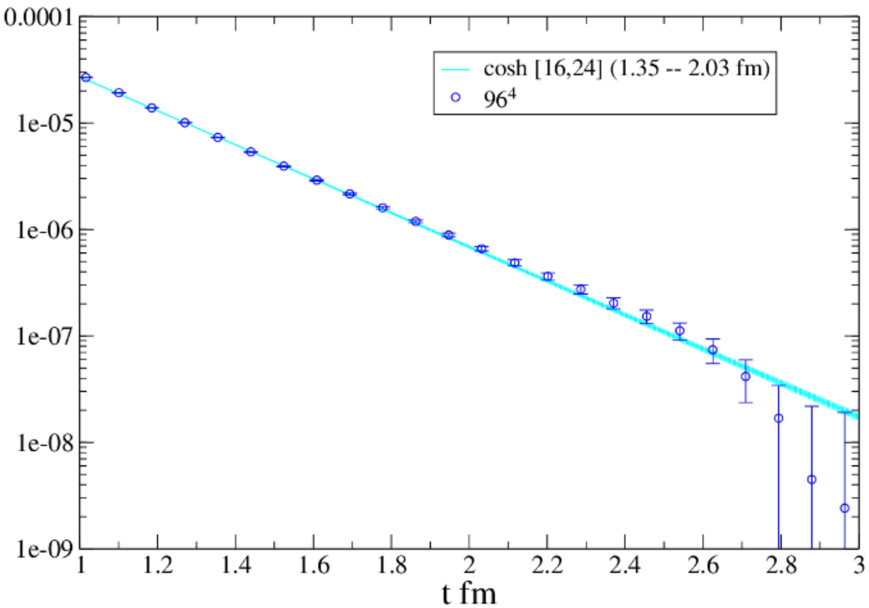
64^4: 87 configs., 179,328 meas (light), 6,247 meas (strange)



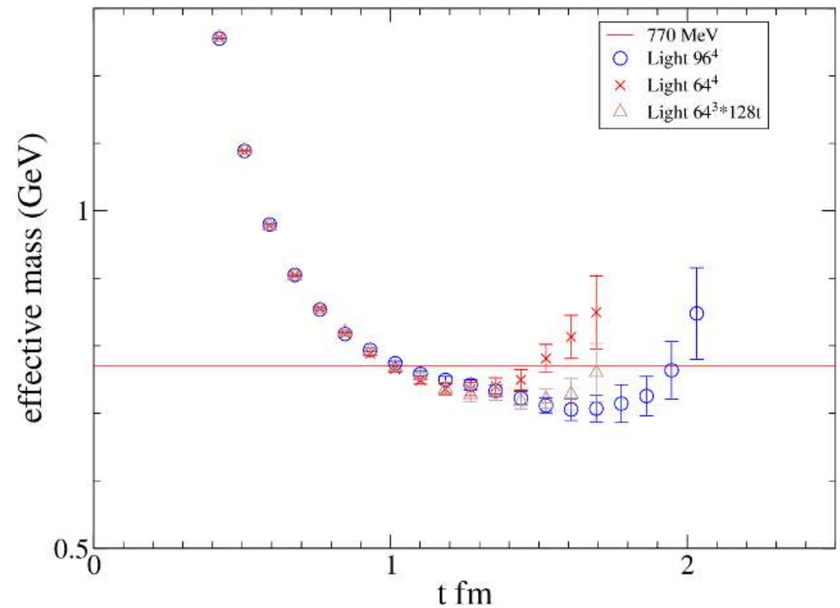
- Statistical error at large t is improved by large volume. $\sim 1/V^{1/2}$

4. Preliminary result
Vector-vector correlator

➢ Propagator



➢ Effective mass



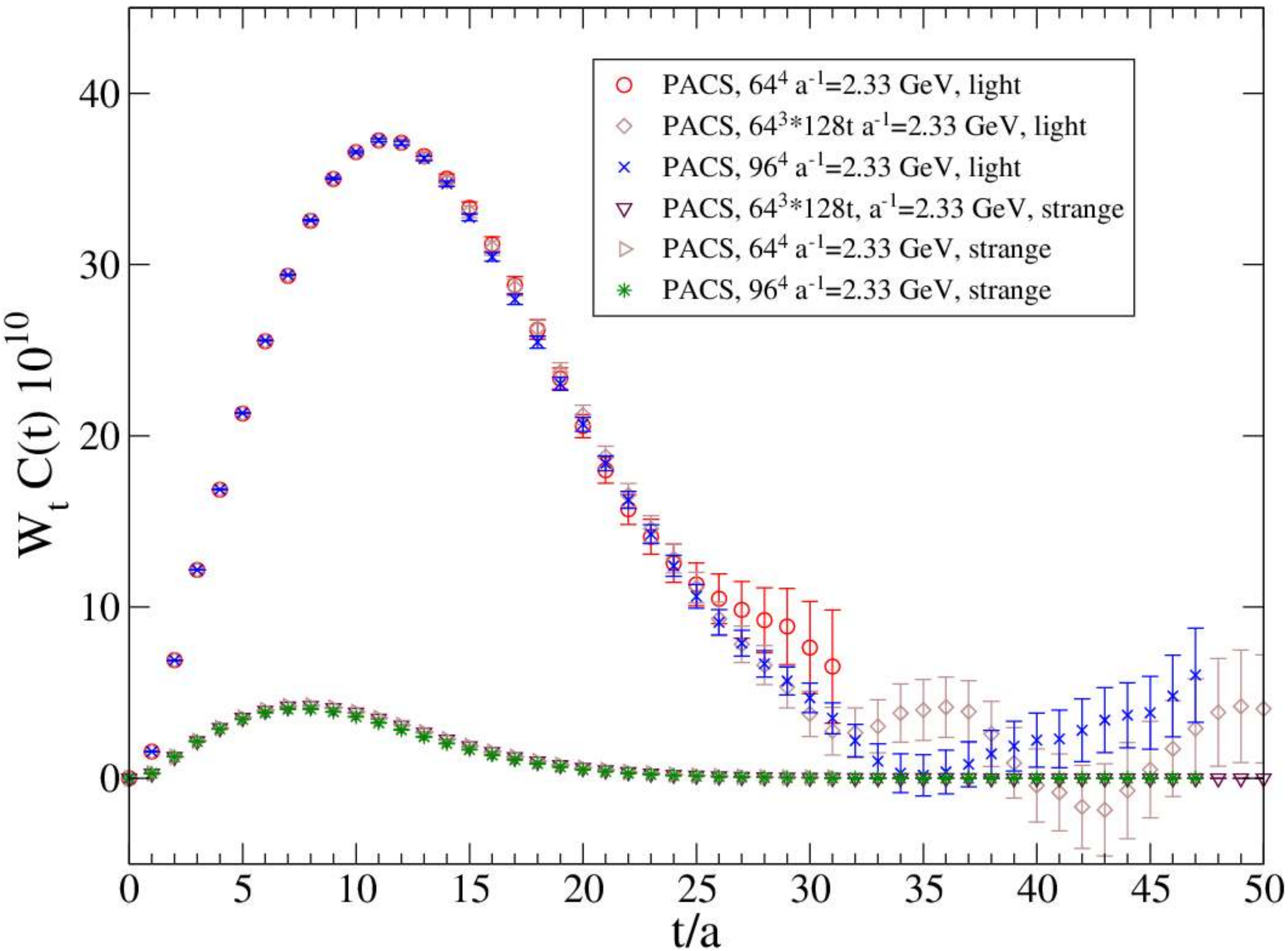
- Separating from single exponential function. Multi-hadron state is visible.
- Good signal up to 2.7 fm.

Plateau appears above 1.3 fm, and its mass is below rho mass.

⇒lattice data implies that Multi-hadron state contributes above 1.3 fm.

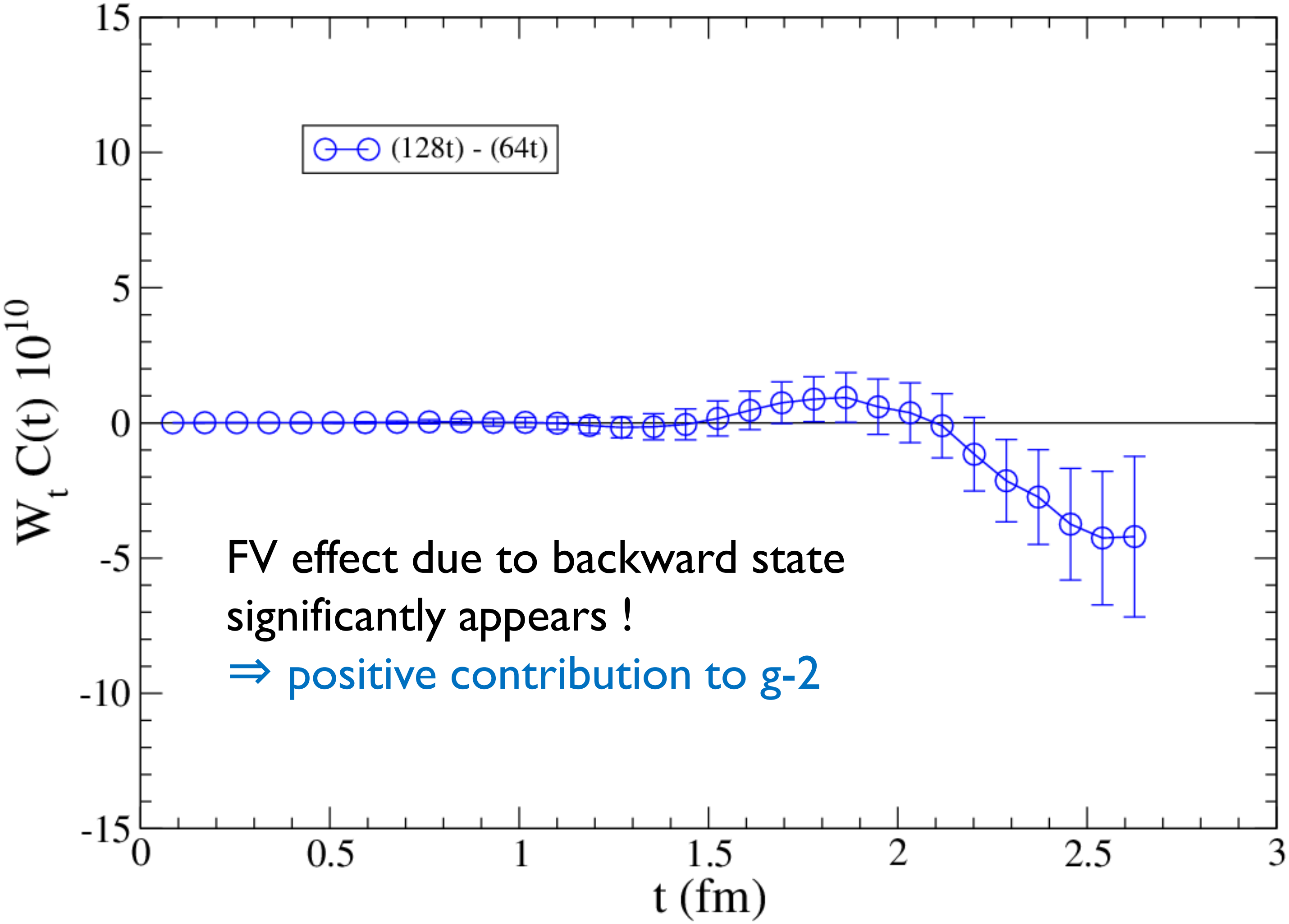
4. Preliminary result

Integrand along temporal direction

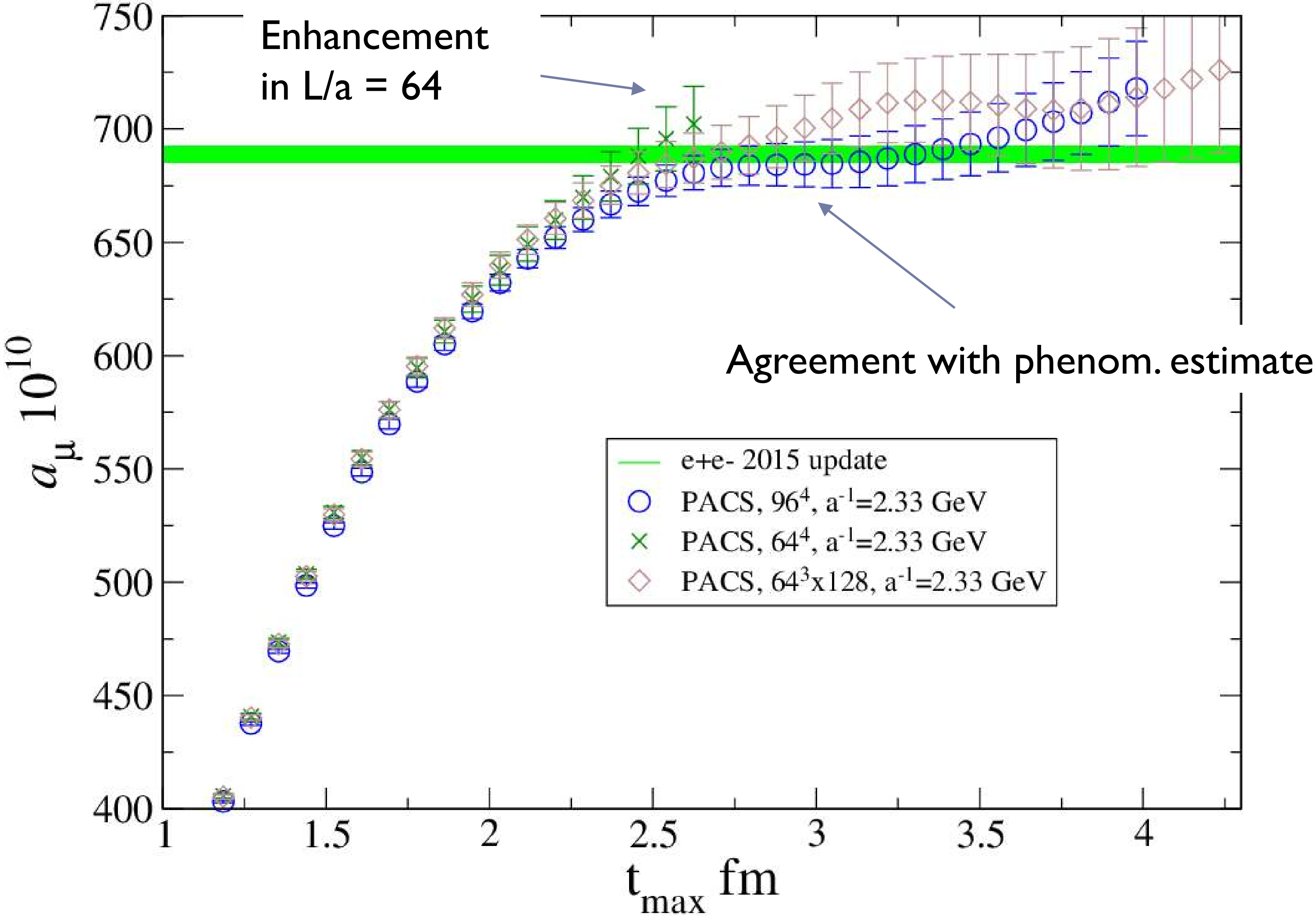


4. Preliminary result

Backward state contribution

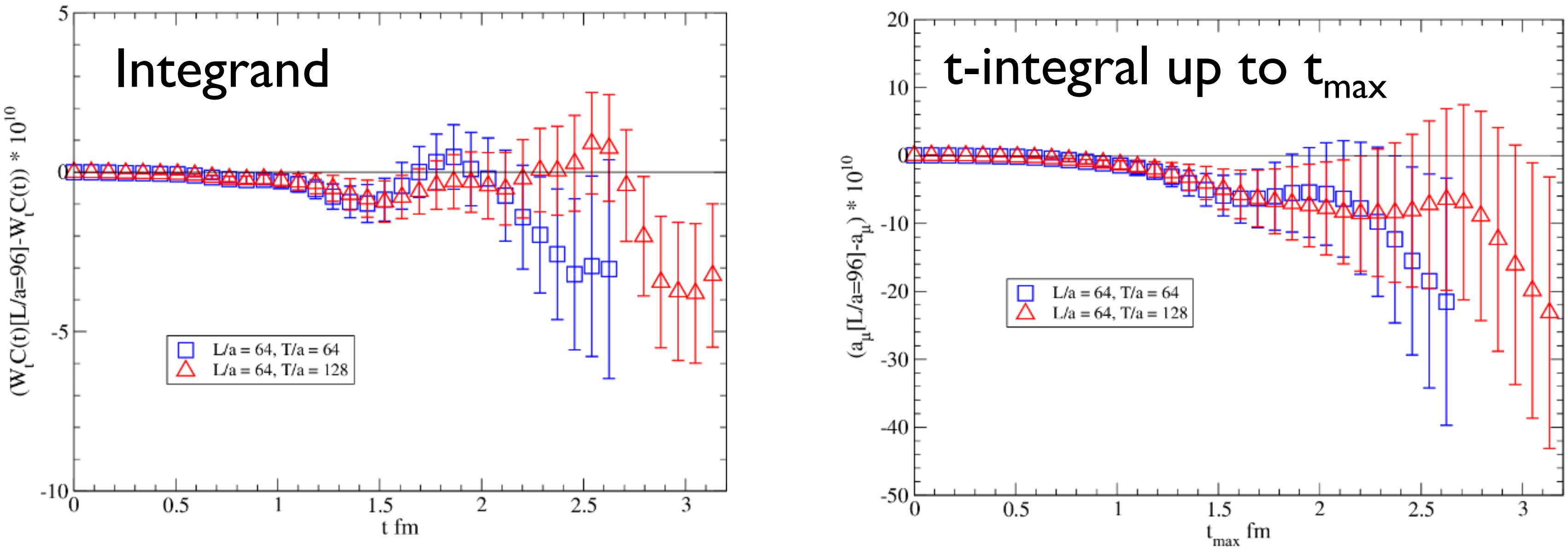


4. Preliminary result
t integral



4. Preliminary result

Finite volume effect

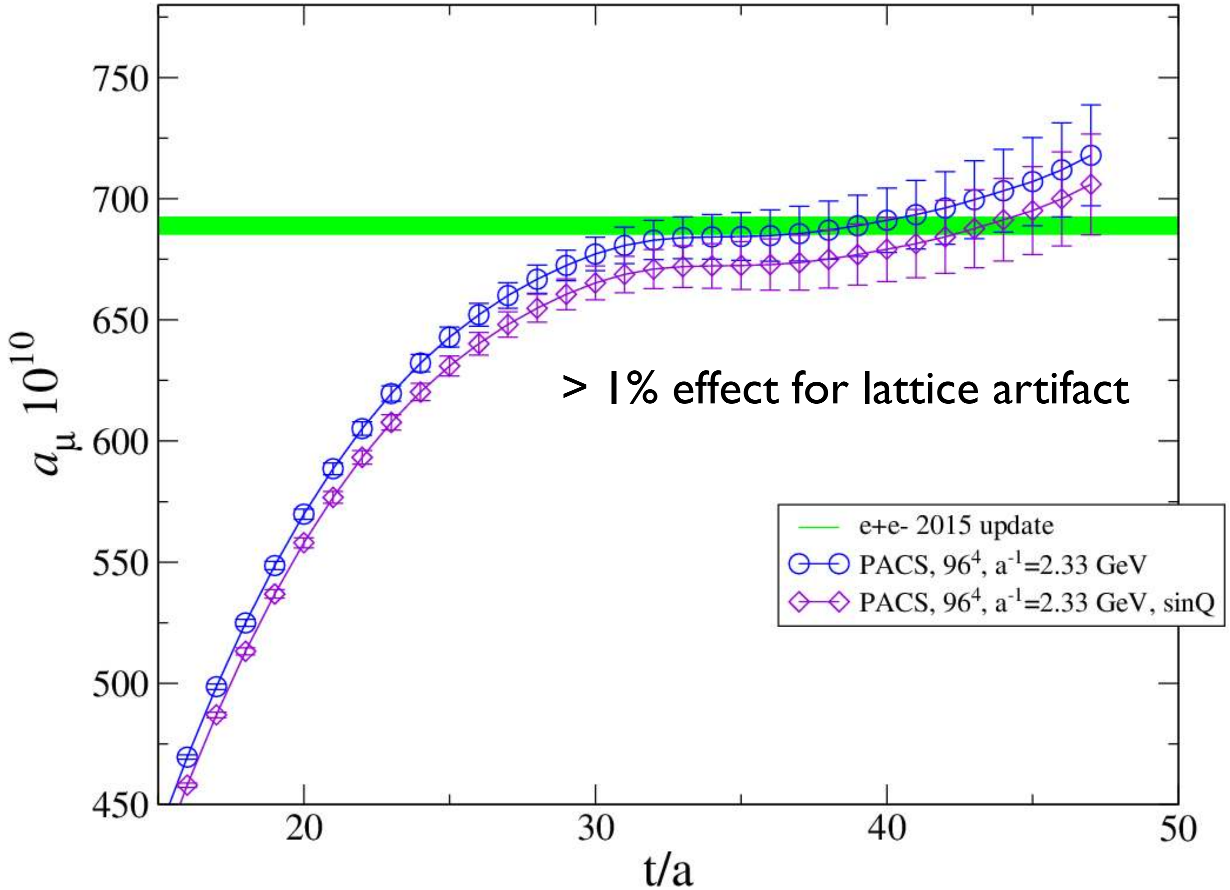


- Slightly negative for t_{max} > 1.3 fm → Δ_{FV}[(L/a=96)-(L/a=64)]~-10, opposite sign from expectation (ChPT etc) Aubin et al., PRD93(2016)

  However pion mass difference, m_π[(L/a=96)–(L/a=64)] = +5 MeV, due to slightly different K_s in two ensembles. For same m_π such a difference would have been reduced by $\Delta a_\mu = +3$ under assumption from ansatz in HPQCD(2016), Mainz (2017)
  ⇒ conservatively ~±2(2)% FV correction in L/a=64 lattice at finite t_{max} ~ 2.5 fm including mass correction.
- Discrepancy between 64^4 and 96^4 (t_{max} > 2.2 fm) → significant backward state effect.

4. Preliminary result

Lattice artifact



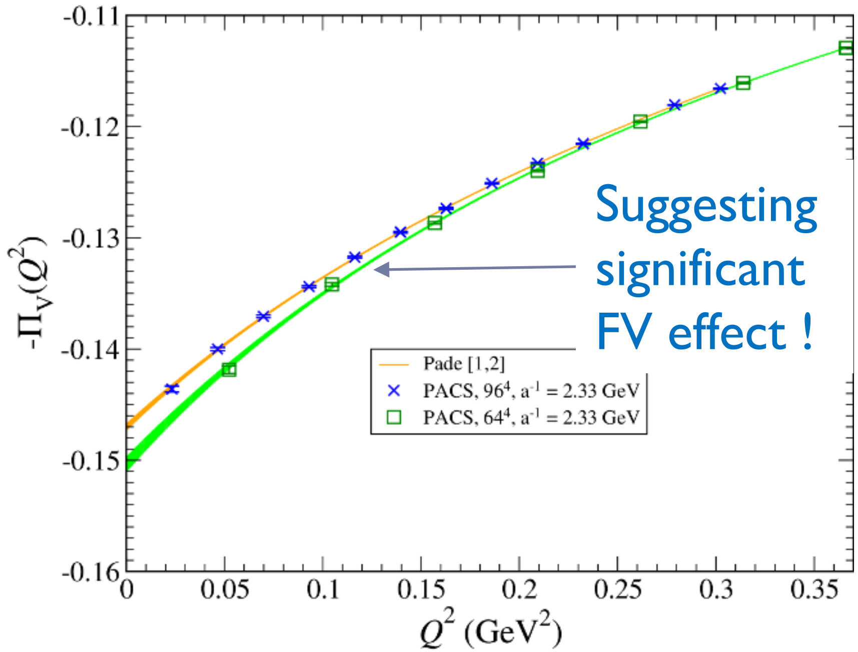
Diff. rep.: $\hat{K}(t) = \frac{2}{m_\mu t^3}\int_0^\infty d\omega \frac{2\hat{\omega}}{\omega^2} K_E(\omega^2)\left[\hat{\omega}^2 t^2 - 4\sin^2(\omega t/2)\right], \quad \hat{\omega} = 2\sin\omega/2$

4. Preliminary result

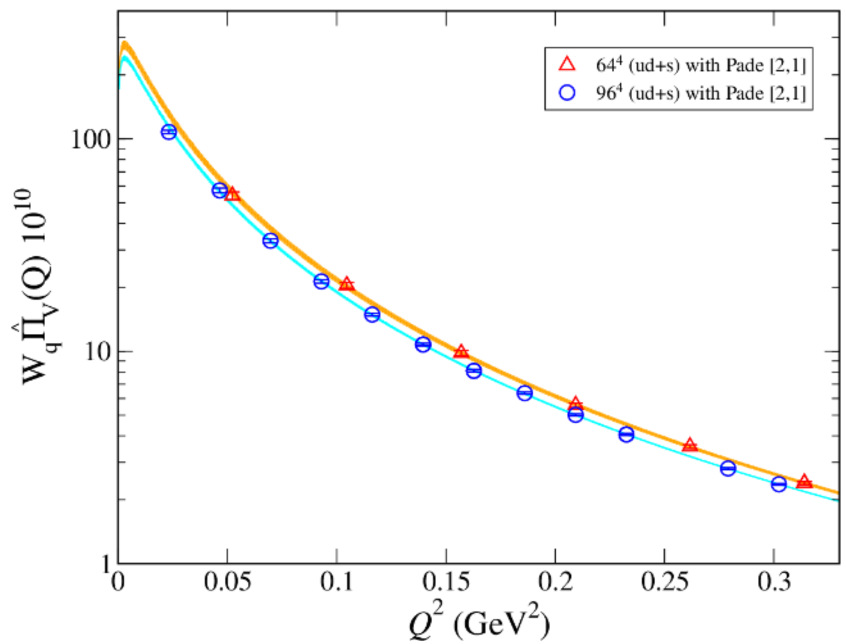
Momentum dependence

- Test of finite volume effect into a_μ with Q^2 integral of VPF

➢ Π(Q)



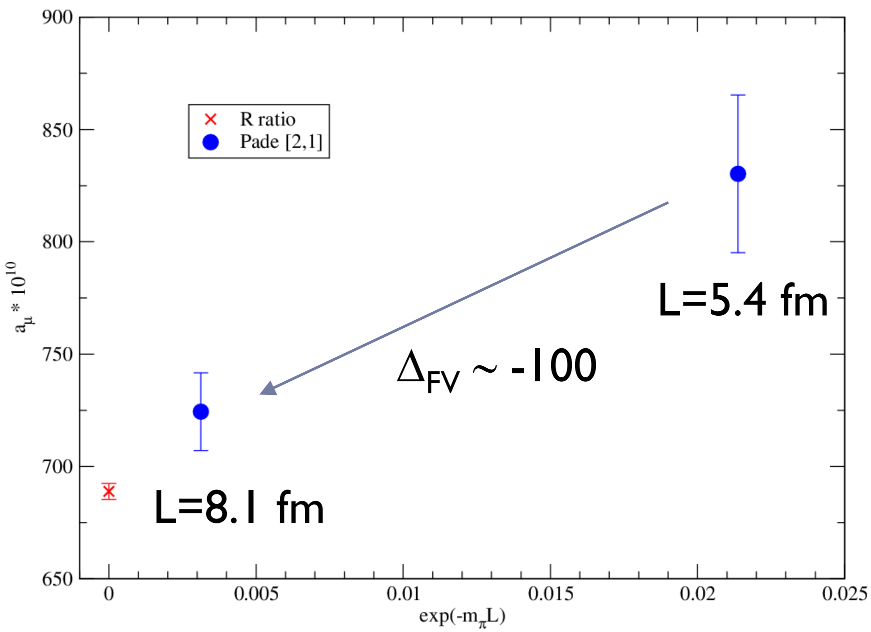
➢ Integrand



$$\Pi^{\text{Pade}}(Q) = \Pi(0) + Q^2\left(A_0\delta_{n,m+1} + \sum_{k=1}^{m} \frac{A_k}{Q^2+B_k}\right)$$

4. Preliminary result
Volume dependence

- Q integral

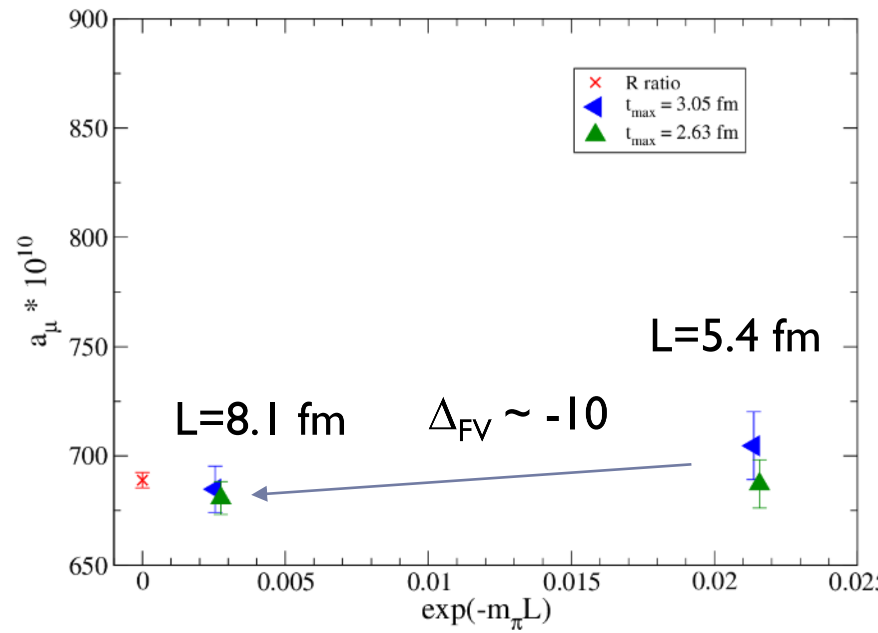


Integral Q^2 = 0 – 0.39 GeV^2

Cf. Mainz 1705.01775

Q_{cut}^2=0.5 GeV^2 (0.19 GeV pion, L=4 fm)
$a_\mu \times 10^{10}$ [ud] = 504(10), $a_\mu \times 10^{10}$ [s] = 47.5(4)

- t integral



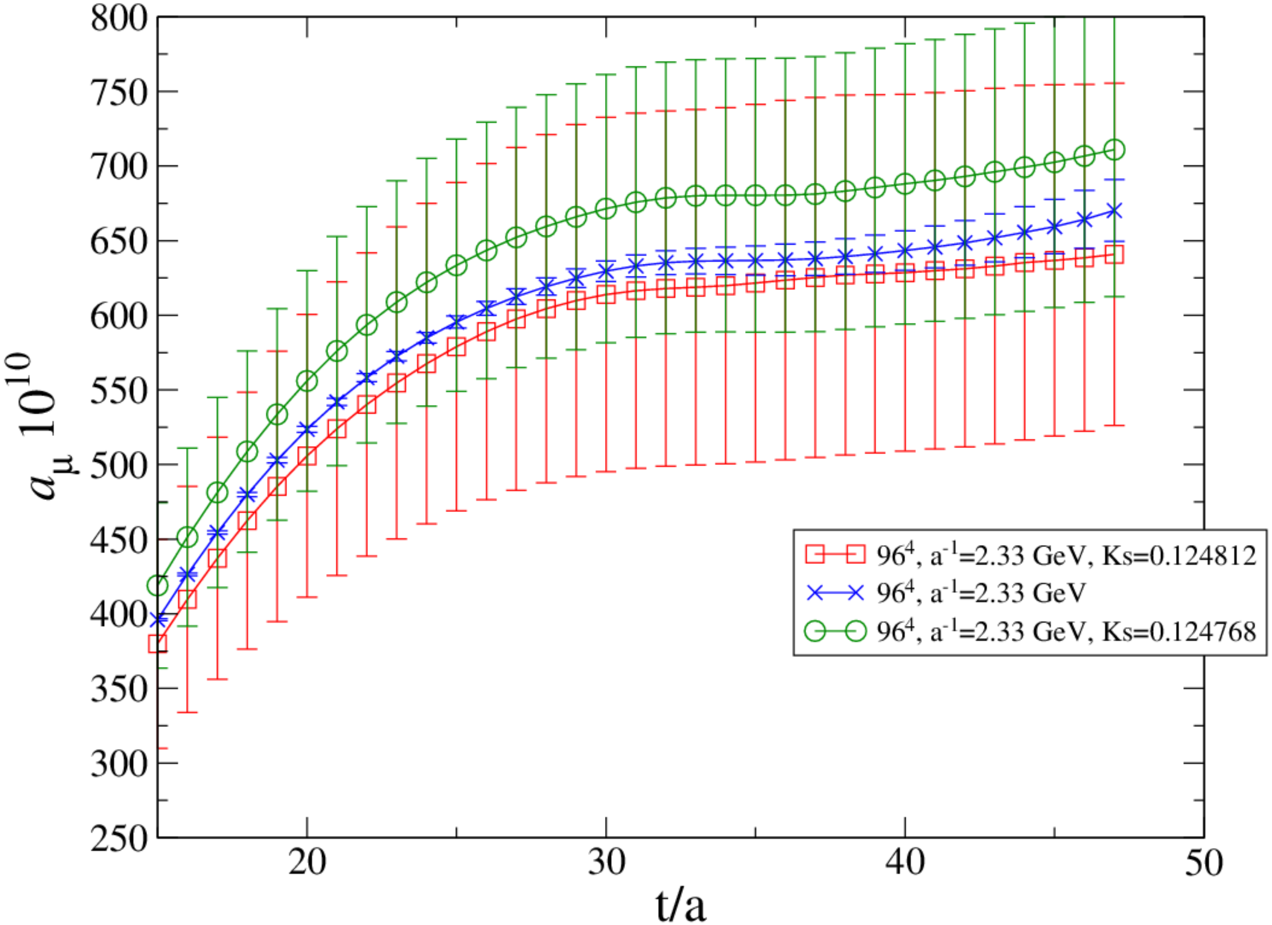
Setting t_{max} is finite, so there is missing effect from t_{max} to infinity.
Note pion mass difference, m_π[(L/a=96)–(L/a=64)] = +5 MeV, due to slightly different K_s in two ensembles.

Summary and future works

- Start HVP computation at physical pion in PACS
  - Direct lattice comparison for FV effect without models.
  - Analysis with both methods, Q-integral and t-integral
  - On 5 fm in physical pion, Q-integral has large FV effect, $\Delta_{FV}[a_\mu(L=8\text{fm})-a_\mu(L=5\text{fm})] < 0$, opposite sign from ChPT.
  - T-integral has small FV (~2%) on L=5fm, but missing effect at $t_{max}\rightarrow\infty$ ⇒ infinite volume limit is important.
  - Study on physical pion is very important to correctly estimate FV uncertainty.
- Future
  - One more large volume (~10 fm) and the infinite volume limit.
  - Continuum limit, Isospin breaking

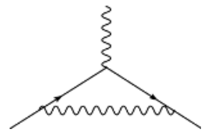
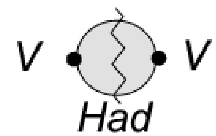
Mass correction

- Reweighting with sea strange kappa value



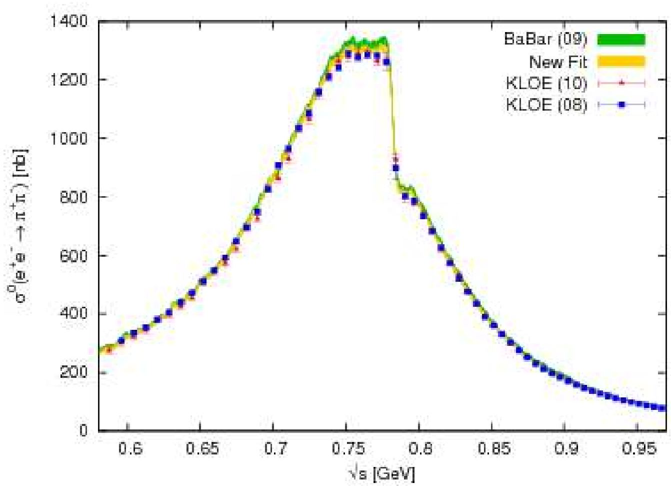
Leading order of hadronic contribution

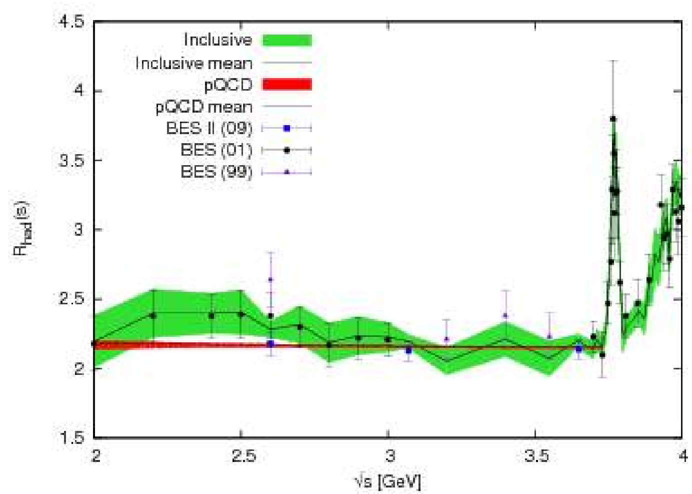
- Hadronic vacuum polarization (HVP)

$$a_\mu^{\mathrm{HLO}} = \int ds$$  × 

$$= \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \left[\int_{m_\pi^2}^{s_{\mathrm{cut}}} ds \frac{K(s)}{s} R_{\mathrm{had}}^{\mathrm{data}}(s) + \int_{s_{\mathrm{cut}}}^{\infty} ds \frac{K(s)}{s} R_{\mathrm{had}}^{\mathrm{pQCD}}(s)\right]$$

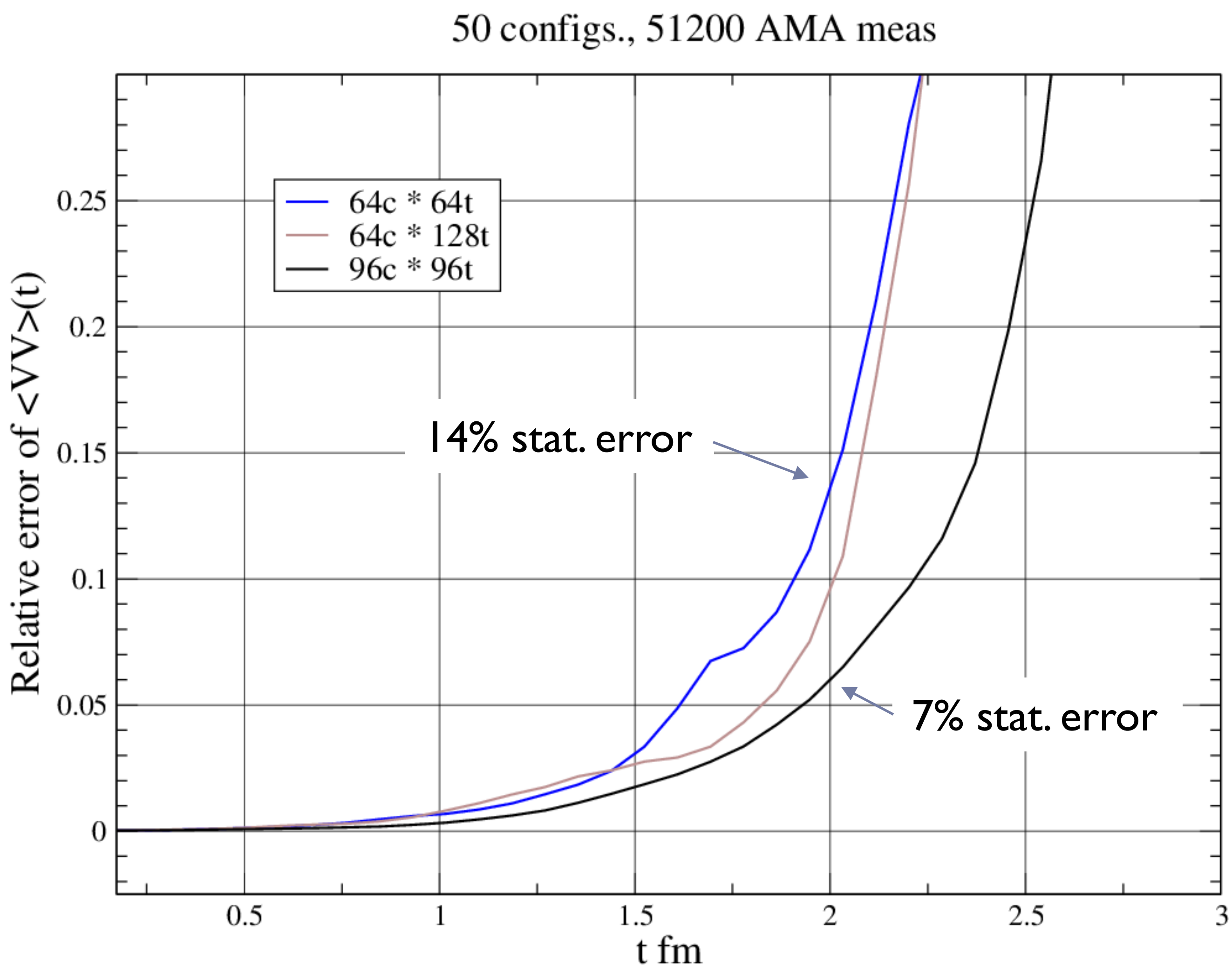
$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (s/m_\mu^2)(1-x)}$$





Hagiwara, et al., J.Phys. G38,085003 (2011)

Volume sum



50 configs., 51200 AMA meas
64c * 64t
64c * 128t
96c * 96t
14% stat. error
7% stat. error
Relative error of <VV>(t)
t fm
0
0.05
0.1
0.15
0.2
0.25
0.5
1
1.5
2
2.5
3

Domain decomposition

Luscher, Comp.Phys.Comm.156 (2004)

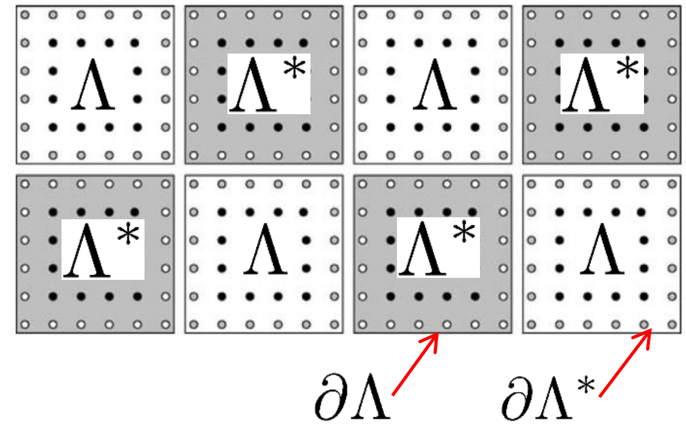
- Preconditioning for iterative solver

$$D_w = \begin{pmatrix} D_\Lambda & \partial D_\Lambda \\ \partial D_{\Lambda^*} & D_{\Lambda^*} \end{pmatrix} \qquad M_{\rm sap} = K \sum_{\nu=0}^{n_{\rm cy}-1} (1 - DK)^\nu$$

$$K = R_\Lambda^T D_\Lambda^{-1} R_\Lambda + R_{\Lambda^*}^T D_{\Lambda^*}^{-1} R_{\Lambda^*} - R_{\Lambda^*}^T D_{\Lambda^*}^{-1} D_{\partial\Lambda^*} D_\Lambda^{-1} R_\Lambda$$

$$R_\Lambda = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad R_{\Lambda^*} = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

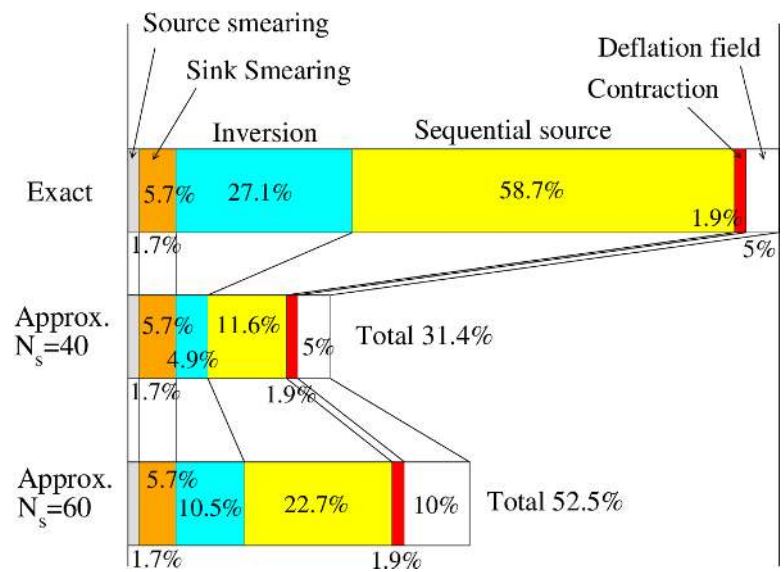
$$x = D^{-1} b \simeq M_{\rm sap} b: \ \text{preconditioner}$$



- Use in a generation of deflation field and projection
- Input parameters
  - Degree of SAP cycle: n_{cy}
  - SAP domain size: Λ_x , Λ_y , Λ_z , Λ_t
  - Precision of A^{-1} and D_Λ^{-1}
  - Number of deflation field: N_s

  Those can control the precision,

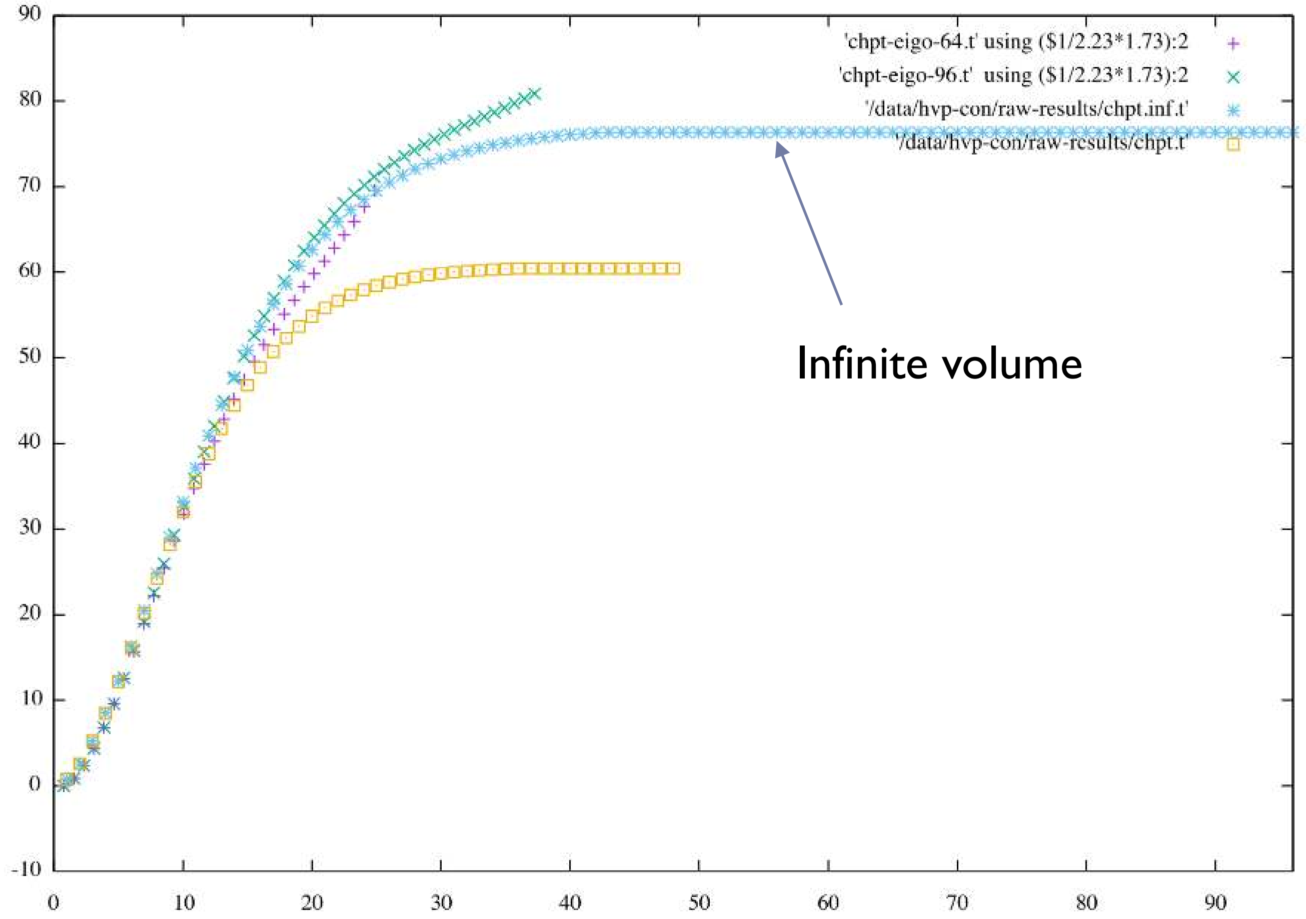
  e.g. small Λ and large n_{cy}



Mainz, NPB914 (2017)

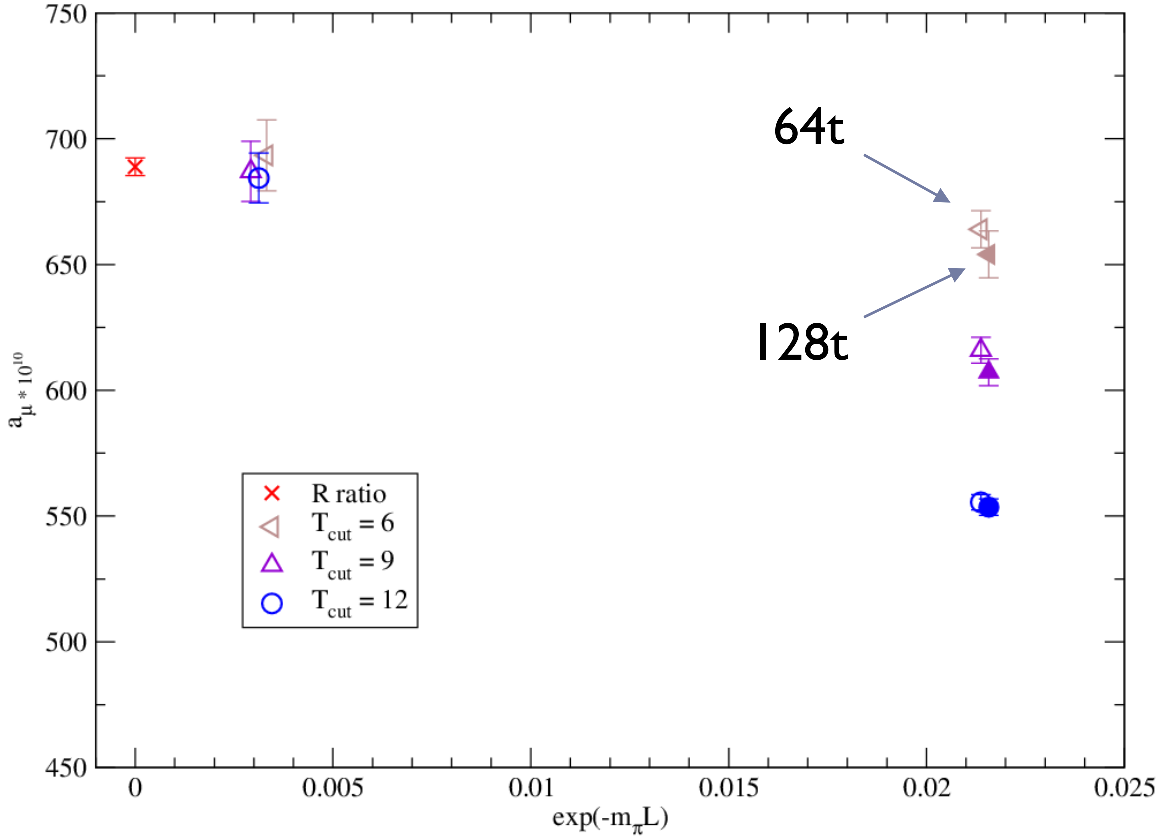
4. Preliminary result

Size effect in one-loop ChPT



4. Preliminary result

T_{cut} dependence



- $T_{cut} = T/2 - t_{max}$: tail in the integral
- The region of integral is changed depending on volume.