Instanton dominance over α_s at low momenta from lattice QCD simulations at $N_f = 0$, $N_f = 2+1$ and $N_f = 2+1+1$.

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Overview

Motivation

Classical solutions of $\mathcal{L}_{\rm QCD}$

$$S = \frac{1}{2}\int d^4x \; G_{\mu\nu}^2 \geqslant 0 \; ; \qquad \frac{\delta S}{\delta G_\nu} = D_\mu G_{\mu\nu} = 0 \qquad \rightarrow \qquad A_I$$

- Chiral anomaly

$$\partial_\mu J_{5,\mu}(x) = \frac{g^2 N_F}{8\pi^2} Tr(G_{\mu\nu}\tilde{G}_{\mu\nu})$$

- Dirac operator zero modes

$$n_R(\psi_0); \, n_L(\gamma_5\psi_0)$$

- Topological charge

$$Q = n_L - n_R$$

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- Topological charge

$$Q = n_L - n_R$$

- Genuinely non-perturbative
- QCD vacuum
- SBχS?
- Deep inelastic scattering?
- Mass generation?
- Confinement?
- Gluon Green functions?

[Rev. Mod. Phys. 70, (1998) 323]

Instantons

Instanton gauge field

Only exact solution [PLB59 (1975) 85]:

$$A_I \equiv A^a_\mu(x) = 2\bar{\eta}^a_{\mu\nu}\frac{x_\nu}{x^2}\phi\left(\frac{x}{\rho}\right)$$

with

$$\phi_{BPST}(z) = \frac{1}{1+z^2}$$

- Multi-instanton solutions are constructed by the sum of those solutions.

$$A = \sum A_I$$

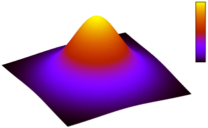
- The profile ϕ is modified at large distances due to instanton *interactions* [NPB245 (1984) 259].

Finite action

$$S_I = \int d^4x S(x) = \frac{8\pi^2}{g^2}$$

topological

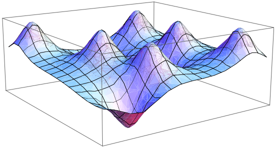
$$Q = \int \partial_\mu K_\mu = \pm 1$$



Gluon Green functions in an instanton background

- Independent pseudo-particle sum ansantz:

$$A_\mu^a(x) = \sum_i 2R_i^{a\alpha}\bar{\eta}^a_{\mu\nu}\frac{x_\nu - z_\nu^i}{(x-z^i)^2}\phi\left(\frac{x-z^i}{\rho_i}\right)$$



in $\langle A^2\rangle_{min}$ Landau gauge.

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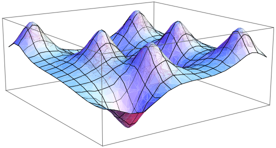

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II Fourier transform

$$\mathbf{A}^{\mathbf{a}}_\mu(\mathbf{x})[\phi(z)] \rightarrow \widetilde{\mathbf{A}}^{\mathbf{a}}_\mu(\mathbf{k})[I(k\rho)] \; ; \qquad I(s) = \frac{8\pi^2}{s} \int_0^\infty dz \; z \; J_2(sz) \; \phi(z)$$

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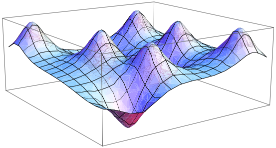

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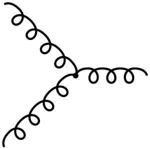
III Neglecting instanton correlations [PRD70, (2004) 114503]

$$\mathbf{G}^{(\mathbf{m})}(\mathbf{k}^2) \sim \mathbf{n}\;\mathbf{k}^{2-\mathbf{m}}\langle\rho^{\mathbf{3m}}\mathbf{I}^{\mathbf{m}}(\mathbf{k}\rho)\rangle$$

with $\langle\cdots\rangle$ the average over instanton sizes.

Computing $\alpha_{\rm MOM}$ from the lattice



- I Gluon propagator

$$\langle \widetilde{A}^a_\mu(k)\widetilde{A}^a_\mu(-k)\rangle \to G^{(2)}(k^2)$$

- II Three gluon vertex

$$\langle \widetilde{A}^a_\mu(k_1)\widetilde{A}^b_\nu(k_2)\widetilde{A}^b_\rho(k_3)\rangle_{k_1^2=k_2^2=k_3^2} \to G^{(3)}(k^2)$$

- III The coupling is defined by the ratio:

c.f. J. Rodríguez-Quintero's talk

$$\alpha_{\rm MOM}(k^2) = \frac{k^6}{4\pi}\frac{(G^{(3)}(k^2))^2}{(G^{(2)}(k^2))^3}$$

Gluon Green functions in an instanton background II

The ratio of Green functions defining $\alpha_{\mathrm{MOM}}(k)$ is then:

$$\alpha_{\mathrm{MOM}}(k^2) = \frac{k^6}{4\pi} \frac{\left(G^{(3)}(k^2)\right)^2}{\left(G^{(2)}(k^2)\right)^3}$$

Gluon Green functions in an instanton background II

The ratio of Green functions defining $\alpha_{\rm MOM}(k)$ is then:

$$\alpha_{\rm MOM}(k^2) = \frac{k^6}{4\pi} \frac{(G^{(3)}(k^2))^2}{(G^{(2)}(k^2))^3} = \frac{k^4}{18\pi n} \frac{\langle \rho^9 I^3(k\rho) \rangle^2}{\langle \rho^6 I^2(k\rho) \rangle^3} = \frac{k^4}{18\pi n} \times \begin{cases} 1 + 48 \frac{\delta\rho^2}{\langle\rho\rangle^2} & k\rho \ll 1 \\ 1 & k\rho \gg 1 \end{cases}$$

Instanton detector

In an instanton background $\alpha_{\rm MOM}(k)$ should exhibit a $\sim k^4$ behavior sensitive to the instanton density n [JHEP 04 (2003) 005].

- This result is independent of the instanton profile ϕ / I
- For small k depends on the width of the instanton distribution, $\delta\rho/\rho$

Lattice set-up

Large-volume $N_f = 0$ configurations:

- Tree-level Symanzik

β	a	V	confs.
3.8	0.285fm	$(13.7\text{fm})^4$	1050
3.9	0.243fm	$(15.6\text{fm})^4$	2000
4.2	0.141fm	$(4.5\text{fm})^4$	420

- Wilson

5.8	0.140fm	$(6.72\text{fm})^4$	960
5.6	0.235fm	$(11.3\text{fm})^4$	1920
5.6	0.235fm	$(12.3\text{fm})^4$	1790

- Iwasaki

2.37	0.140fm	$2.8^3 \times 5.6\text{fm}^4$	200

Details for the lattice setup in: [**PLB 760 (2016) 354**]

- Twisted mass $N_F = 2+1+1$ configurations ($m_\pi \sim 297\text{MeV}$)

β	a	V	confs.
1.95	0.083fm	$4.0^3 \times 7.9\text{fm}^4$	200

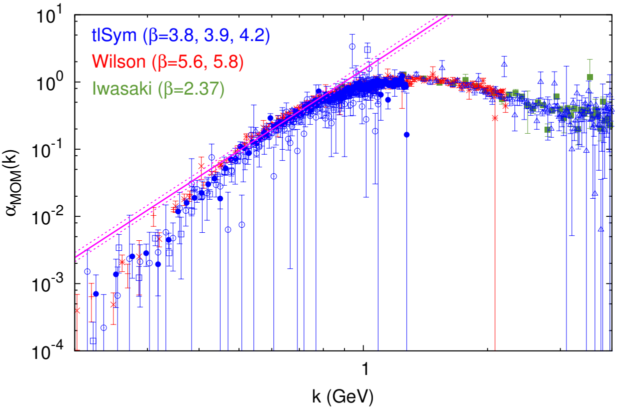
ETM Collaboration: [**JHEP 06 (2010) 111**]

- Domain-Wall $N_F = 2+1$ configurations ($m_\pi \sim$ *physical*)

β	a	V	confs.
2.37	0.0626fm	$(2.00\text{fm})^4$	590
2.25	0.0835fm	$(5.34\text{fm})^4$	330
2.13	0.1139fm	$(5.48\text{fm})^4$	350

RBC & UKQCD collaborations: [**PRD 93 (2016) 074505**]

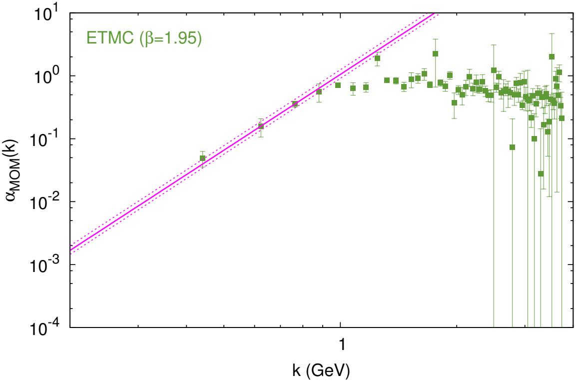
$N_F = 0$ lattice results for $\alpha_{\rm MOM}$



Instanton prediction $\alpha_{\rm MOM}(k^2) \sim k^4/18\pi n$ works for $k \in (0.3, 0.9){\rm GeV}^{-1}$

$N_F = 2 + 1 + 1$ twisted mass

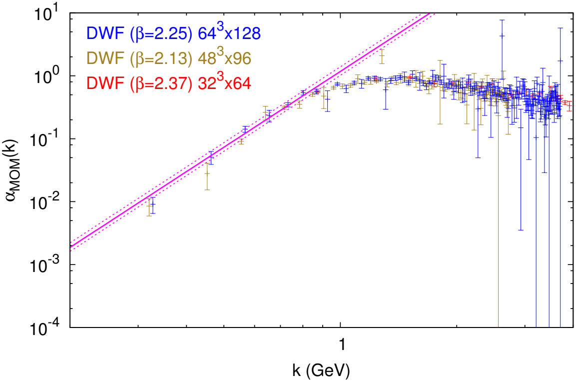
ETMC configurations $\beta = 1.95$



$$\frac{n_{2+1+1}}{n_0} \approx 1.5(1)$$

$N_F = 2+1$ Domain Wall fermions

UKQCD configurations, DW $\beta = 2.13$, $\beta = 2.25$, & $\beta = 2.37$,



$$\frac{n_{2+1}}{n_0} \approx 1.3(1)$$

- Instanton k^4 law works for $0.3 GeV \lesssim k \lesssim 1 GeV$
- Density grows with the number of dynamical flavours

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- Density grows with the number of dynamical flavours
- What if we eliminated quantum fluctuations?

Wilson Flow

- Flow Equation:

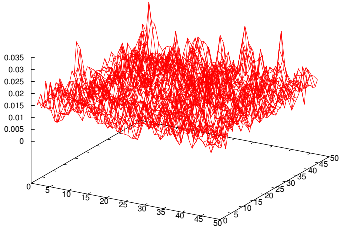
$$\partial_\tau B_\mu(\tau, x) = D_\nu G_{\mu\nu}(\tau, x) \ ,$$

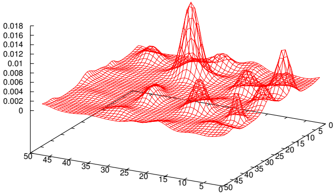
  with $B_\mu(0, x) = A_\mu(x)$

- In the lattice:
  $\partial_\tau V_\mu(x, \tau) = -g_0^2 \partial_{x,\mu} S[V] V_\mu(x, \tau)$
  and $V_\mu(x, \tau) = U_\mu(x)$ with S the gauge action.

- Has a smoothing effect of radius $\sqrt{8\tau}$ [JHEP 08 (2010) 071]

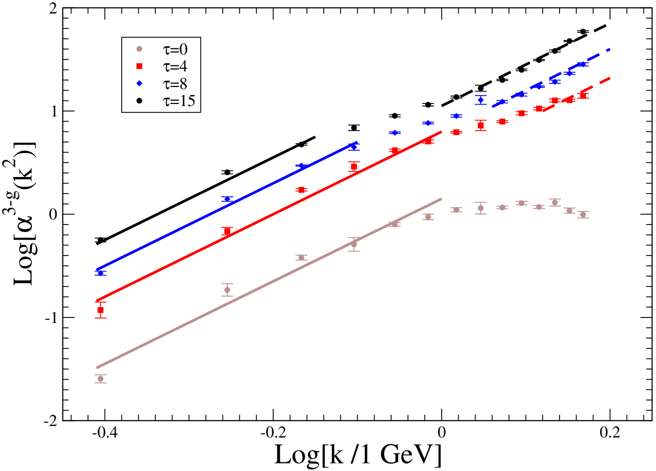
- Flow time usually expressed in terms of t_0 defined by $\sqrt{8t_0} = 0.3fm$





$\alpha_{\mathrm{MOM}}(k)$ after Wilson Flow

Asymptotic freedom disappears and k^4-law emerges [PLB760 (2016) 354]



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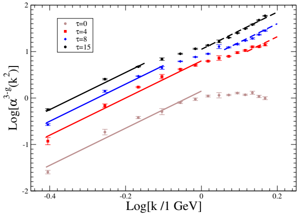
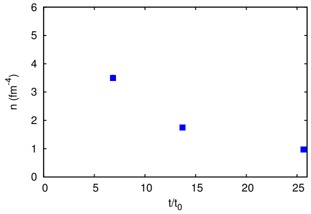




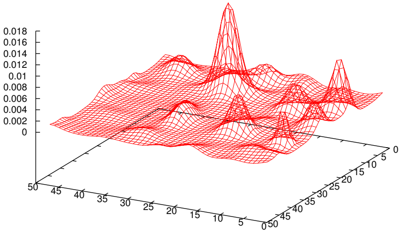
- Different slopes at small and large k are associated to the instanton distribution.

$$\alpha_{\mathrm{MOM}}(k^2) = \frac{k^4}{18\pi n} \times \begin{cases} 1 + 48\frac{\delta\rho^2}{\langle\rho\rangle^2} & k\rho \ll 1 \\ 1 & k\rho \gg 1 \end{cases}$$

- At large momenta we can obtain instanton density n.
- At small momenta the slope is a factor $c = 1 + 48\frac{\delta\rho^2}{\langle\rho\rangle^2}$ times larger.
- If $\frac{\delta\rho^2}{\langle\rho\rangle^2} \approx 0.014$ [PRD58 (1998) 014505], $c \approx 1.6$ and $n = 12(1)\mathrm{fm}^{-4}$

Locating instantons after Wilson Flow

Wilson flow eliminates short-range fluctuations



Action density after Wilson flow.

Topological charge density

For BPST instantons,

$$Q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma} G_{\mu\nu} G_{\mu\nu}$$

behave as:

$$Q(x) = \frac{6}{\pi^2\rho^4}\left(\frac{\rho^2}{x^2+\rho^2}\right)^4$$

For modified profiles this shape remains unchanged for small x

Locating instantons after Wilson Flow

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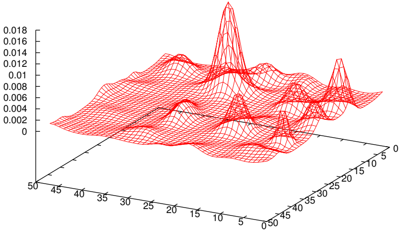
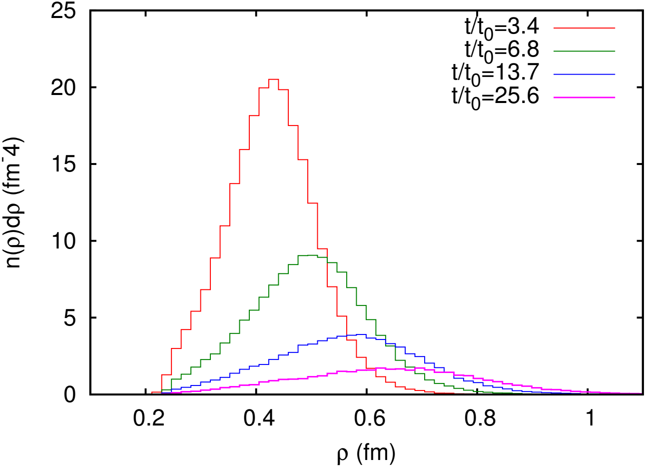


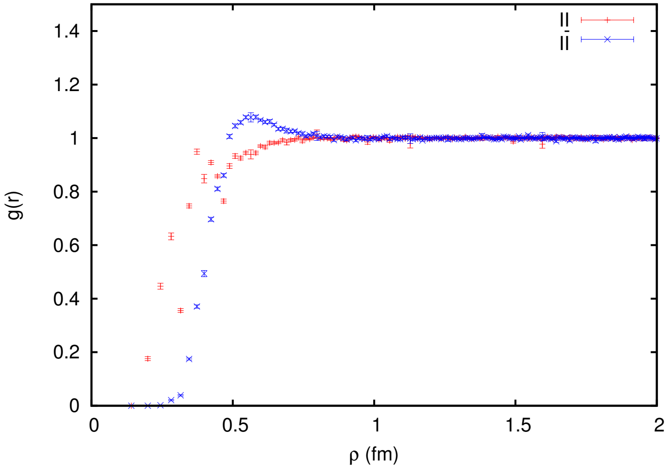
Action density after Wilson flow.

- Search for local extremes $x / Q(x) > Q(x \pm \hat{\mu})$
- Fit $Q(x \pm \hat{\mu})$ to BPST (like PRD 88 (2013) 034501)
- Check self-duality
- Discard close pairs when $r_{ij}^2 < \rho_i \rho_j$

Instanton sizes (tlSym 32^4, $\beta = 4.2$)



Radial distribution function (tlSym 32^4, $\beta = 4.2$ at $t/t_0 = 3.4$)



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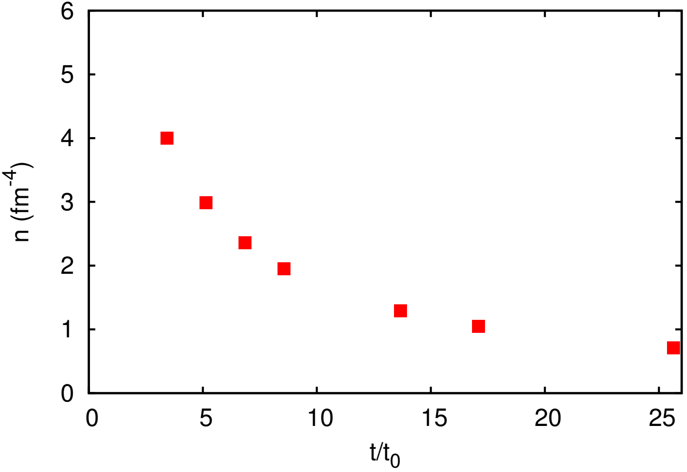
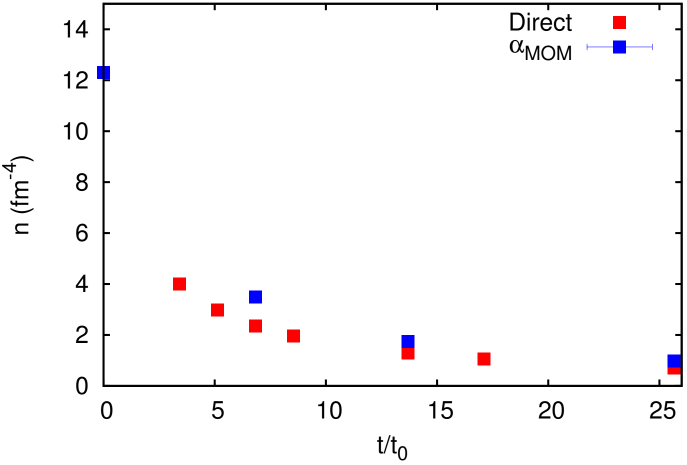


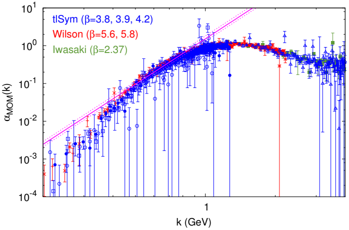
Direct counting vs $\alpha_{\rm MOM}(k^2)$ (tISym 32^4, $\beta = 4.2$)



Conclusions & outlook

Conclusions

- Instantons explain the behavior of α_{MOM} in the IR
- After Wilson flow instantons dominate also at large momenta
- Instanton density increases with N_F
- Determined n and ρ after WF



Perspectives

- Instanton prediction for other Green functions
- Investigate the profile function
- Understand how WF gets rid of Λ_{QCD}