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Numerical investigation of QED finite-volume effects for meson mass and HVP

James Harrison

University of Southampton
RBC/UKQCD

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The RBC & UKQCD collaborations

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Luchang Jin
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
David Murphy
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang

University of Connecticut

Tom Blum
Dan Hoying
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Tobias Tsang
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

Peking University

Xu Feng

University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda

York University (Toronto)

Renwick Hudspith

Introduction

- We have made an exploratory calculation of isospin-breaking corrections to the HVP [V. Guelpers, Thursday, 17:30; arXiv:1706.05293].
- QED finite volume effects (FVE) must be taken into account for a physical calculation of the HVP.
- 2-loop analytical calculation of HVP FVE has not yet been carried out.
- We use lattice scalar QED, as a quicker numerical calculation of FVE and as a cross-check for the analytical result.
- Cross-checked method against known results for meson mass FVE.
- Preliminary results for FV behaviour of HVP.

Isospin breaking

- Isospin-breaking effects enter at $\mathcal{O}\left(\alpha, \frac{m_u - m_d}{\Lambda_{QCD}}\right) \sim 1\%$

- Precision in QCD now approaching 1% for several quantities.
  - e.g. hadronic contribution to muon $g-2$: 1% precision or better required to compete with determination from $e^+e^- \to$ hadrons, and to prepare for upcoming experiments at Fermilab and J-PARC.

- Isospin is significant systematic uncertainty

- Electromagnetism and light quark mass difference need to be included.

QED in finite volume

- For sufficiently large volume, QCD FVE are exponentially suppressed.
- QED is long-range due to massless photon, so QED FVE are much larger than for QCD.
- Typically scale with inverse powers of L, not exponential.
- QED FVE can be comparable in magnitude to QED corrections [Borsanyi et al., arXiv:1406.4088].
- FVE must be accounted for in any lattice calculation involving QED.

QED finite volume effects

- FVE are not very sensitive to higher modes (discretisation, hadron structure).
- Can study these analytically using an effective theory, with point-like hadrons
  - Scalar QED (pseudoscalar mesons)
  - Spinor QED (spin-1/2 baryons)
- In momentum space, integrals become discrete sums in finite volume.
- Derive relations between finite-volume quantites and their infinite-volume counterparts.
- FVE have been calculated for hadron masses [Davoudi & Savage, arXiv:1402.6741; Borsanyi et al., arXiv:1406.4088] and leptonic decay amplitudes [Lubicz et al., arXiv:1611.08497].

QED finite volume effects

Alternative: lattice scalar QED.

- Generate $U(1)$ gauge configurations
- Calculate scalar propagators and expectation values
- Repeat for several volumes
- Fit polynomial in $1/L$ to extract coefficients

Offers a quick numerical method for obtaining FVE, which is generally applicable to a wide range of observables.

Scalar QED on the lattice

Discretised scalar QED action:

$$
\begin{aligned}
S\left(\phi, A_\mu\right) &= \frac{a^4}{2}\sum_x \phi^*(x)\,\Delta\phi(x) + S_\gamma\left(A_\mu\right) \\
\Delta &= -\sum_\mu D^*_\mu D_\mu + m^2 \\
D_\mu f(x) &= a^{-1}\left[e^{ieaA_\mu(x)} f(x + a\hat{\mu}) - f(x)\right]
\end{aligned}
$$

Quenched theory: set scalar determinant = 1 in path integral.

Sampling gauge field configurations

Non-compact photon action:

$$
\begin{aligned}
S_\gamma\left(A_\mu\right) &= \frac{a^4}{4}\sum_x\sum_{\mu,\nu}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \\
\partial_\mu f\left(x\right) &= a^{-1}\left[f\left(x+a\hat{\mu}\right) - f\left(x\right)\right]
\end{aligned}
$$

- Feynman gauge.
- In momentum space, $\tilde{A}_\mu\left(k\right)$ is Gaussian - cheap to sample gauge configurations [Duncan, Eichten & Thacker, arXiv:hep-lat/9602005].
- Subtract zero mode using QED_L scheme [Uno & Hayakawa, arXiv:0804.2044].

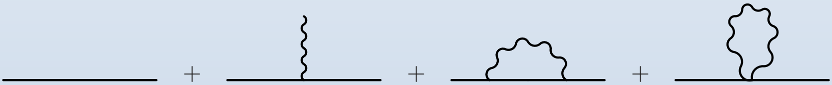
Calculating the scalar propagator

Alternative to CG, making use of FFT.

Expand scalar propagator around $e = 0$:

$$\Delta = -\sum_{\mu} D_\mu^* D_\mu + m^2 = \Delta_0 + e\Delta_1 + e^2\Delta_2 + \mathcal{O}\left(e^3\right)$$

$$\Delta^{-1} = \Delta_0^{-1} - e\Delta_0^{-1}\Delta_1\Delta_0^{-1} + e^2\left[\Delta_0^{-1}\Delta_1\Delta_0^{-1}\Delta_1\Delta_0^{-1} - \Delta_0^{-1}\Delta_2\Delta_0^{-1}\right] + \mathcal{O}\left(e^3\right)$$



Calculating the scalar propagator

$$\Delta_0^{-1} = \mathcal{F}^{-1}\frac{1}{\hat{k}^2+m^2}\mathcal{F}$$

$$\Delta_0^{-1}\Delta_1\Delta_0^{-1} = -ia^{-1}\sum_\mu \mathcal{F}^{-1}\left[\frac{1}{\hat{k}^2+m^2}\mathcal{F}A_\mu\mathcal{F}^{-1}\frac{e^{iak_\mu}}{\hat{k}^2+m^2} - \frac{e^{-iak_\mu}}{\hat{k}^2+m^2}\mathcal{F}A_\mu\mathcal{F}^{-1}\frac{1}{\hat{k}^2+m^2}\right]\mathcal{F}$$

$$\Delta_0^{-1}\Delta_2\Delta_0^{-1} = \frac{1}{2}\sum_\mu \mathcal{F}^{-1}\left[\frac{1}{\hat{k}^2+m^2}\mathcal{F}A_\mu^2\mathcal{F}^{-1}\frac{e^{iak_\mu}}{\hat{k}^2+m^2} - \frac{e^{-iak_\mu}}{\hat{k}^2+m^2}\mathcal{F}A_\mu^2\mathcal{F}^{-1}\frac{1}{\hat{k}^2+m^2}\right]\mathcal{F}$$

where $\mathcal{F}$ represents the Fourier transform.

Finite volume corrections - meson mass

Scalar mass FV corrections are known:

$$m^2(L) \sim m_\infty^2 - \alpha \frac{\kappa}{L}\left(m_0 + \frac{2}{L}\right)$$

$$\kappa = 2.837297(1)$$

in QED_L, up to terms exponentially suppressed in $m_0 L$ [Borsanyi et al., arXiv:1406.4088]. m_0 is bare mass, m_∞ is infinite-volume mass.

We can use this as a validity check of our method.

Finite volume corrections - meson mass

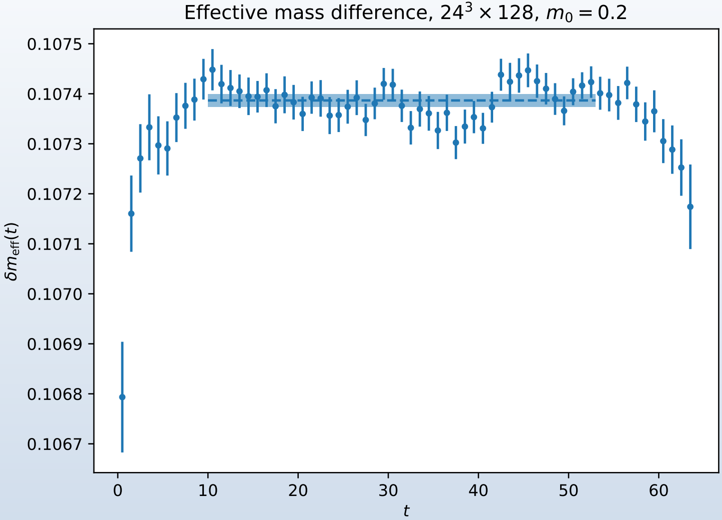
We define an effective mass difference from ratio of charged and free propagators (neglect backward propagating states for simplicity of presentation):

$$\delta m_{\text{eff}}(t) = \frac{\Delta^{-1}\left(t,\vec{0}\right)}{\Delta_0^{-1}\left(t,\vec{0}\right)} - \frac{\Delta^{-1}\left(t+1,\vec{0}\right)}{\Delta_0^{-1}\left(t+1,\vec{0}\right)}$$

Can compare to perturbation theory:

$$\frac{\Delta^{-1}\left(t,\vec{0}\right)}{\Delta_0^{-1}\left(t,\vec{0}\right)} = c + te^2\left(\frac{T+\Sigma}{2m_0}\right)$$

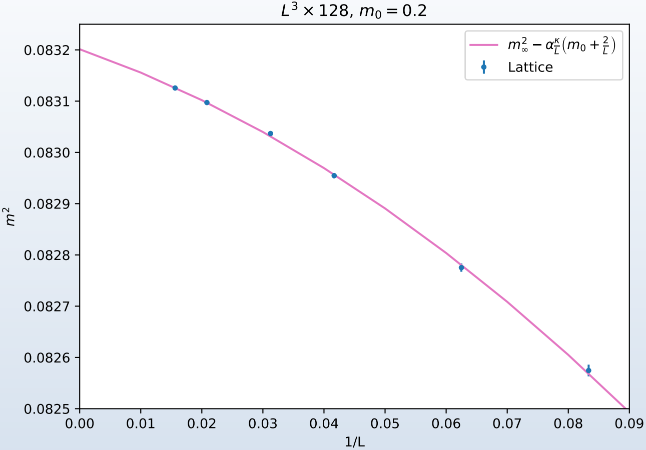
Finite volume corrections - meson mass



Finite volume corrections - meson mass

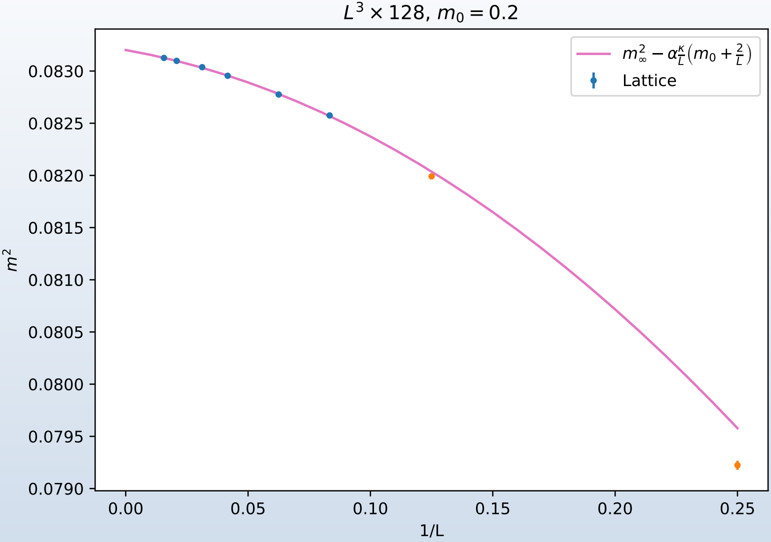
- Eight volumes, from $L = 4$ to $L = 64$
- $T = 128$ for all L
- 100 configurations per volume
- Computed on a single KNL
- Largest volume took $\sim$ 1 day

Finite volume corrections - meson mass



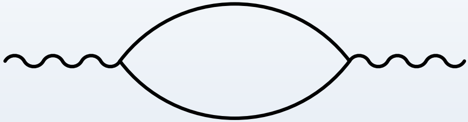
Good agreement between lattice and analytical:
χ^2/d.o.f. = 0.44 (note: this is not a fit).

Finite volume corrections - meson mass



Finite volume corrections - HVP

Leading effective field theory contribution to HVP is scalar bubble diagram.



Vacuum polarisation tensor:

$$C_{\mu\nu}(x) = \left\langle V_\mu(x) V_\nu(0) \right\rangle$$

$$\Pi_{\mu\nu}(Q) = a^4 \sum_x e^{-iQ\cdot x} C_{\mu\nu}(x) - a^4 \sum_x C_{\mu\nu}(x)$$

$$\Pi_{\mu\nu}(Q) = \left(\delta_{\mu\nu}\hat{Q}^2 - \hat{Q}_\mu \hat{Q}_\nu\right) \Pi\left(\hat{Q}^2\right)$$

Finite volume corrections - HVP

Conserved vector current:

$$V_\mu(x) = a^2 \left[\phi^*(x)\, e^{ieaA_\mu(x)} \phi(x + a\hat{\mu}) - \phi^*(x + a\hat{\mu})\, e^{-ieaA_\mu(x)} \phi(x)\right]$$

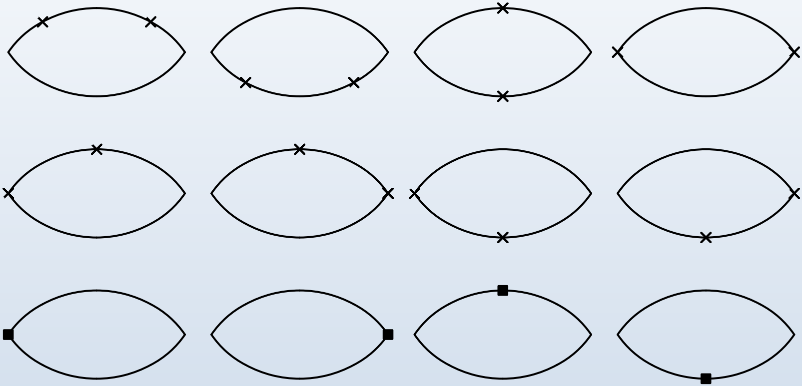
Contact term at $x = 0$ due to non-local currents:

$$T_\mu(x) = \phi^*(x)\, e^{ieaA_\mu(x)} \phi(x + a\hat{\mu}) + \phi^*(x + a\hat{\mu})\, e^{-ieaA_\mu(x)} \phi(x)$$

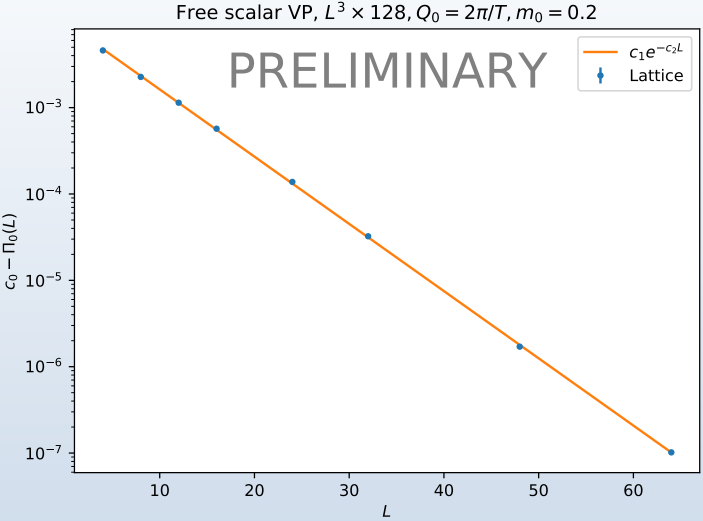
Contact term is removed by subtracting zero-mode $\tilde{\Pi}_{\mu\nu}(0)$.

Finite volume corrections - HVP

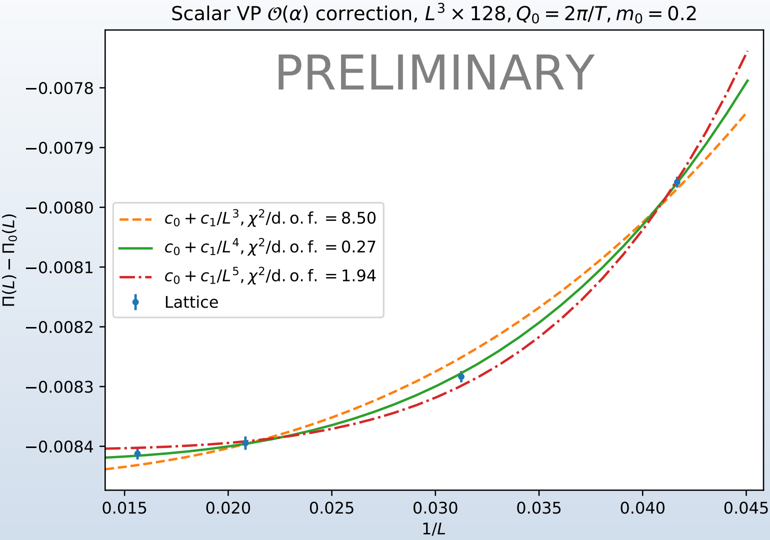
$\mathcal{O}(\alpha)$ contributions to scalar VP:



Finite volume corrections - HVP



Finite volume corrections - HVP



Summary

- We want to calculate QED FVE for the HVP.
- As a first step before analytical calculation, we use lattice scalar QED to calculate FVE
- Simulations are very cheap ($\sim$ 1 day on a single KNL)
- We successfully reproduce known results for meson mass FVE
- Preliminary data suggest leading QED FV behaviour of HVP may be $\mathcal{O}\left(1/L^4\right)$
- This technique is more generally applicable to a wide range of quantities

Acknowledgements

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