Dynamics of entanglement entropy of interacting fermions in a 1D driven harmonic trap

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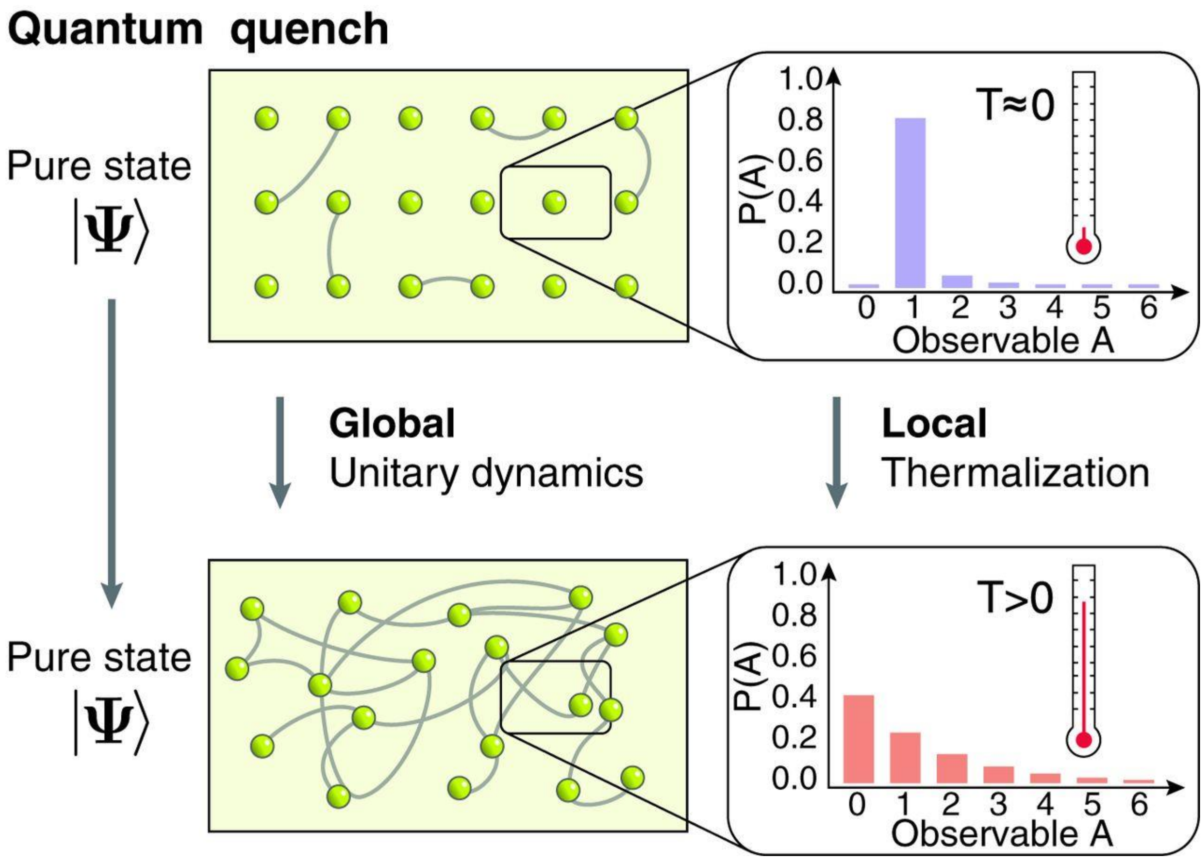
Lattice2017

Experiments show local thermal behavior in globally pure states



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A.M. Kaufman *et al., Science* 19 Aug 2016: Vol. 353, Issue 6301, pp. 794-800

Information and entropy

10010110 10111001

…

$2^8 = 256$ strings

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- What if we restrict to a subset of 8 strings?

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000 111 001 110
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$2^3 = 8$ strings

$$\log_2 (\text{\# of strings}) = \text{\# of bits}$$

- How much information is needed to specify a particular string?

Information and entropy

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$2^3 = 8$ strings

$\log_2(\#\text{ of strings}) = \#\text{ of bits}$

- How much information is needed to specify a particular string?

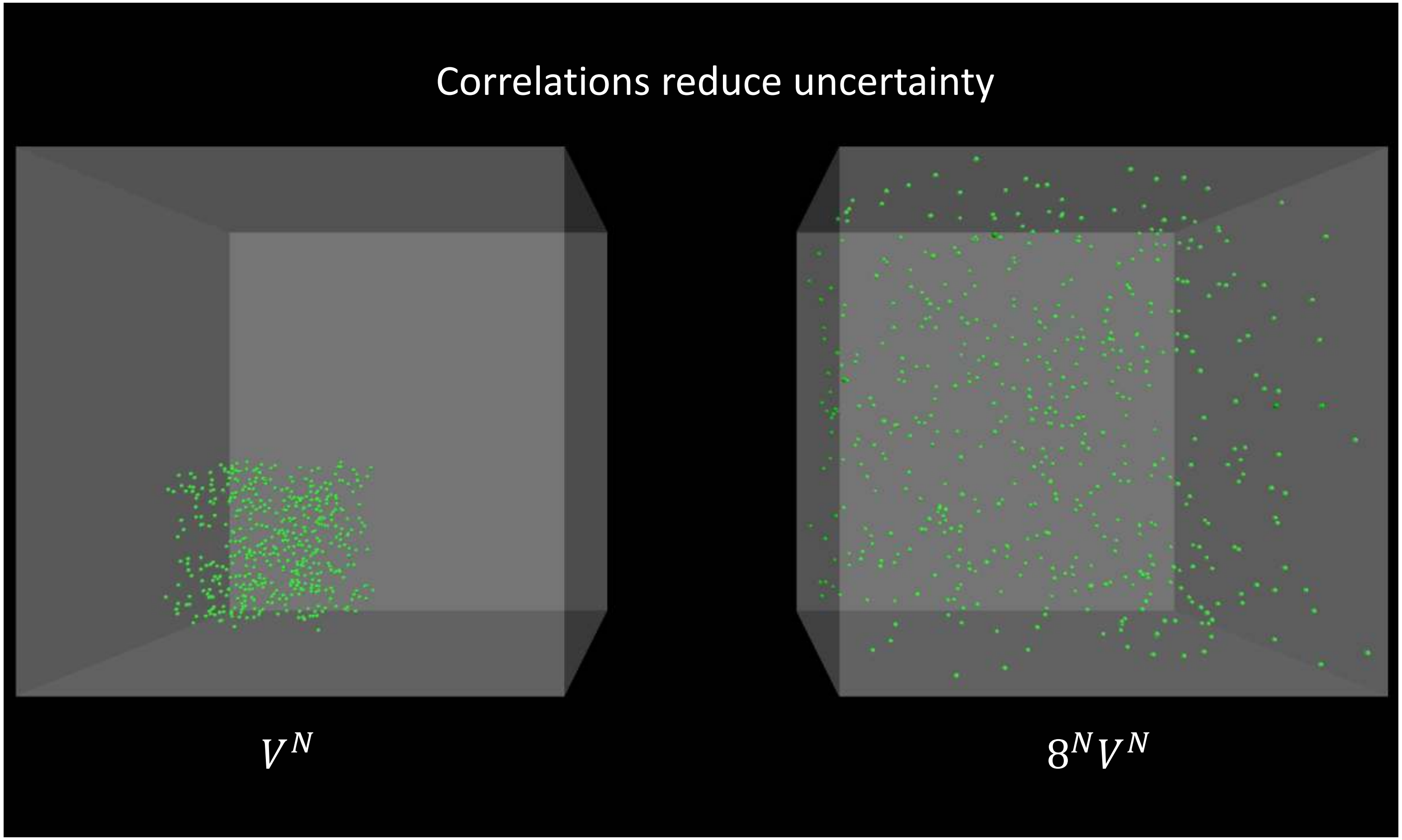
$$S_S = -\sum_x p(x)\log p(x)$$

For 8 strings, $p(x) = \frac{1}{8}$:

$S_S = -\log_2 \frac{1}{8} = 3$ bits

(not necessarily a whole number)

Information and entropy



Extension to quantum mechanics

von Neumann:

$$S_{vN} = -\mathrm{Tr}(\hat{\rho}\log\hat{\rho})$$

Rényi:

$$S_n = \frac{1}{1-n}\log\mathrm{Tr}\,\hat{\rho}^n$$

$$S_2 = -\log\mathrm{Tr}\,\hat{\rho}^2$$

Pure state: $S_2 = 0 \quad (T = 0)$

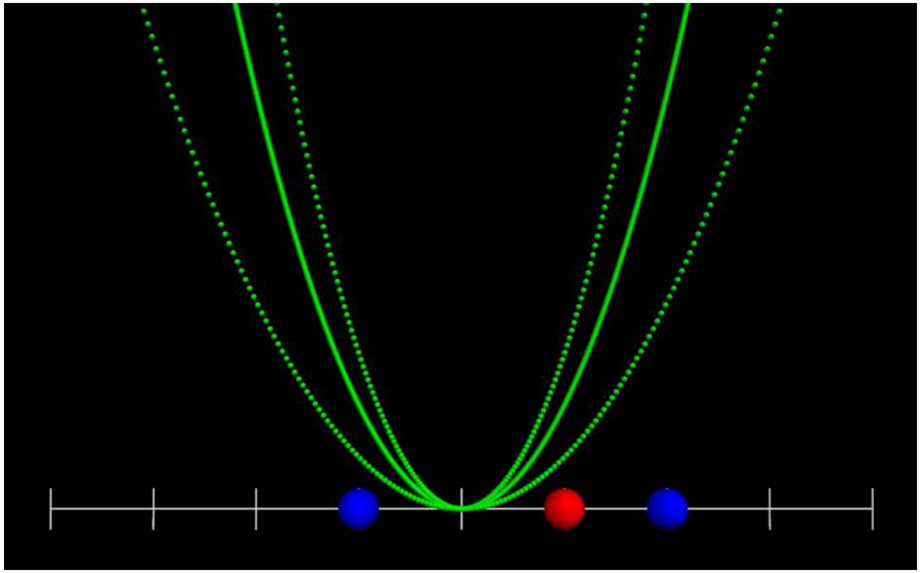
Mixed with equal weights: $S_2 = \log N$
$= \log(\mathrm{rank}\,\hat{\rho})$

Maximally mixed: $S_2 = \log V_{\mathcal{H}} \quad (T \to \infty)$

$$\frac{1}{\mathrm{Tr}\hat{\rho}^2} \sim \text{weighted \# of states contributing to } \hat{\rho}$$

System of interest

$$\hat{H} = \frac{\hat{p}_{\uparrow,\downarrow}{}^2}{2m} - g\delta(\hat{x}_\uparrow - \hat{x}_\downarrow) + \frac{1}{2}m\omega_0^2\hat{x}_{\uparrow,\downarrow}{}^2\left(1 + \alpha\sin\frac{t}{T}\right)$$



Transfer matrix:

$$\hat{\mathcal{T}} = \exp\left(-\tau\hat{H}\right)$$

Ground state projection:

$$\lim_{N\to\infty}\hat{\mathcal{T}}^N|\Psi\rangle = |\Psi_0\rangle$$

Wick rotation:

$$\tau \to i\delta t$$

Real-time evolution:

$$\hat{\mathcal{T}}|\Psi(t)\rangle = |\Psi(t+\delta t)\rangle$$

Not Monte Carlo;
numerically exact

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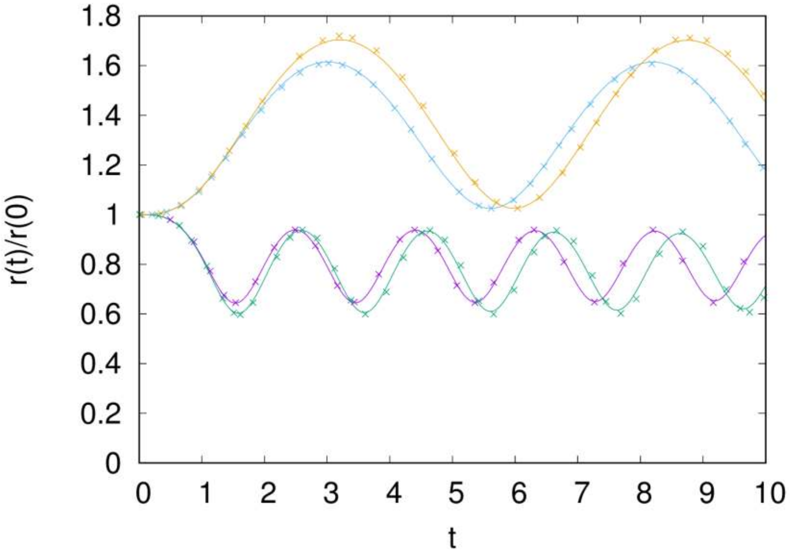
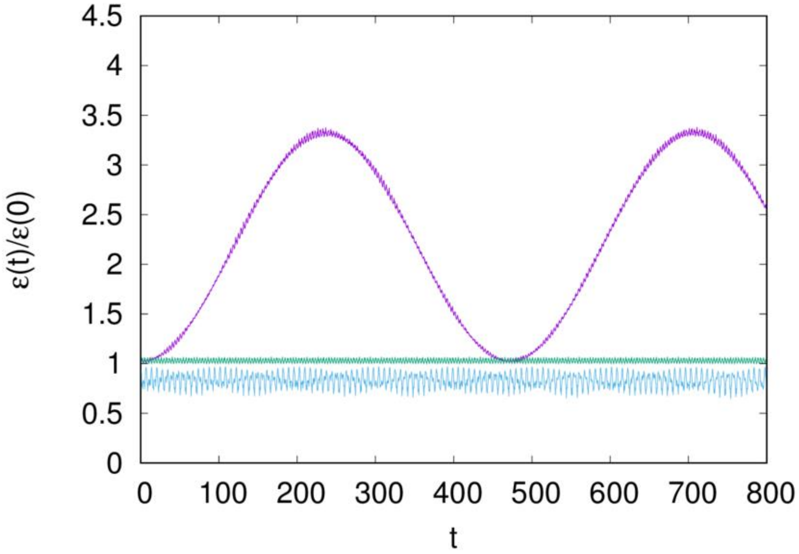
WJ Porter and JE Drut, Phys. Rev. B **94**, 165112, 2016.

Agreement with previous work



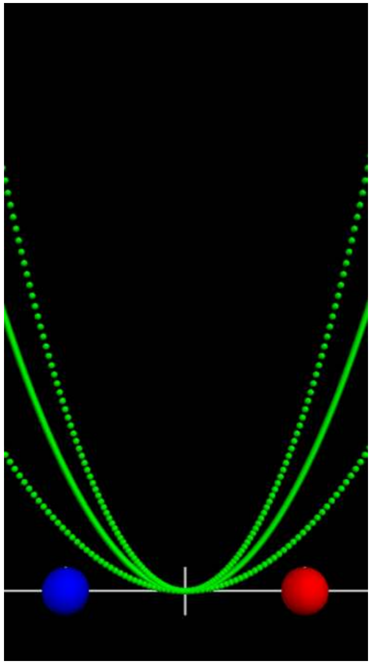


$$\omega^2(t) = \begin{cases} e^{\pm t}, & t < 1 \\ e^{\pm 1}, & t \geq 1 \end{cases}$$

$$\omega^2(t) = \omega_0^2 \left(1 + 0.5 \sin \frac{t}{T}\right)$$

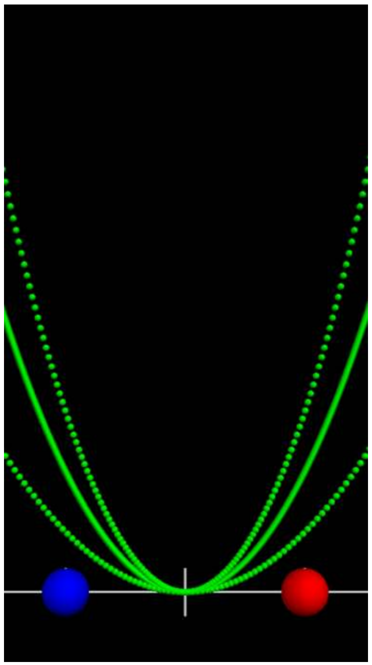
Ebert, Martin *et al.* Annalen Phys. 528 (2016) no.9-10, 693-704 arXiv:1512.06628

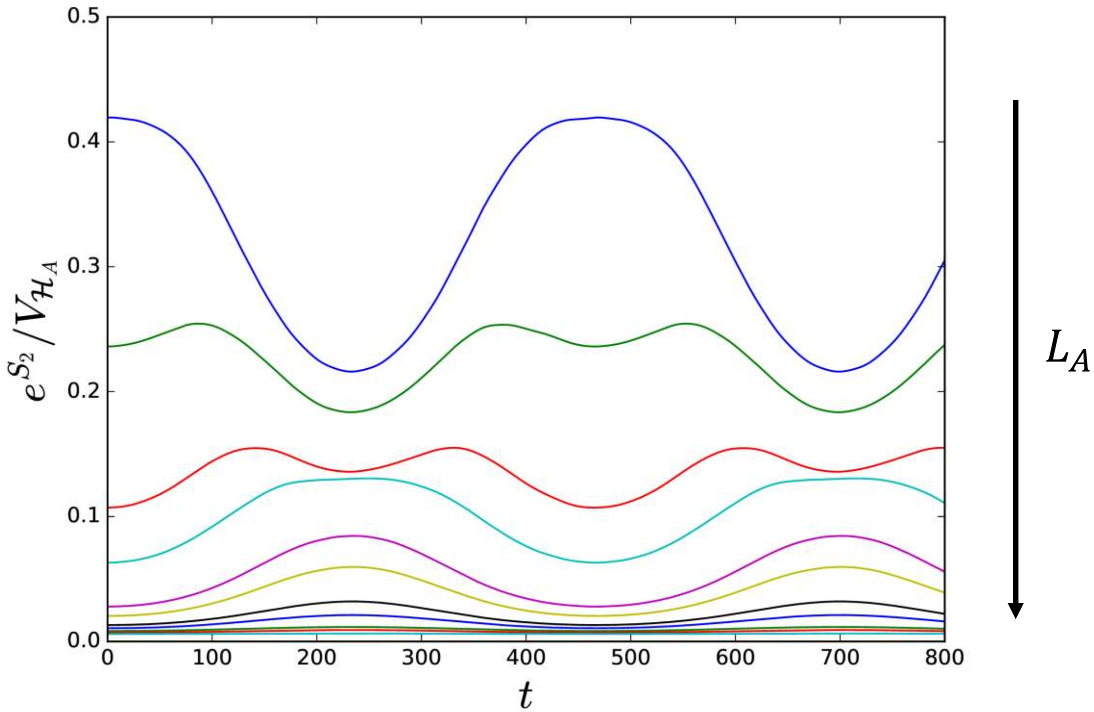
Small subsystems appear thermal



$$\hat{\rho}_A = \mathrm{Tr}_{\bar{A}}\hat{\rho}$$

Small subsystems appear thermal



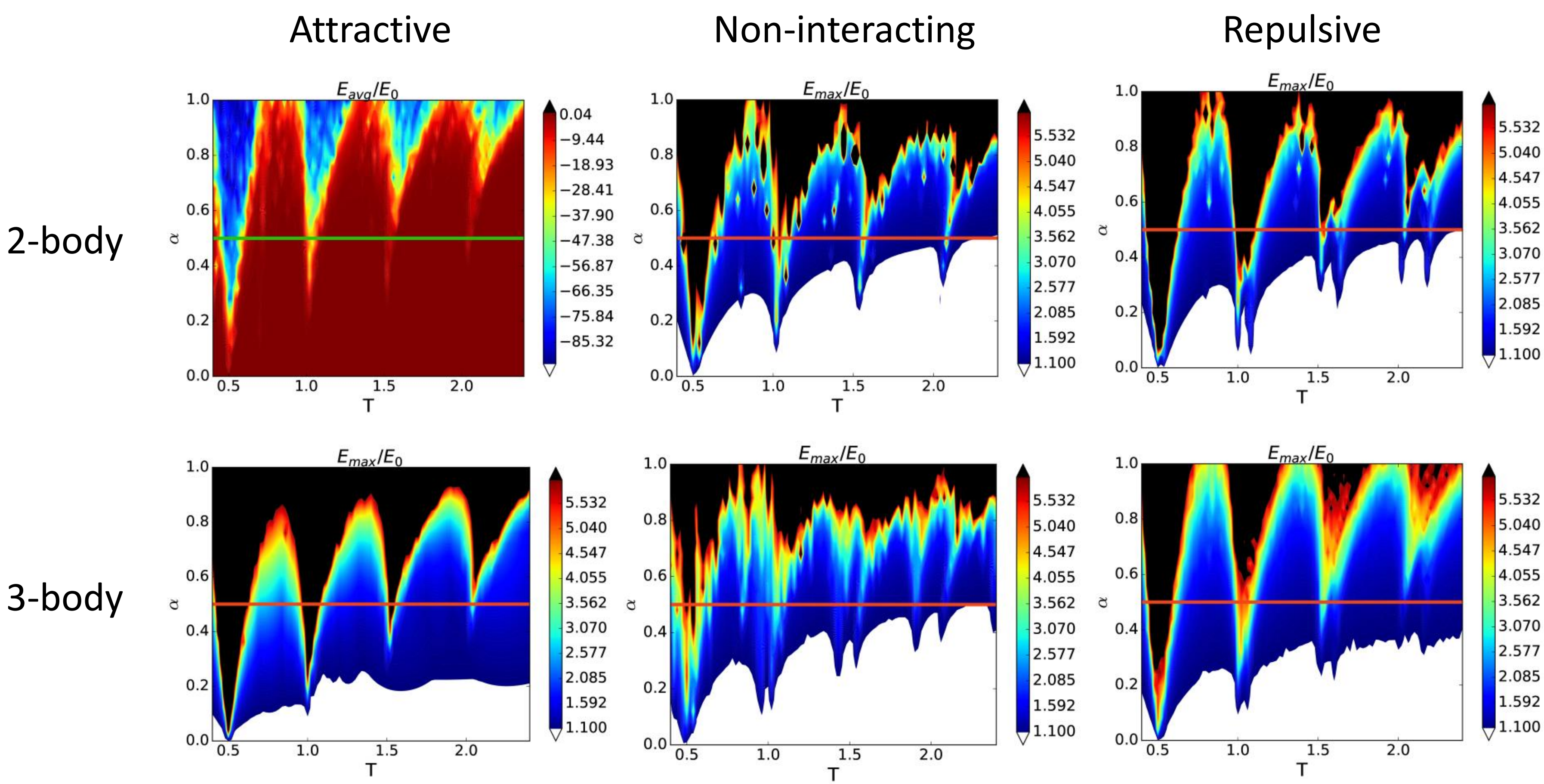


$\hat{\rho}_A = \mathrm{Tr}_{\bar{A}}\hat{\rho}$

Fraction of Hilbert space occupied indicates quasi-temperature à la $\exp(-\beta\Delta E)$

$$V_{\mathcal{H}_A} = \left[\sum_{k=0}^{N_\uparrow}\binom{L_A}{k}\right]\left[\sum_{k=0}^{N_\downarrow}\binom{L_A}{k}\right]$$

Energy reveals unstable regions

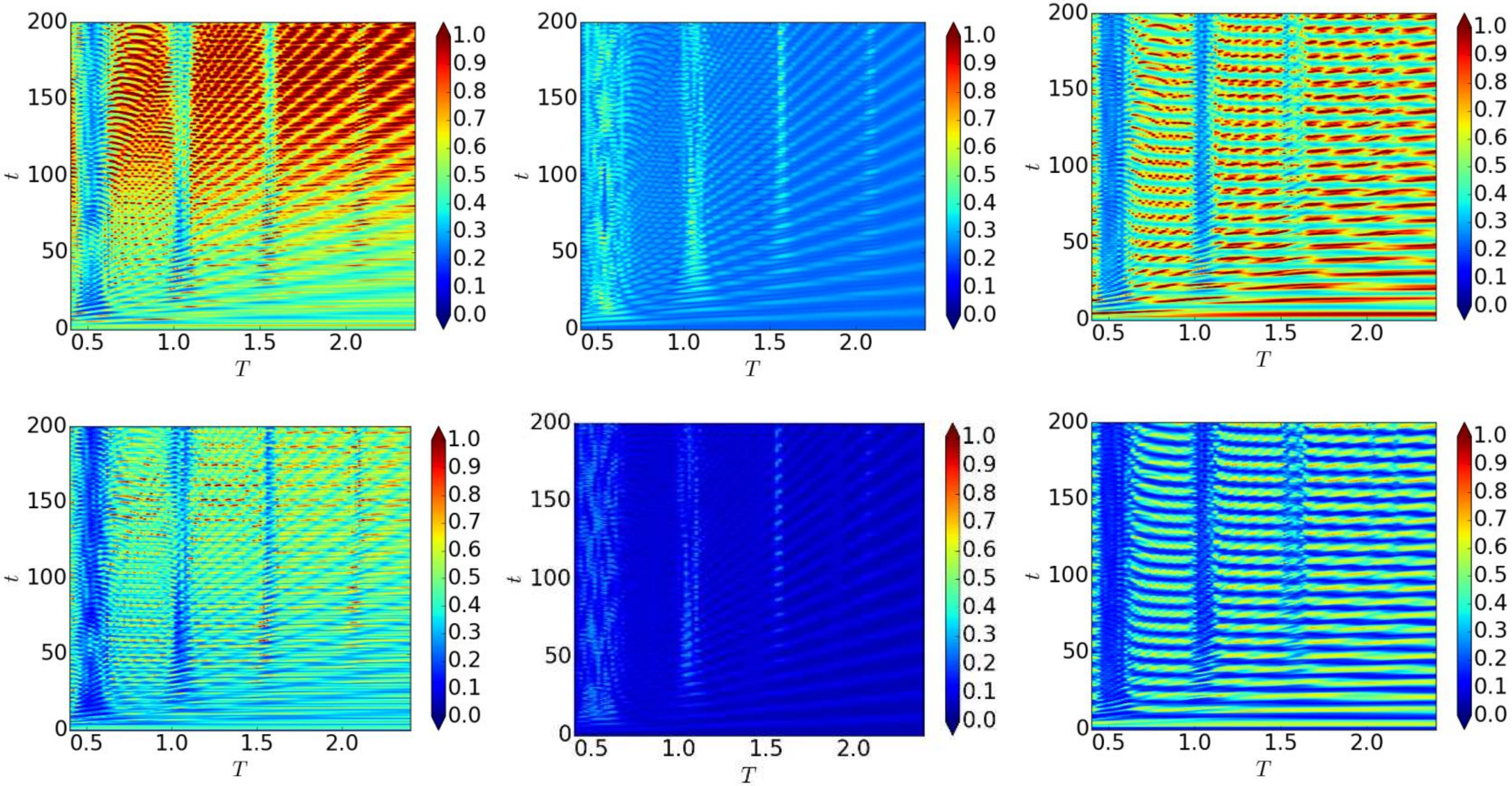


Hilbert space occupation around resonances

Attractive | Non-interacting | Repulsive

$L_A = 2$

$L_A = 3$



Summary and acknowledgements

- Entanglement causes locally thermal behavior in a globally pure state
  - Measured by entanglement entropy, fraction of Hilbert space
- Driven behavior is coupling-dependent
- Off-resonant driving yields non-trivial entanglement structure

UNC collaborators:

- Joaquín Drut
- William J. Porter
- **Christopher Shill**
- **Andrew Loheac**