Update on SU(2) gauge theory with $N_F = 2$ fundamental flavours

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Motivation - Composite Higgs

Naturalness problem - Higgs mass expected to receive contributions of order Λ
Why is the Higgs so light?

- Could be a pseudo-Goldstone boson
- Higgs transforms under $SU(2)_L \times SU(2)_R \sim SO(4)$ global custodial symmetry
- other Goldstone bosons become longitudinal components of W and Z
- Minimal composite Higgs model $SO(5) \to SO(4)$, but no known UV completion
- Next-to-minimal composite Higgs $SO(6) \to SO(5)$ - this model

SU(2) with 2 fundamental flavours

SU(2) model, 2 Dirac fermions in fundamental representantion [1402.0233].
$SU(2) = Sp(2) \sim SO(3)$ smallest non-abelian Lie group.

$$\mathcal{L} = -\frac{1}{4} F^{a\mu\nu} F^{a}_{\mu\nu} + i\bar{U}\gamma^{\mu} D_{\mu} U + i\bar{D}\gamma^{\mu} D_{\mu} D$$

Fundamental representation of SU(2) is pseudo-real $\rightarrow$ we can construct a flavour multiplet

$$Q = \begin{pmatrix} u_L \\ d_L \\ -i\sigma^2 C \bar{u}_R^T \\ -i\sigma^2 C \bar{d}_R^T \end{pmatrix}$$

$\mathcal{L}$ is symmetric under SU(4) flavour group (locally isomorphic to SO(6)).

SU(2) with 2 fundamental flavours

SU(4) symmetry is broken spontaneously by a fermion condensate $\Sigma^{ab} = \langle Q^a (i\sigma_c^2) C Q^b \rangle$ [1109.3513] to the subgroup which leaves it invariant:

$$U^T \Sigma U = \Sigma \quad U \in Sp(4) \sim SO(5).$$

This produces 5 Goldstone bosons ("pions").
On the lattice we add an explicit mass term:

$$-m(\bar{u}u + \bar{d}d) = \frac{m}{2} Q^T(-i\sigma^2) CEQ + h.c.$$

$$E = \begin{pmatrix} 0 & 1_2 \\ -1_2 & 0 \end{pmatrix}$$

Phenomenology

Under EW Q_L transform as $(\mathbf{2}, 0)$, U_R as $(\mathbf{1}, 1/2)$ and D_R as $(\mathbf{1}, -1/2)$.

The model contains a choice of inequivalent vacua. Of particular interest are Σ_B and Σ_H

	$\Sigma_B = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$	$\Sigma_H = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
EW symmetry	unbroken	broken
model	composite Higgs	Technicolor
pions	$W^{\pm}$, Z, H + 1 extra	$W^{\pm}$, Z + 2 extra
Higgs	pion	scalar resonance

The vacuum can also be a superposition of the two:

$$\Sigma_0 = \cos\theta \Sigma_B + \sin\theta \Sigma_H$$

$$f_{PS} \sin\theta = v = 246\mathrm{GeV}$$

This is the minimal UV-complete model, which can achieve this

Spectrum - what we expect

Quantum numbers

- Flavour

  Quarks transform in fundamental of Sp(4). Mesons can therefore be

  $$\mathbf{4} \otimes \mathbf{4} = \mathbf{10} \oplus \mathbf{5} \oplus \mathbf{1}$$

  specifically

  - Symmetric combination (dim 10): $\varphi^a \chi^b + \varphi^b \chi^a$
  - Antisymmetric, 'E-traceless' (dim 5): $\varphi^a \chi^b - \varphi^b \chi^a - \frac{1}{2} E^{ab} E^{cd} \varphi^c \chi^d$
  - 'E-trace' (dim 1): $E^{ab} \varphi^a \chi^b$

- Parity

  To have a parity transformation which commutes with flavour transformations we need the intrinsic parity of fermions to be imaginary:

  $$\mathbf{P} U(x) \mathbf{P}^{-1} = i\gamma^0 U(x^P)$$
  $$\mathbf{P} D(x) \mathbf{P}^{-1} = i\gamma^0 D(x^P)$$

Decay products

Pions transform under 5-dimensional representation of Sp(4)

$$\mathbf{5} \otimes \mathbf{5} = \mathbf{14} \oplus \mathbf{10} \oplus \mathbf{1}$$

$$\mathbf{5} \otimes \mathbf{5} \otimes \mathbf{5} = \mathbf{35} \oplus \mathbf{35} \oplus \mathbf{30} \oplus \mathbf{10} \oplus \mathbf{5} \oplus \mathbf{5} \oplus \mathbf{5}$$

- Mesons in **10** representation can decay to both 2 and 3 pions
- Mesons in **5** representation can decay to both 3 pions but *not* to 2 pions
- Mesons in **1** representation can decay to both 2 pions but *not* to 3 pions

Under parity, all pions are odd.

What we expect

Flavour	J^P	operator	decay products
5	0^-	$\bar{U}\gamma^5 D$	stable
1	0^-	$\bar{U}\gamma^5 U + \bar{D}\gamma^5 D$	stable
1	0^+	$\bar{U}U + \bar{D}D$	2 π s-wave
5	0^+	$\bar{U}D$	stable
1	1^-	$\bar{U}\gamma^i\gamma^5 U + \bar{D}\gamma^i\gamma^5 D$	stable
10	1^+	$\bar{U}\gamma^i D$	2π p-wave, 3π p-wave
5	1^-	$\bar{U}\gamma^i\gamma^5 D$	3 π p-wave
10	1^-	$\bar{U}\gamma^0\gamma^5\gamma^i D$	3π p-wave

We assume that the pions are the only potentially allowed decay modes.

Lattice setup

- Wilson plaquette gauge action
- (unimproved) Wilson fermion action, $Z_2 \times Z_2$ noise sources

beta	volume	$-am_0$
1.8	$16^3 \times 32$	1.0, 1.089, 1.12, 1.14, 1.15
1.8	$24^3 \times 32$	1.155, 1.157
2.0	$16^3 \times 32$	0.85, 0.9, 0.94, 0.945
2.0	$32^3 \times 32$	0.947, 0.949, 0.952, 0.956, 0.957, 0.959
2.0	$32^3 \times 64$	0.958, 0.9585
2.2	$16^3 \times 32$	0.6, 0.65, 0.68, 0.7
2.2	$32^3 \times 32$	0.72, 0.735, 0.75
2.2	$48^3 \times 48$	0.76, 0.763
2.3	$32^3 \times 32$	0.575, 0.6, 0.625, 0.65, 0.675
2.3	$48^3 \times 48$	0.68, 0.685

new mass increased volume increased statistics

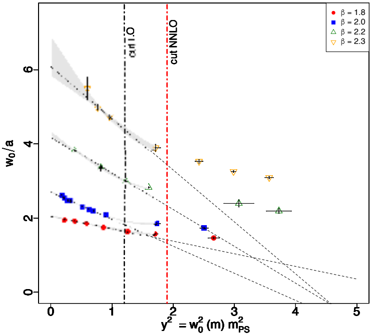
Scale setting

$$W(T) = t\frac{d}{dt}\langle t^2 E(t)\rangle$$

We define w_0 by

$$W(w_0^2) = 1$$

beta	w_0/a
1.8	2.068(26)
2.0	2.711(17)
2.2	4.318(58)
2.3	6.369(514)



Renormalization

RI'-MOM scheme - nothing new here, move along...

$$\langle \bar{\psi}(p) O_R(x,\mu) \psi(-p) \rangle_{amp} \Big|_{p^2=\mu^2} = \langle O_R(x,\mu) \rangle_{tree}$$

$$O(x) = Z_O(\mu) O_R(x,\mu)$$

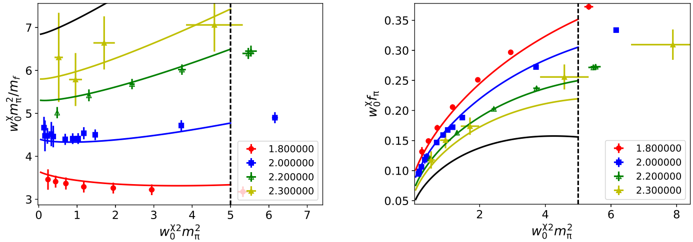
$$Z_q = \frac{1}{4N_c} Tr\left(S^{-1}(p)(D(p))^{-1}\right)\Big|_{p^2=\mu^2}$$

β	Z_A	Z_V	Z_P/Z_S	$Z_P^{RI'}(w_0^2 p^2 = 7)$
1.8	0.77911(4)(9)	0.5599(4)(40)	0.2809(48)(45)	0.2051(36)(66)
2.0	0.8072(3)(5)	0.6356(2)(26)	0.4080(25)(27)	0.2907(16)(72)
2.2	0.8267(2)(23)	0.6973(2)(30)	0.5655(16)(121)	0.3803(8)(49)
2.3	0.8449(23)(72)	0.7280(19)(80)	0.6799(260)(440)	0.4201(136)(13)

Fitting the chiral parameters: F and B_0

Fit to χPT parameters using f_{PS} or m_π^2/m_{PCAC} as a function of m_π^2.

$$f_\pi = F(1 + Am_\pi^2 \log m_\pi^2 + Bm_\pi^2 + C/w_0 + Dm_\pi^2/w_0)$$
$$m_\pi^2/m_f = 2B(1 + A'm_\pi^2 \log m_\pi^2 + B'm_\pi^2 + C'/w_0 + D'm_\pi^2/w_0)$$



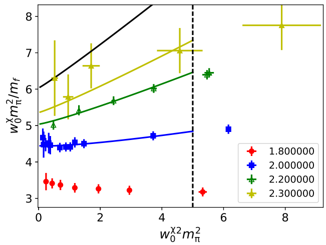
$F = 0.045(10) \quad \chi^2/dof = 2.01$
$B = 3.42(14) \quad \chi^2/dof = 0.90$

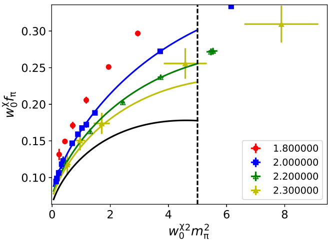
Fitting the chiral parameters: F and B_0 - excluding coursest ensembles

Fit to χPT parameters using f_{PS} or m_π^2/m_{PCAC} as a function of m_π^2 - excluding $\beta = 1.8$.

$$f_\pi = F(1 + Am_\pi^2 \log m_\pi^2 + Bm_\pi^2 + C/w_0 + Dm_\pi^2/w_0)$$

$$m_\pi^2/m_f = 2B(1 + A'm_\pi^2 \log m_\pi^2 + B'm_\pi^2 + C'/w_0 + D'm_\pi^2/w_0)$$



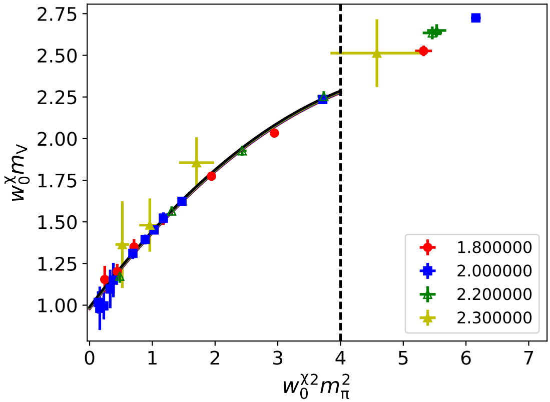


$F = 0.0632(55) \quad \chi^2/dof = 0.456$

$B = 3.01(11) \quad \chi^2/dof = 0.328$

Chiral and continuum extrapolation of vector meson

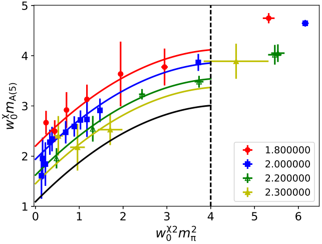
$$w_0^\chi m = w_0^\chi m^\chi + A(w_0^\chi m_\pi)^2 + B(w_0^\chi m_\pi)^4 + C\frac{a}{w_0}$$

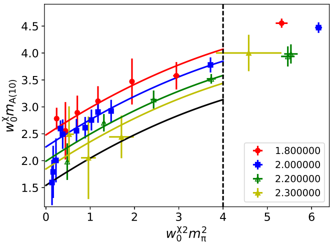


$m_V^\chi = 0.989(27)$ $\chi^2/dof = 0.378$

Chiral and continuum extrapolation of axial mesons

$$w_0^\chi m = w_0^\chi m^\chi + A(w_0^\chi m_\pi)^2 + B(w_0^\chi m_\pi)^4 + C\frac{a}{w_0}$$





$m_{A5}^\chi = 1.09(15)$ $\quad \chi^2/dof = 0.306$

$m_{A10}^\chi = 1.54(18)$ $\quad \chi^2/dof = 0.500$

- Worked out the spectrum of SU(2) model
- New results in good agreement with the previous ones
- χPT coefficients and meson masses in chiral limit:

B	F	m_V	m_A (5)	m_A (10)
3.01(11)	0.0632(55)	0.989(27)	1.09(15)	1.54(18)

Future work:

- Renormalization
- Flavour singlet spectrum (σ, η etc.)