Calculation of $\bar{B} \to D^* \ell \bar{\nu}$ form factor at zero-recoil using the Oktay-Kronfeld action

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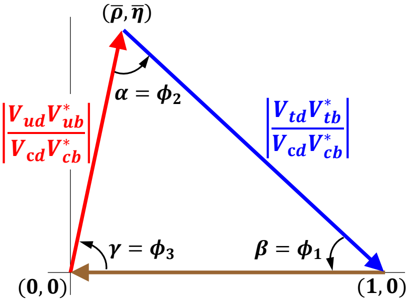
Why V_{cb}?

- Standard model evaluation of $|\epsilon_K^{\rm SM}|$ using inputs determined from lattice QCD : $\hat{B}_K$, (exclusive) $|V_{cb}|$, and etc. has a deviation from the experimental value [W. Lee, poster session]

$$\Delta\epsilon_K = \epsilon_K^{\rm Exp} - \epsilon_K^{\rm SM} = 3.3\sigma,$$

- Constraint on the apex of CKM unitarity triangle



$|V_{cb}|$ from the exclusive decay $\bar{B} \to D^* l \bar{\nu}$

$$\frac{d\Gamma}{dw}(\bar{B} \to D^* l \bar{\nu}) = \frac{G_F^2 |V_{cb}|^2 M_B^5}{4\pi^3} r^{*3}(1-r^*)^2(w^2-1)^{\frac{1}{2}} \eta_C |\eta_{EW}|^2 \chi(w) |\mathcal{F}(w)|^2$$

- $w = v_B \cdot v_{D^{(*)}}$, $r^{(*)} = \frac{M_{D^{(*)}}}{M_B}$
- η_C: Coulomb attraction, $\eta_{EW} = 1.0066$: the one-loop electroweak correction
- $\chi(w)$: phase-space factor
- $\mathcal{F}(w)$: Form factor ($\leftarrow$LATTICE)

Decay mode	$\|V_{cb}\| \times 10^3$	Reference
$B \to D\ell\bar{\nu}$	40.49(97)	PRD94, 094008 (2016)
$B \to D^*\ell\bar{\nu}$	38.71(47)(59)	arXiv:1612.07233
$B \to X_c\ell\bar{\nu}$	42.00(64)	PLB763, 60 (2016)

Heavy quarks on the lattice: Fermilab method

- The most updated version of V_{cb} calculation is done using the Fermilab action to control the c, b heavy quark discretization errors. It is generalized version of the Wilson clover action [El-Khadra, Kronfeld, and Mackenzie, PRD55, 3933 (1997)]

$$S_{Fermilab} = S_0 + S_E + S_B$$

$$S_0 = a^4 \sum_x \bar{\psi}(x) \left[m_0 + \gamma_4 D_4 - \frac{1}{2}\Delta_4 + \zeta\left(\gamma \cdot \boldsymbol{D} - \frac{1}{2} r_s \Delta^{(3)}\right)\right] \psi(x)$$

$$S_E = -\frac{1}{2} c_E \zeta a^5 \sum_x \bar{\psi}(x) \boldsymbol{\alpha} \cdot \boldsymbol{E} \psi(x), \quad S_B = -\frac{1}{2} c_B \zeta a^5 \sum_x \bar{\psi}(x) i \boldsymbol{\Sigma} \cdot \boldsymbol{B} \psi(x).$$

- The Wilson term breaks the chiral symmetry explicitly, and the mass gets additive renormalization.
  → We tune the bare mass m_0 to the physical quark, nonperturbatively.

Oktay-Kronfeld action

- The OK action is an improved version of the Fermilab action such that the bilinear operators are tree-level matched to QCD through $\mathcal{O}(\lambda^3)$ in HQET power counting where $\lambda \sim a\Lambda \sim \Lambda/(2m_Q)$ [Oktay and Kronfeld, PRD78, 014504 (2008)].

$$S_{OK} = S_{Fermilab} + S_{new}.$$

$$S_{new} = a^4 \sum_x \bar{\psi}(x) \Big[c_1 a^2 \sum_i \gamma_i D_i \Delta_i + c_2 a^2 \{\gamma \cdot \boldsymbol{D}, \Delta^{(3)}\} + c_3 a^2 \{\gamma \cdot \boldsymbol{D}, i\boldsymbol{\Sigma} \cdot \boldsymbol{B}\} + c_{EE} a^2 \{\gamma_4 D_4, \boldsymbol{\alpha} \cdot \boldsymbol{E}\} + c_4 a^3 \sum_i \Delta_i^2 + c_5 a^3 \sum_{i \neq j} \{i\Sigma_i B_i, \Delta_j\} \Big] \psi(x)$$

- The matching determines c_B, c_E, $c_{1,\cdots,5}$ and c_{EE} as a function of m_0.

$$am_0 = \frac{1}{2u_0} \left(\frac{1}{\kappa} - \frac{1}{\kappa_{\text{crit}}} \right)$$

We have a tree-level value for the κ_{crit}

$$\kappa_{\text{crit}}^{\text{tree}} = [2u_0(1 + 3\zeta r_s + 18c_4)]^{-1} = 0.053850 \qquad (\zeta = r_s = 1)$$

where $u_0 = 0.86372$ for MILC HISQ lattice (a12m310, $24^3 \times 64$)

Fermilab method

We write non-relativistic dispersion relation,

$$E(\mathbf{p}) = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{(\mathbf{p}^2)^2}{8M_4^3} - \frac{a^3 W_4}{6} \sum_i p_i^4 + \cdots .$$

- M_1: rest mass
- M_2: kinetic mass $\rightarrow$ Tuning to the physical mass
- M_4: quartic mass
- W_4: Lorentz symmetry breaking term

(Example) Tree-level relation between the bare quark mass m_0 and the kinetic quark mass m_2

$$\frac{1}{am_2} = \frac{2\zeta^2}{am_0(2 + am_0)} + \frac{r_s \zeta}{1 + am_0}$$

Nonperturbative determination of κ_{crit}

- $M_2(\kappa, \kappa_{\text{crit}})$: Light kinetic meson mass (600∼950 MeV)
- $m_2(\kappa, \kappa_{\text{crit}})$: kinetic quark mass

Let us suppose the meson mass relation

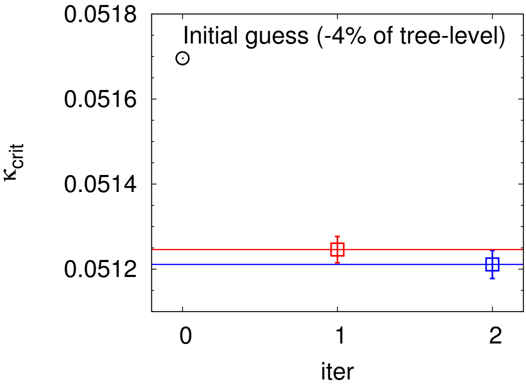
$$M_2^2 = A + Bm_2 + Cm_2^2.$$

The fitting using the true value of κ_{crit} will give $A = 0$. Note that the action depends on both κ and κ_{crit}. We determine κ_{crit} iteratively, as follows.

1. Start with an initial guess $\kappa'_{\text{crit}} = 0.96\kappa^{\text{tree}}_{\text{crit}}$
2. Determine the OK action coefficients using κ'_{crit}
3. Produce 2-pt pion correlators, and determine kinetic meson mass $M_2(\kappa, \kappa'_{\text{crit}})$ using various κ in the range (600∼950 MeV)
4. Find κ_{crit} such that fitting in terms of $m_2(\kappa, \kappa_{\text{crit}})$ gives $A = 0$.
5. Update $\kappa'_{\text{crit}} = \kappa_{\text{crit}}$ and go to the step 2.

Nonperturbative determination of κ_{crit}: result

$N_f = 2+1+1$ MILC HISQ ensemble (a12m310),
$N_{\text{conf}} = 130$, point source



$$\kappa_{\text{crit}} = 0.051211(33)(4)$$

where the errors are statistical and systematic (fit ambiguity).

κ tuning using D_s and B_s masses

- $M_2(\kappa, \kappa_{\text{crit}})$: Heavy-light meson mass
- $m_2(\kappa, \kappa_{\text{crit}})$: kinetic quark mass

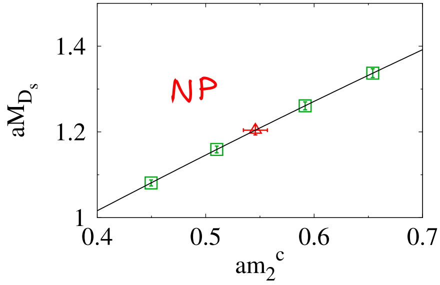
We use the HQET expansion of heavy-light meson masses M_2 as a fitting function:

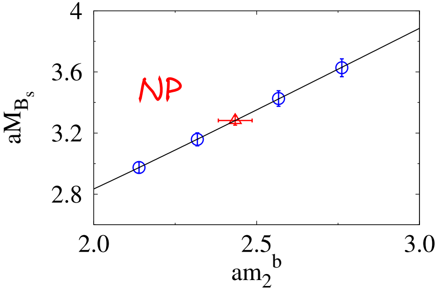
$$aM_2 = am_2 + d_0 + \frac{d_1}{am_2} + \frac{d_2}{(am_2)^2}.$$

1. Determine the OK action coefficients using charm and bottom type κ values with nonperturbative κ_{crit}.
2. Produce 2-pt B_s, D_s correlators, and determine $M_2(\kappa, \kappa_{\text{crit}})$
3. Determine the coefficients d_0, d_1 and d_2 using least-χ^2 fitting
4. Find m_2^{tuned} that gives the physical meson mass $M^{\text{Phys}} = M_2(m_2^{\text{tuned}})$.
5. obtain κ^{tuned} such that $m_2^{\text{tuned}} = m_2(m_0^{\text{tuned}})$ and $m_0^{\text{tuned}} = m_0(\kappa^{\text{tuned}}, \kappa_{\text{crit}})$.

κ tuning using D_s and B_s masses: results

- $N_f = 2+1+1$ MILC HISQ ensemble (a12m310)
- HISQ propagators ($am_s = 0.0509$) with point source
- OK propagators ($\kappa_{\rm crit} = 0.051211$) with covariant Gaussian smearing $(\sigma, N) = (1.5, 5)$.





$$\kappa_c = 0.048524(33)(43), \qquad \kappa_b = 0.04102(14)(9)$$

κ tuning using D_s and B_s masses: results

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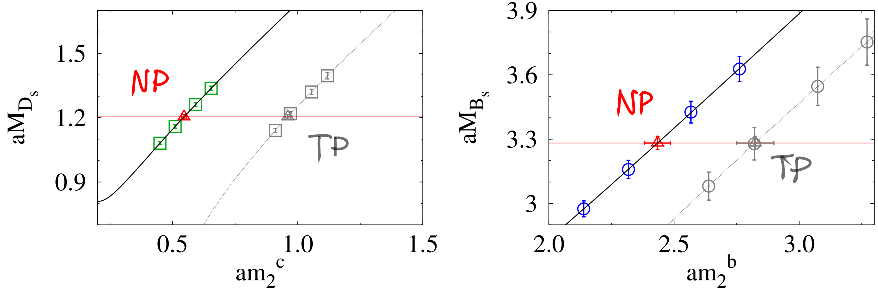
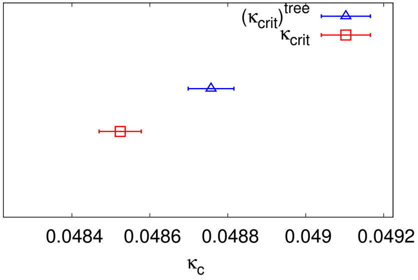


$$\kappa_c = 0.048524(33)(43), \qquad \kappa_b = 0.04102(14)(9)$$

Effect of κ_{crit} on the κ tuning

We compare the tuning results using two different critical hopping parameters: the tree-level value $(\kappa_{\text{crit}})^{\text{tree}}$ and the true nonperturbative value κ_{crit}:

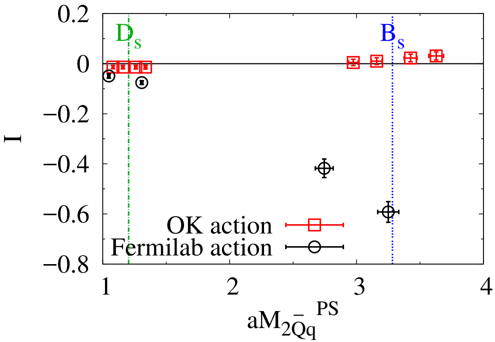


Inconsistency

We calculate the inconsistency parameter I [Collins, Edwards, Heller, and Sloan, NPB Proc. Suppl. 47, 455 (1996)] to see $\mathcal{O}(\mathbf{p}^4)$ improvement in the OK action.

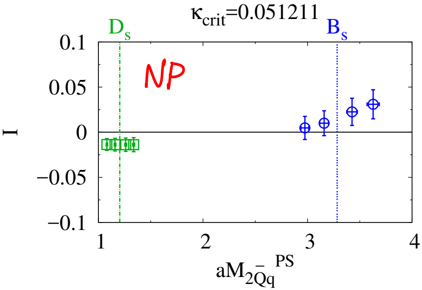
$$I \equiv \frac{2\delta M_{\bar{Q}q} - (\delta M_{\bar{Q}Q} + \delta M_{\bar{q}q})}{2M_{2\bar{Q}q}} = \frac{2\delta B_{\bar{Q}q} - (\delta B_{\bar{Q}Q} + \delta B_{\bar{q}q})}{2M_{2\bar{Q}q}}$$

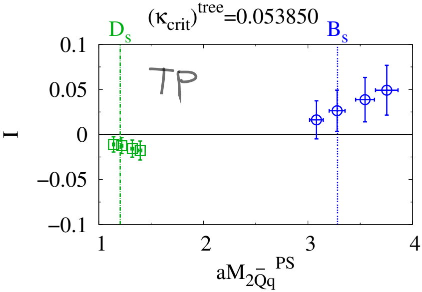
where binding energy $M_{1\bar{Q}q} = m_{1\bar{Q}} + m_{1q} + B_{1\bar{Q}q}$ and so on.



[Yong-Chull Jang et al., arXiv:1701.00345v1]

Inconsistency





Heavy-heavy current for $B \to D^*$

Effective Hamiltonian for the tree-level weak interaction is

$$H_W = \frac{G_F}{\sqrt{2}} V_{cb} [\bar{c}\gamma_\mu(1-\gamma_5)b][\bar{e}\gamma^\mu(1-\gamma_5)\nu_e] + \text{h.c.}.$$

Hadronic matrix elements of vector and axial vector currents

$$\frac{1}{\sqrt{m_B m_{D^*}}}\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle = h_V(w)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\frac{1}{\sqrt{m_B m_{D^*}}}\langle D^*(p',\epsilon)|A^\mu|\bar{B}(p)\rangle = -i h_{A_1}(w)(w+1)\epsilon^{*\mu} + i h_{A_2}(w)(\epsilon^*\cdot v)v^\mu + i h_{A_3}(w)(\epsilon^*\cdot v)v'^\mu$$

$$\frac{d\Gamma}{dw}(\bar{B}\to D^* l\bar{\nu}) = \frac{G_F^2|V_{cb}|^2 M_B^5}{4\pi^3} r^{*3}(1-r^*)^2(w^2-1)^{\frac{1}{2}}\eta_C|\eta_{EW}|^2\chi(w)|\mathcal{F}(w)|^2$$

In general, $\bar{B}\to D^*$ form factor $\mathcal{F}(w)$ is composed of h_V, h_{A_1}, h_{A_2}, and h_{A_3}. At zero recoil ($w=1$, $\mathbf{v}=\mathbf{v}'$), it is reduced to the single form factor: $\chi(1)=1$ and $\mathcal{F}(1) = h_{A_1}(1)$.

$\bar{B} \to D^*$ at zero recoil: $h_{A_1}(1)$ and R

[C. Bernard et al. (Fermilab Lattice and MILC collab.), PRD79, 014506 (2009)]

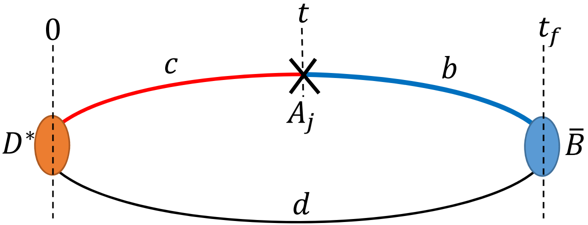
$$|h_{A_1}(1)|^2 = \frac{\langle D^*|A^j_{cb}|\bar{B}\rangle\langle\bar{B}|A^j_{bc}|D^*\rangle}{\langle D^*|V^4_{cc}|D^*\rangle\langle\bar{B}|V^4_{bb}|\bar{B}\rangle}$$

On the lattice, we calculate

$$\left|\frac{h_{A_1}(1)}{\rho_{A_j}}\right|^2 \leftrightarrow R(t,t_f) \equiv \frac{C^{B\to D^*}_{A_1}(t,t_f)\,C^{D^*\to B}_{A_1}(t,t_f)}{C^{B\to B}_{V_4}(t,t_f)\,C^{D^*\to D^*}_{V_4}(t,t_f)}$$

where ρ_{A_j} is the matching factor and $C(t,t_f)$ are the lattice 3-point functions.

Lattice 3-point functions



$$C_{A_1}^{B\to D^*}(t, t_f) = \sum_{\vec{x},\vec{y}} \langle O^\dagger_{D^*}(0) A_1^{cb}(\vec{y}, t) O_B(\vec{x}, t_f)\rangle \qquad (0 < t < t_f)$$

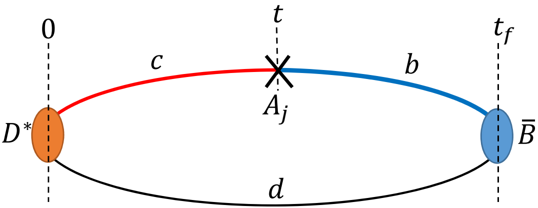
Interpolating operators

$$O_B = \bar{\psi}_b \gamma_5 \psi_l, \qquad O_{D^*} = \bar{\psi}_c \gamma_j \psi_l$$

Improved current

$$A_j^{cb} = \bar{\Psi}_c \gamma_j \gamma_5 \Psi_b,$$

Lattice 3-point functions: current improvement

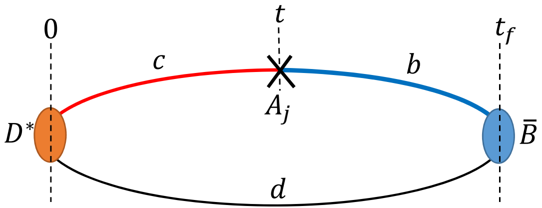


$$A_j^{cb} = \bar{\Psi}_c \gamma_j \gamma_5 \Psi_b,$$

The $\mathcal{O}(\lambda^3)$ improved field with 9 nonzero rotation parameters (d_i): [Jaehoon Leem, Poster session]

$$\begin{aligned}\Psi(x) = e^{M_1/2}\Big[1 \\ &+ d_1 \boldsymbol{\gamma}\cdot\boldsymbol{D} + d_2 \Delta^{(3)} + i d_B \boldsymbol{\Sigma}\cdot\boldsymbol{B} &&\to \mathcal{O}(\lambda^1) \\ &+ d_E \boldsymbol{\alpha}\cdot\boldsymbol{E} + d_{rE}\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} &&\to \mathcal{O}(\lambda^2) \\ &+ d_3 \gamma_i D_i \Delta_i + d_4 \{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \Delta^{(3)}\} &&\to \mathcal{O}(\lambda^3) \\ &+ d_5 \{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \boldsymbol{\Sigma}\cdot\boldsymbol{B}\} + d_{EE}\{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\}\Big]\psi(x).\end{aligned}$$

Lattice 3-point functions: t and t_f dependence



$$C_{A_1}^{B\to D^*}(t,t_f)=\sum_{\vec{x},\vec{y}}\langle O_{D^*}^\dagger(0)A_1^{cb}(\vec{y},t)O_B(\vec{x},t_f)\rangle \qquad (0<t<t_f)$$

$$=A_{00}^{B\to D^*}e^{-M_B^{(0)}t_f}e^{(M_B^{(0)}-M_{D^*}^{(0)})t}\left[1+c^{B\to D^*}(t,t_f)+\cdots\right]$$

and we define the leading effects of the opposite parity

$$c^{B\to D^*}(t,t_f)\equiv(-1)^{(t_f-t)/a}\frac{A_{10}^{B\to D^*}}{A_{00}^{B\to D^*}}e^{-\Delta M_B(t_f-t)}+(-1)^{t/a}\frac{A_{01}^{B\to D^*}}{A_{00}^{B\to D^*}}e^{-\Delta M_{D^*}t}$$
$$+(-1)^{t_f/a}\frac{A_{11}^{B\to D^*}}{A_{00}^{B\to D^*}}e^{-\Delta M_{D^*}t}e^{-\Delta M_B(t_f-t)},$$

where $\Delta M_B=M_B^{(1)}-M_B^{(0)}$, $\Delta M_{D^*}=M_{D^*}^{(1)}-M_{D^*}^{(0)}$.

Double ratio $R(t, t_f)$, $\bar{R}(t, t_f)$

$$\left|\frac{h_{A_1}(1)}{\rho_{A_j}}\right|^2 \leftrightarrow R(t, t_f) \equiv \frac{C_{A_1}^{B\to D^*}(t, t_f) C_{A_1}^{D^*\to B}(t, t_f)}{C_{V_4}^{B\to B}(t, t_f) C_{V_4}^{D^*\to D^*}(t, t_f)}$$

$$= \frac{A_{00}^{B\to D^*} A_{00}^{D^*\to B}}{A_{00}^{B\to B} A_{00}^{D^*\to D^*}}[1 + c^{B\to D^*}(t, t_f) + c^{D^*\to B}(t, t_f) - c^{B\to B}(t, t_f) - c^{D^*\to D^*}(t, t_f) \cdots].$$

[C. Bernard et al. (Fermilab Lattice and MILC collab.), PRD79, 014506 (2009)]

$$\bar{R}(t, t_f) \equiv \frac{1}{2} R(t, t_f) + \frac{1}{4} R(t, t_f + 1) + \frac{1}{4} R(t + 1, t_f + 1)$$

$$= \frac{A_{00}^{B\to D^*} A_{00}^{D^*\to B}}{A_{00}^{B\to B} A_{00}^{D^*\to D^*}}[1 + \bar{c}^{B\to D^*}(t, t_f) + \bar{c}^{D^*\to B}(t, t_f) - \bar{c}^{B\to B}(t, t_f) - \bar{c}^{D^*\to D^*}(t, t_f) + \cdots]$$

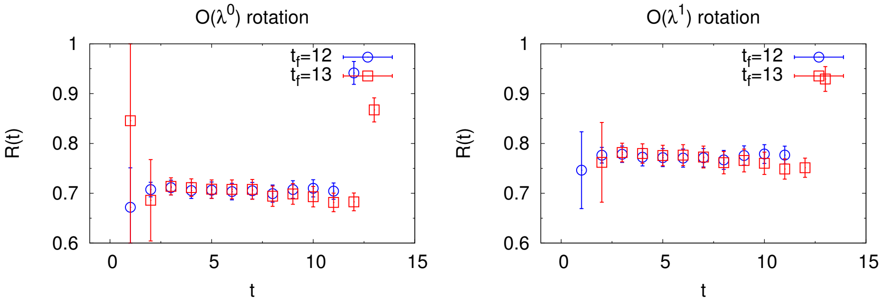
Simulation details

- 3 source-time slices with coherent sequential inversion
- HISQ propagators with point source
- OK propagators with covariant Gaussian smearing $(\sigma, N) = (1.5, 5)$.

Ensemble	Volume	$a\hat{m}'/am'_s/am'_c$	$M_\pi^P L$	N_{conf}	N_{conf}^{meas*}	N_{conf}^{meas**}
a15m310	$16^3 \times 48$	0.0013/0.065/0.838	3.78	1021	-	-
a15m220	$24^3 \times 48$	0.0064/0.064/0.828	3.99	1000	-	-
a15m130	$32^3 \times 48$	0.00235/0.0647/0.831	3.30	1020	-	-
a12m310	$24^3 \times 64$	0.0102/0.0509/0.635	4.54	1040	300	100
a12m220	$32^3 \times 64$	0.00507/0.0507/0.628	4.29	1000	-	-
a12m130	$48^3 \times 64$	0.00184/0.0507/0.628	3.88	1000	-	-
a09m310	$32^3 \times 96$	0.0074/0.037/0.440	4.50	1011	100	-
a09m220	$48^3 \times 96$	0.00363/0.0363/0.430	4.71	1000	-	-
a09m130	$64^3 \times 96$	0.0012/0.0363/0.432	3.66	1031	-	-

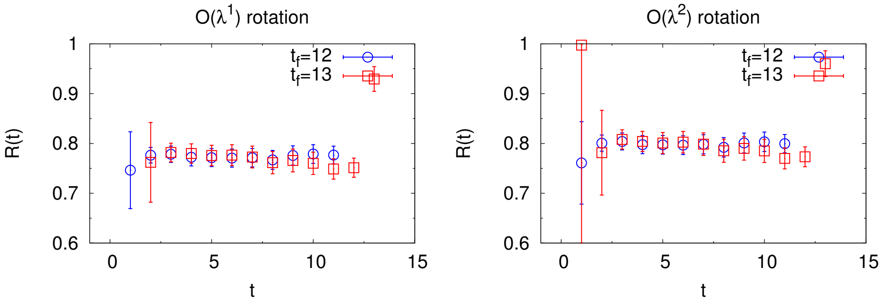
- (*) Number of measured configurations using tree-level $\kappa_{\text{crit}}^{\text{tree}}$
- (**) Number of measured configurations using nonperturbative κ_{crit}

Double ratio $R(t, t_f)$: a12m310



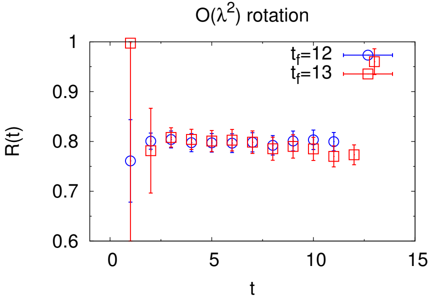
(LEFT) Double ratio R with unimproved current operator
(RIGHT) Double ratio R with $\mathcal{O}(\lambda)$ improved current operator

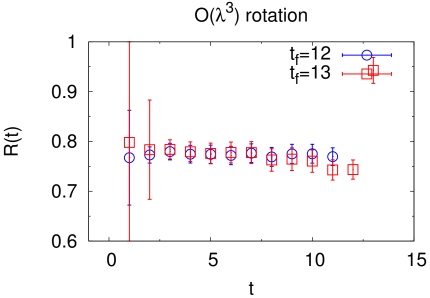
Double ratio $R(t, t_f)$: a12m310



(LEFT) Double ratio R with $\mathcal{O}(\lambda)$ improved current operator
(RIGHT) Double ratio R with $\mathcal{O}(\lambda^2)$ improved current operator

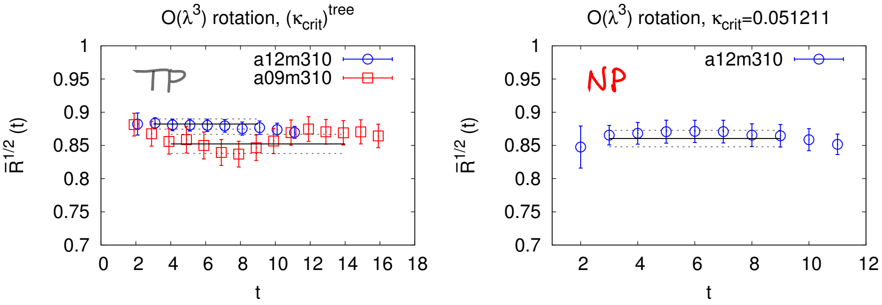
Double ratio $R(t, t_f)$: a12m310





(LEFT) Double ratio R with $\mathcal{O}(\lambda^2)$ improved current operator
(RIGHT) Double ratio R with $\mathcal{O}(\lambda^3)$ improved current operator

$\bar{R}^{1/2}(t, t_f) \leftrightarrow h_{A_1}(1)/\rho_{A_j}$: preliminary result



Ensemble	Dimension	κ_{crit}	t_f	$h_{A_1}(1)/\rho_{A_j}$
a09m310	$32^3 \times 96$	$0.053207\cdots$ (tree)	17, 18	0.852(14)
a12m310	$24^3 \times 64$	$0.053850\cdots$ (tree)	12, 13	0.8822(76)
a12m310	$24^3 \times 64$	0.051211 (nonperturbative)	12, 13	0.860(13)

Summary

- We determine a nonperturbative κ_{crit} for the OK action.
- Using nonperturbative κ_{crit}, we tune the κ_b and κ_c.
- We produced V_{cb} related 3-point functions, and obtained $h_{A_1}(1)/\rho_{A_j}$.

To be done

- Perturbative calculation of ρ_{A_j}
- Chiral-continuum extrapolation
- Extending measurement to fine, superfine, and ultrafine ensembles.
- Accumulating more statistics