Computing Nucleon EDM on a Lattice

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Taku Izubuchi, Hiroshi Ohki, ***Sergey Syritsyn***

based on arXiv:1701.07792
(accepted to PRD as Editor's suggestion)

LATTICE 2017
Granada, Spain, June 18-24, 2017

BROOKHAVEN NATIONAL LABORATORY
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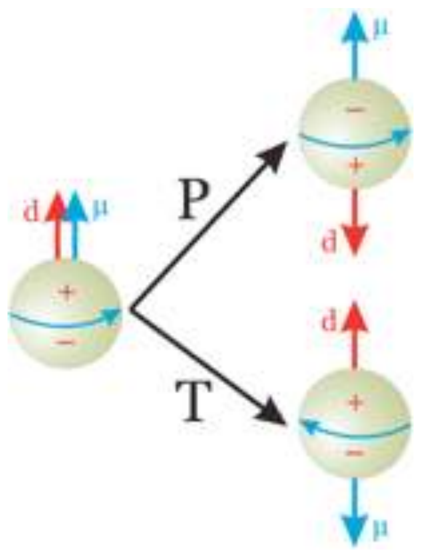
Outline

- Nucleon EDM: theory and experiment
- Nucleon EDM calculations on a lattice, corrected
- nEDM induced by quark chromo-EDM
- EDMs in Background Electric Field

Electric Dipole Moments of Nucleons



Motivations to search for new CP-odd interactions

- Evidence for SM Extensions
- Baryogenesis Requirement
- Strong CP problem (θ_{QCD})

$$\vec{d}_N = d_N \frac{\vec{S}}{S}$$

$$\mathcal{H} = -\vec{d}_N \cdot \vec{E}$$

$$\langle N|V_\mu(q)|N\rangle = \bar{u}_N \Big[\gamma_\mu F_1(q^2) + i\frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \frac{F_2(q^2)}{2m_N} + (2im_N\gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2)\frac{F_A(q^2)}{m_N^2} + \frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_N}\Big] u_N$$

$\not{P}$ anapole form factor

$\not{P}$, $\not{CP}$ EDM form factor

Role of Lattice QCD : connect quark/gluon-level (effective) operators to hadron/nuclei matrix elements and interactions

$$\mathcal{L}_{eff} = \sum_n \frac{c_n}{\Lambda^{d_n-4}} \mathcal{O}_n^{(d_n)}$$

$$\begin{cases} \mathcal{L}^{(4)} = \theta \frac{g^2}{32\pi^2} G\tilde{G} & \text{(QCD theta-angle)} \\ \mathcal{L}^{("6")} = \sum_q \left[d_q\, \bar{q}(F\cdot\sigma)\gamma_5 q + \tilde{d}_q \bar{q}(G\cdot\sigma)\gamma_5 q\right] & \text{Quark (chromo-)EDM} \\ \ldots & \text{(3-gluon, 4-quark, etc)} \end{cases}$$

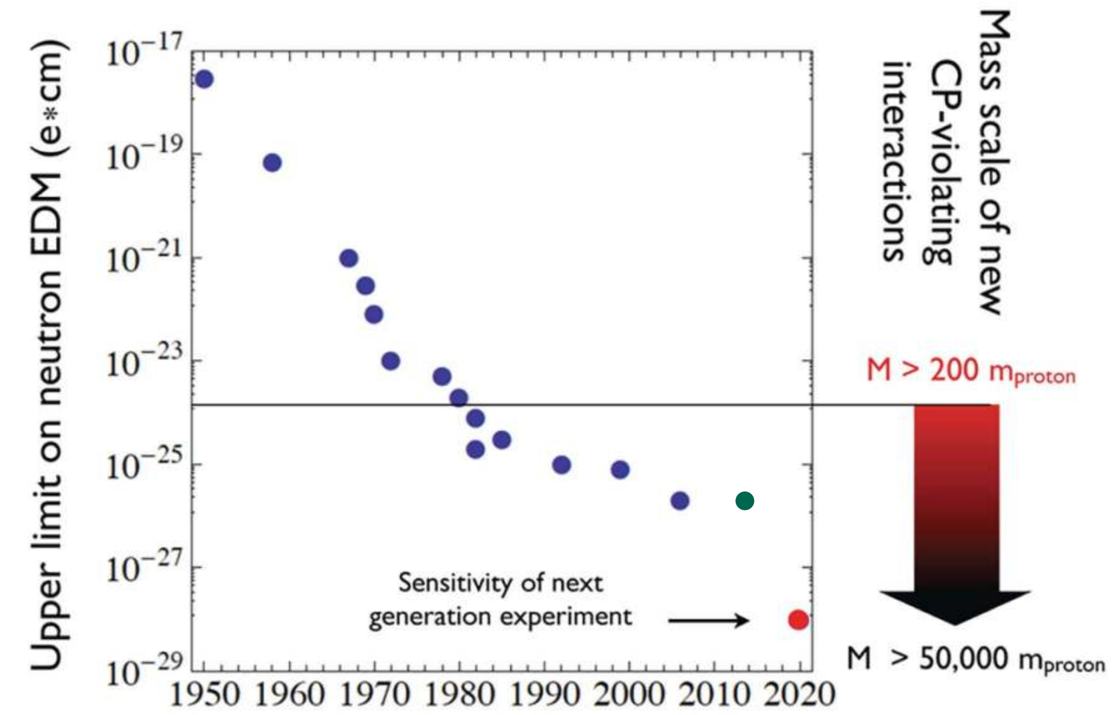
$\Rightarrow$

$$d_{n,p}$$
$$F_3^{n,p}(Q^2)$$

Experimental Outlook: Neutron EDM

	10^{-28} *e* cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	< *0.001*

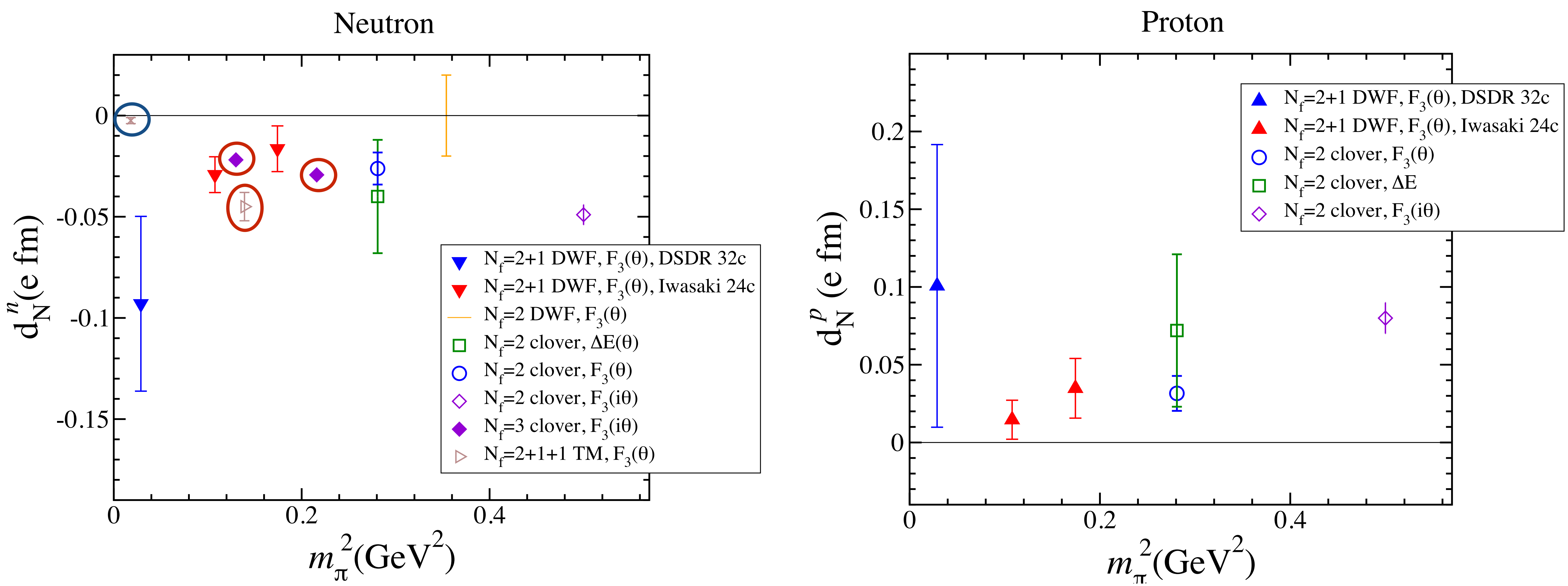
[B.Filippone's talk, KITP 2016]



nEDM sensitivity :

- 1–2 years : next best limit
- 3–4 years : x10 improvement
- 7–9 years : x100 improvement

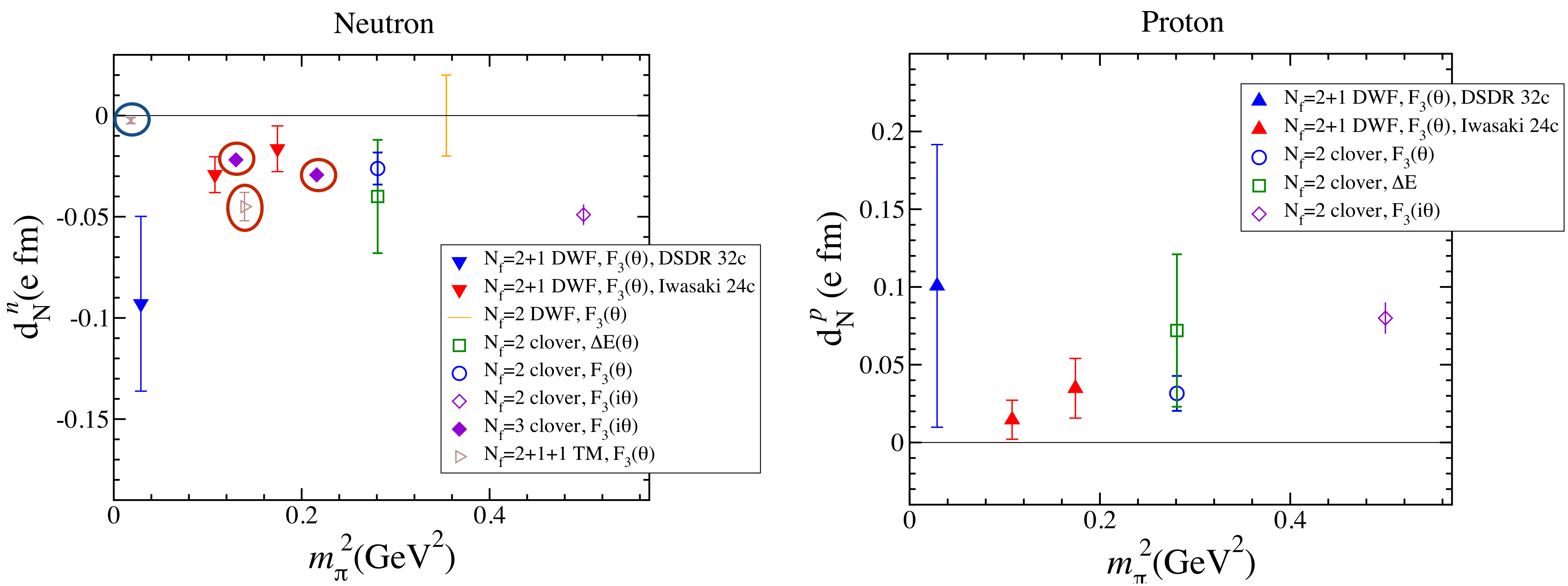
θ_{QCD}-induced Nucleon EDM



[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- Phenomenology: $|d_n| \simeq \theta_{QCD} \times$(few 10^{-3} e fm) $\Longrightarrow |\theta_{QCD}| \lesssim 1.5\times10^{-10}$
- Lattice : $|d_n| \simeq \theta_{QCD}\times$(few 10^{-2} e fm) $\Longrightarrow$ tighter constraint on θ_{QCD} ?

θ_{QCD}-induced Nucleon EDM



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Unfortunately, there is a problem:
misinterpreted mixing between electric and magnetic moments

Nucleon "Parity Mixing" on/off a Lattice

Lattice nucleon operator $N = u\,[u^T C\gamma_5 d]$

Ground state in CPv vacuum $\langle \mathrm{vac}|N|p,\sigma\rangle_{\not{CP}} = \tilde{u}_{p,\sigma} = e^{i\alpha\gamma_5}\,u_{p,\sigma}$

Solutions to
$(\not{\partial} + m_N e^{-2i\alpha\gamma_5})\tilde{u}_p = 0$
$\gamma_4\tilde{u} = e^{-2i\alpha\gamma_5}\tilde{u}$ at rest

Solutions to
$(\not{\partial} + m_N)u_p = 0$
$\gamma_4 u = u$ at rest

Nucleon propagator
$$\langle N(t)\bar{N}(0)\rangle_{\not{CP}} = e^{-E_N t} e^{i\alpha\gamma_5}\frac{-i\not{p}_{\mathcal{E}} + m_N}{2E_N}e^{i\alpha\gamma_5}$$
$$\sim \frac{-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}}{2E_N} = \sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma}$$

The mixing phase α has to be calculated and removed by field redefinition
(Similar issues appear in EFT (ChPT) calculations with effective due to baryon CPv $\propto \overline{B}\,\gamma_5\,B$*)*

Nucleon "Parity Mixing" : EDM and aMDM

Nucleon-current correlator spin structure in the **original** works [S.Aoki et al (2005), ...]

$$\langle N_{p'}\, J^\mu\, \bar{N}_p \rangle_{\not{CP}} \overset{?}{\sim} \underbrace{\Big(\sum_{\sigma'} \tilde{u}_{\sigma'} \bar{\tilde{u}}_{\sigma'}\Big)_{p'} \Gamma^\mu \Big(\sum_{\sigma} \tilde{u}_{\sigma} \bar{\tilde{u}}_{\sigma}\Big)_{p}}_{\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]}$$

Correct spin structure [SNS, S.Aoki, et al (2017)]

$$\langle N_{p'}\, J^\mu\, \bar{N}_p \rangle_{\not{CP}} \sim \sum_{\sigma',\sigma} \tilde{u}_{p',\sigma'} \underbrace{\langle p',\sigma' | J^\mu | p,\sigma\rangle}_{\bar{u}_{p',\sigma'}\Gamma^\mu u_{p,\sigma} \neq \bar{\tilde{u}}_{p',\sigma'}\Gamma^\mu \tilde{u}_{p,\sigma}} \bar{\tilde{u}}_{p,\sigma}$$

Chiral rotation results in "rotation" in the $F_{2,3}$ plane

$$e^{i\alpha\gamma_5}\Gamma^\mu e^{i\alpha\gamma_5} \leftrightarrow \Gamma^\mu$$

$$e^{2i\alpha}(\text{“}F_2\text{”} + i\text{“}F_3\text{”}) = (F_2 + iF_3)_{\text{true}}$$

... and spurious contributions to electric dipole moment $F_3(0)$

$$\begin{cases} \text{“}F_2\text{”} &= [\cos(2\alpha)F_2 + \sin(2\alpha)F_3]_{\text{true}} \\ \text{“}F_3\text{”} &= [\cos(2\alpha)F_3 - \sin(2\alpha)F_2]_{\text{true}} \end{cases}$$

... so for small CPv

$$\text{“}F_3\text{”} \approx [F_3]_{\text{true}} - 2\alpha[F_2]_{\text{true}}$$

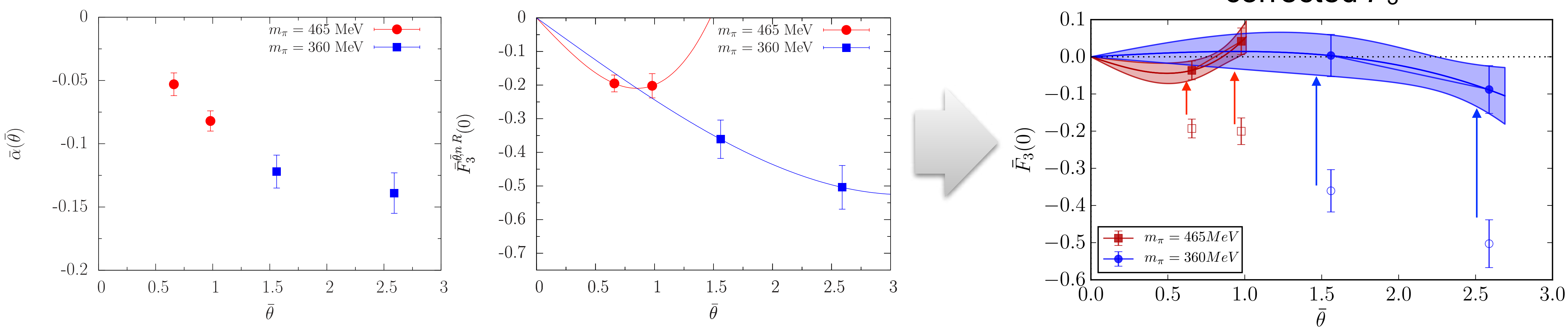
$$\text{“}d_{n,p}\text{”} \approx [d_{n,p}]_{\text{true}} - 2\alpha\frac{\kappa_{n,p}}{2m_N}$$

Previous Lattice Results on θ_{QCD}-induced nEDM

Correction is simple: $[F_3]_{\rm true} = \text{“}F_3\text{”} + 2\alpha F_2$

- [F. Guo *et al* (QCDSF), PRL115:062001 (2015)]

  dynamical calculations with finite imag. θ^I angle



- [C.Alexandrou *et al* (ETMC), PRD93:074503 (2016)]

  d_n=–0.045(06) *e* fm (~7.5σ) → +0.008(6) *e* fm (1.3σ)

  + zero result confirmed by the authors

- [E.Shintani *et al*, D78:014503 (2008)],

  uniform Minkowski-real bg. electric field: **not affected** by the spinor "parity mixing"

  d_n=–0.040(28) *e* fm (~1.4σ) at m_π≈530 MeV; *Precision is insufficient for comparison*

After removing spurious contributions, θ_{QCD}-induced nEDM on a lattice is much smaller
The conflict with phenomenology value and m_q scaling disappears

Nucleon "Parity Mixing" : EDM and aMDM

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\cancel{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

Correct assignment of $F_{2,3}$ [SNS, S.Aoki, *et al* (2017) arXiv:1701.07792]

- coupling of spin to E,B in the forward limit $p, p' \to (\vec{p}^{(\prime)}, m_N)$

$$\langle H_{\rm int}\rangle = eA_\mu\langle J^\mu\rangle = -\frac{eG_M(0)}{2m_N}\vec{\Sigma}\cdot\vec{H} - \frac{eF_3(0)}{2m_N}\vec{\Sigma}\cdot\vec{E}$$

$$\mu = \frac{e}{2m_N}G_M(0), \quad d = \frac{e}{2m_N}F_3(0)$$

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$$\mu = \frac{e}{2m_N}G_M(0), \qquad d = \frac{e}{2m_N}F_3(0)$$

- parity of the vector current matrix element: $F_{1,2}$ P,T-even, F_3 P,T-odd

$$\begin{cases} u_{\vec{p},\sigma} \to u_{-\vec{p},\sigma} = \gamma_4\, u_{\vec{p},\sigma} \\ F_{1,2} \to (+F_{1,2}), \quad F_3 \to (-F_3) \end{cases}$$

vs.

$$\begin{cases} \tilde{u}_{\vec{p},\sigma} \to \tilde{u}_{-\vec{p},\sigma} = e^{2i\alpha\gamma_5}\gamma_4\, \tilde{u}_{\vec{p},\sigma} \\ e^{2i\alpha}(``F_2" + i``F_3") \to e^{-2i\alpha}(``F_2" - i``F_3") \end{cases}$$

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- poles of the Dirac equation with CPv nucleon mass in bg. electric & magnetic fields

$$\mathcal{L}_N = \bar{N}\left[i\not{\partial} - me^{-2i\alpha\gamma_5} - Q\gamma_\mu A^\mu - (\tilde{\kappa} + i\tilde{\zeta}\gamma_5)\frac{1}{2}F_{\mu\nu}\frac{\sigma^{\mu\nu}}{2m_N}\right]N$$

$$E_N(\vec{p}=0) - m_N = -\frac{\kappa}{2m_N}\vec{\Sigma}\cdot\vec{H} - \frac{\zeta}{2m_N}\vec{\Sigma}\cdot\vec{E} + O(\kappa^2, \zeta^2)$$

where $\kappa + i\zeta = e^{2i\alpha\gamma_5}(\tilde{\kappa} + i\tilde{\zeta})$

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- poles of the Dirac equation with CPv nucleon mass in bg. electric & magnetic fields

$$\mathcal{L}_N = \bar{N}\big[i\not{\partial} - me^{-2i\alpha\gamma_5} - Q\gamma_\mu A^\mu - (\tilde{\kappa} + i\tilde{\zeta}\gamma_5)\frac{1}{2}F_{\mu\nu}\frac{\sigma^{\mu\nu}}{2m_N}\big]N$$

$$\rightarrow E_N(\vec{p}=0) - m_N = -\frac{\kappa}{2m_N}\vec{\Sigma}\cdot\vec{H} - \frac{\zeta}{2m_N}\vec{\Sigma}\cdot\vec{E} + O(\kappa^2, \zeta^2)$$

where $\kappa + i\zeta = e^{2i\alpha\gamma_5}(\tilde{\kappa} + i\tilde{\zeta})$

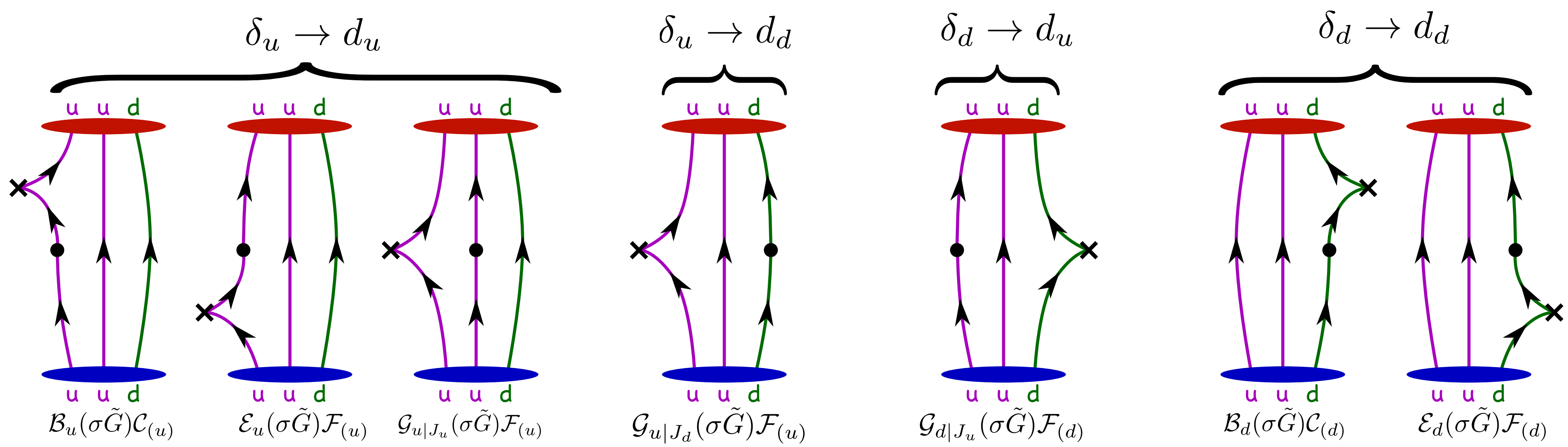
- Numerical test: compare EDFF with mass shift in a uniform bg. electric field

This work: Quark chromo-EDM

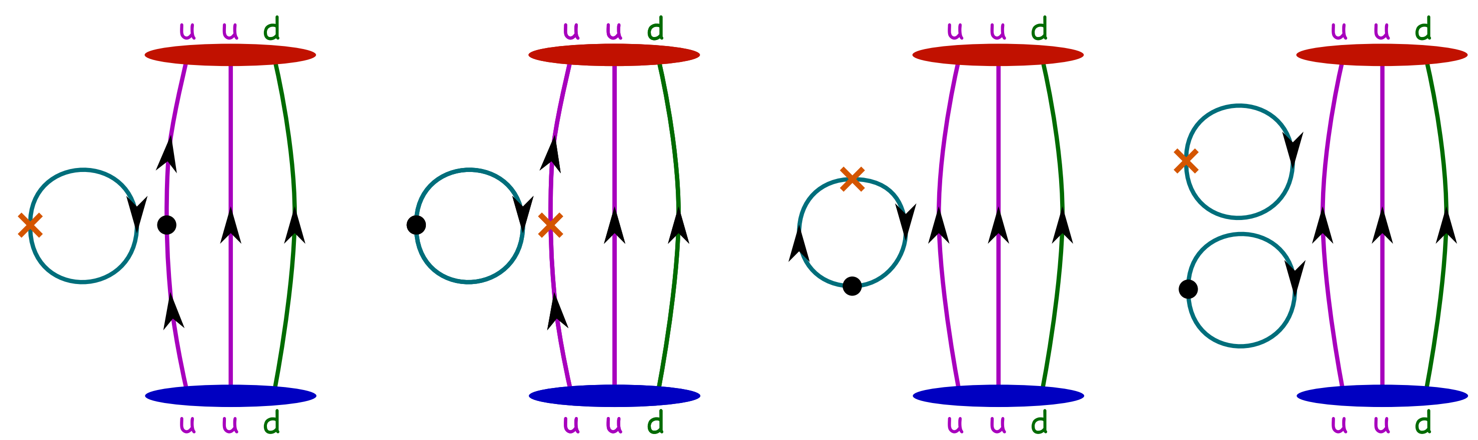
$$\mathcal{L}^{(5)} = \sum_q \tilde{d}_q\, \bar{q}(G\cdot\sigma)\gamma_5 q \quad \longrightarrow \quad \begin{cases} \langle N(y)\,\bar{N}(0) \int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle \\ \langle N(y)\,[\bar{\psi}\gamma^\mu\psi]_z\,\bar{N}(0) \int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle \end{cases}$$

First calculations : [T.Bhattacharya et al(LANL, LATTICE'15,'16]

- So far: Only quark-connected insertions

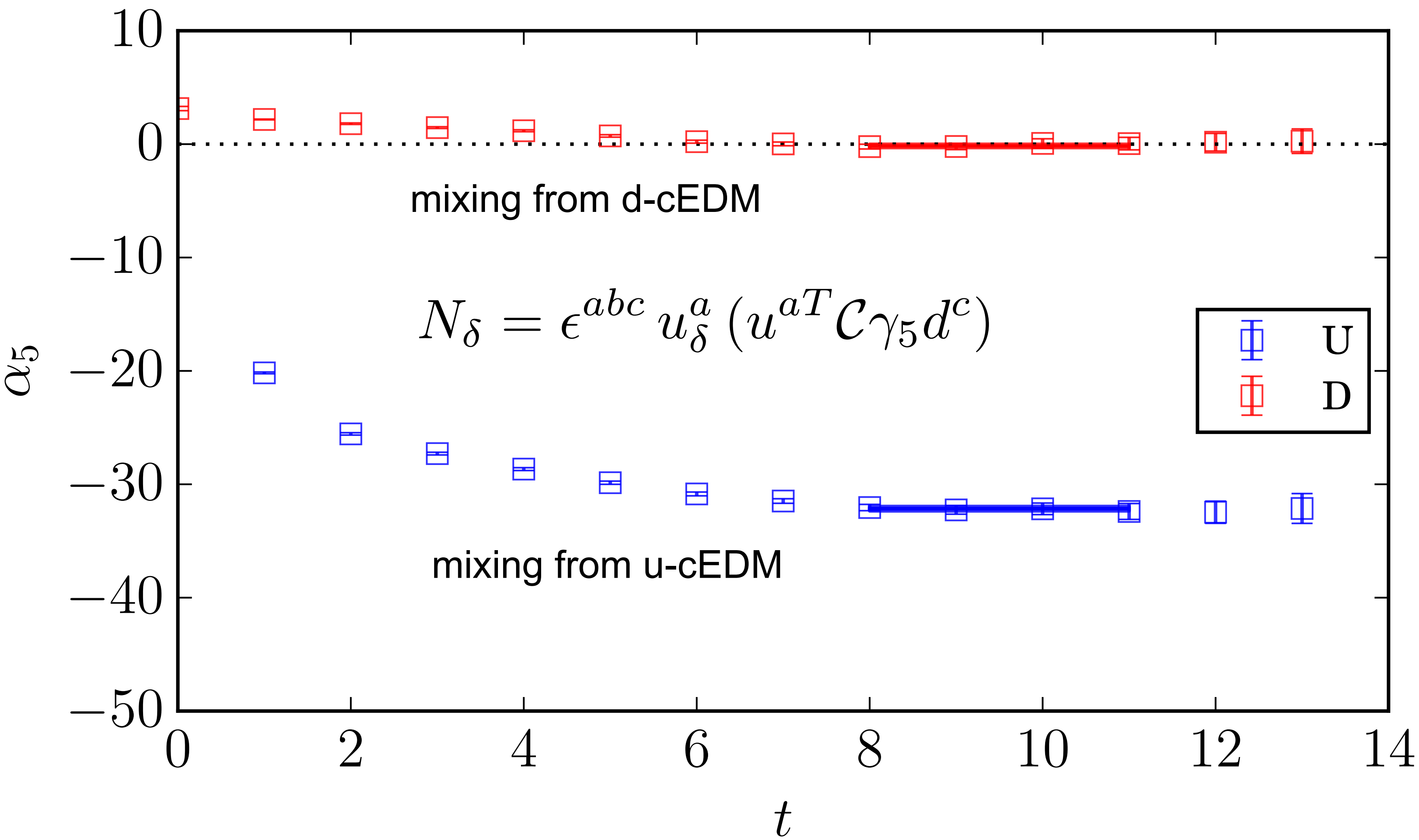


- Future (hopefully): Single- and double-disconnected diagrams (contribute to isosinglet cEDM, mix with θ-term)



Parity Mixing (Proton)

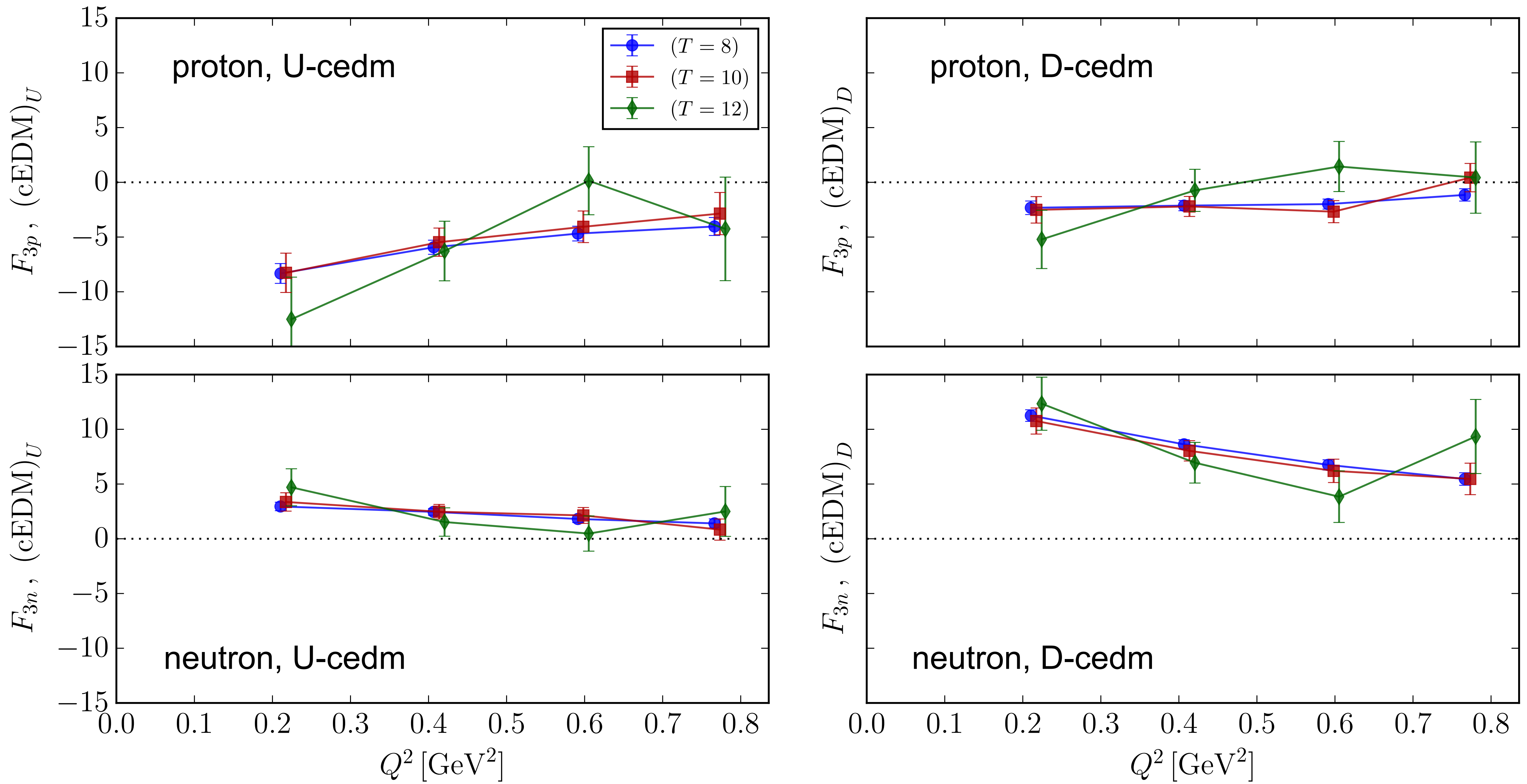
$$\langle N(t)\bar{N}(0)\rangle_{\not{CP}} = \frac{-i\not{p} + m_N e^{2i\alpha_5\gamma_5}}{2m_N} e^{-E_N t}$$



$$\hat{\alpha}_5 = \frac{\alpha_5}{\tilde{d}} = -\frac{\mathrm{ReTr}\left[T^+\gamma_5 \cdot C_{2pt}^{\overline{CP}}(t)\right]}{\mathrm{ReTr}\left[T^+ \cdot C_{2pt}^{CP}(t)\right]}\,, \quad t\to\infty$$

Need large mixing angle to see the effect of the correction

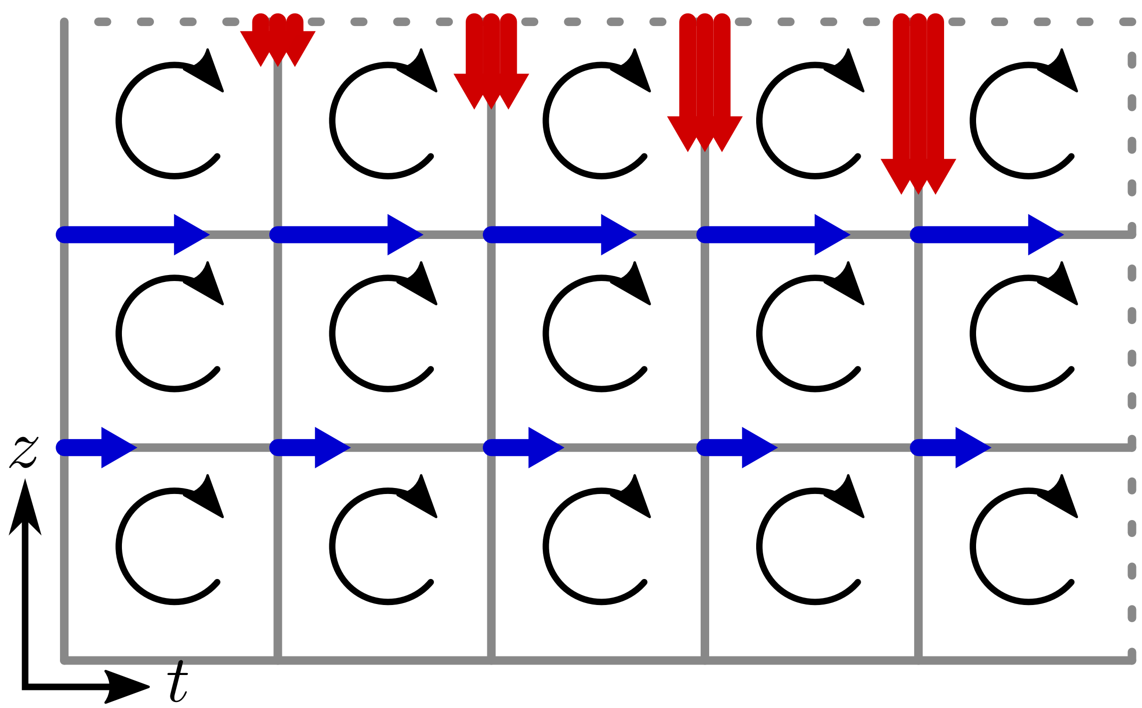
Current Results on cEDM-induced nEDM



- connected-only
- bare cEDM operators on a lattice (no renormalization/mixing subtraction)
- statistics = 10,500 samples on 24^3x64 m_π = 330 MeV DW ensemble

Background Electric Field

Accessing magnetic and electric moments at Q^2=0
Imag.Minkowski/Real Euc. electric field on a lattice
[W.Detmold et al (2009)] : calculation of hadron polarizabilities



$$U_\mu \to e^{iqA_\mu} U_\mu$$

$$A_z(z,t) = n\,\mathcal{E}_{\rm min} \cdot t$$

$$A_t(z, t = L_t - 1) = -n\,\mathcal{E}_{\rm min} \cdot L_t z$$

Full flux through the "side" of the periodic box

$$= q\Phi = 2\pi \cdot n$$

Constant Electric field has to be quantized,

$$\mathcal{E}_{\rm min} = \frac{1}{|q_d|}\frac{2\pi}{L_x L_t}$$

Electric field on a 24^3x64 lattice

$$\mathcal{E} = \frac{6\pi}{L_x L_t} \approx 0.037\ \mathrm{GeV}^2$$
$$\approx 186\ \mathrm{MV/fm}$$

Unambiguous determination of EDM from the energy shift

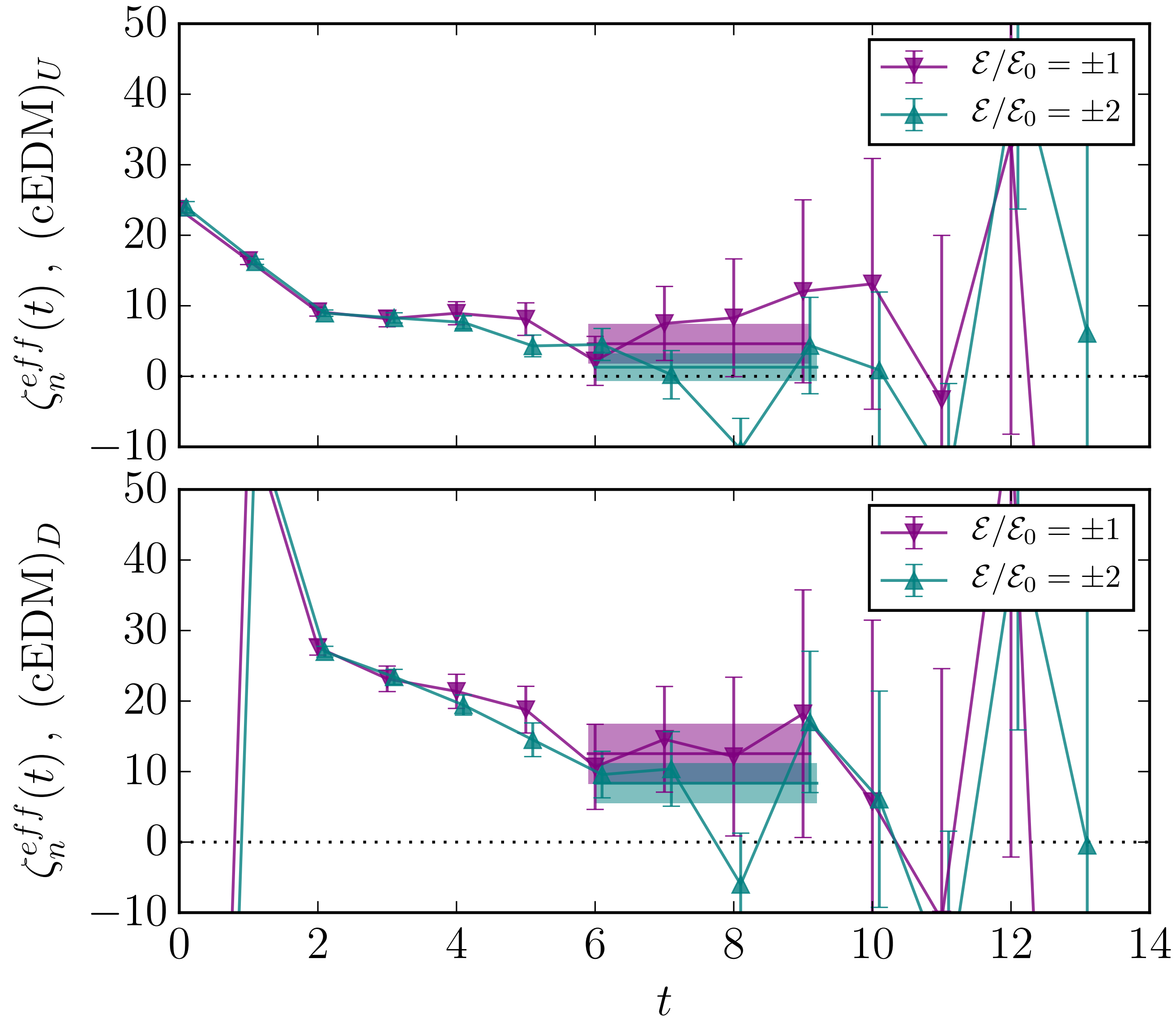
CP-odd Neutron Energy Shift

$$\langle N(t)\bar{N}(0)\,\mathcal{O}_{\overline{CP}}\rangle_{\mathcal{E}} \sim e^{-E_N t}\left[A - d_N \mathcal{E}_z \Sigma_z t\right]$$

$$f(t,\mathcal{E}) = \frac{\mathrm{ReTr}\left[\Sigma_z\cdot\langle N(t)\bar{N}(0)\,\mathcal{O}_{\overline{CP}}\rangle_{\mathcal{E}}\right]}{\mathrm{ReTr}\left[\langle N(t)\bar{N}(0)\rangle_{\mathcal{E}}\right]} \sim A + d_N \mathcal{E} t$$

$$\zeta_n^{\mathrm{eff}}(t) = 2m_N d_n^{\mathrm{eff}}(t) = \frac{2m_N}{\mathcal{E}_z}\frac{df}{dt}$$

Only Q=0 (neutron) has a simple correlator form



- $(2.7\ \mathrm{fm})^3$ x (7.3 fm)
- m_π=330 MeV
- ~3,200 samples

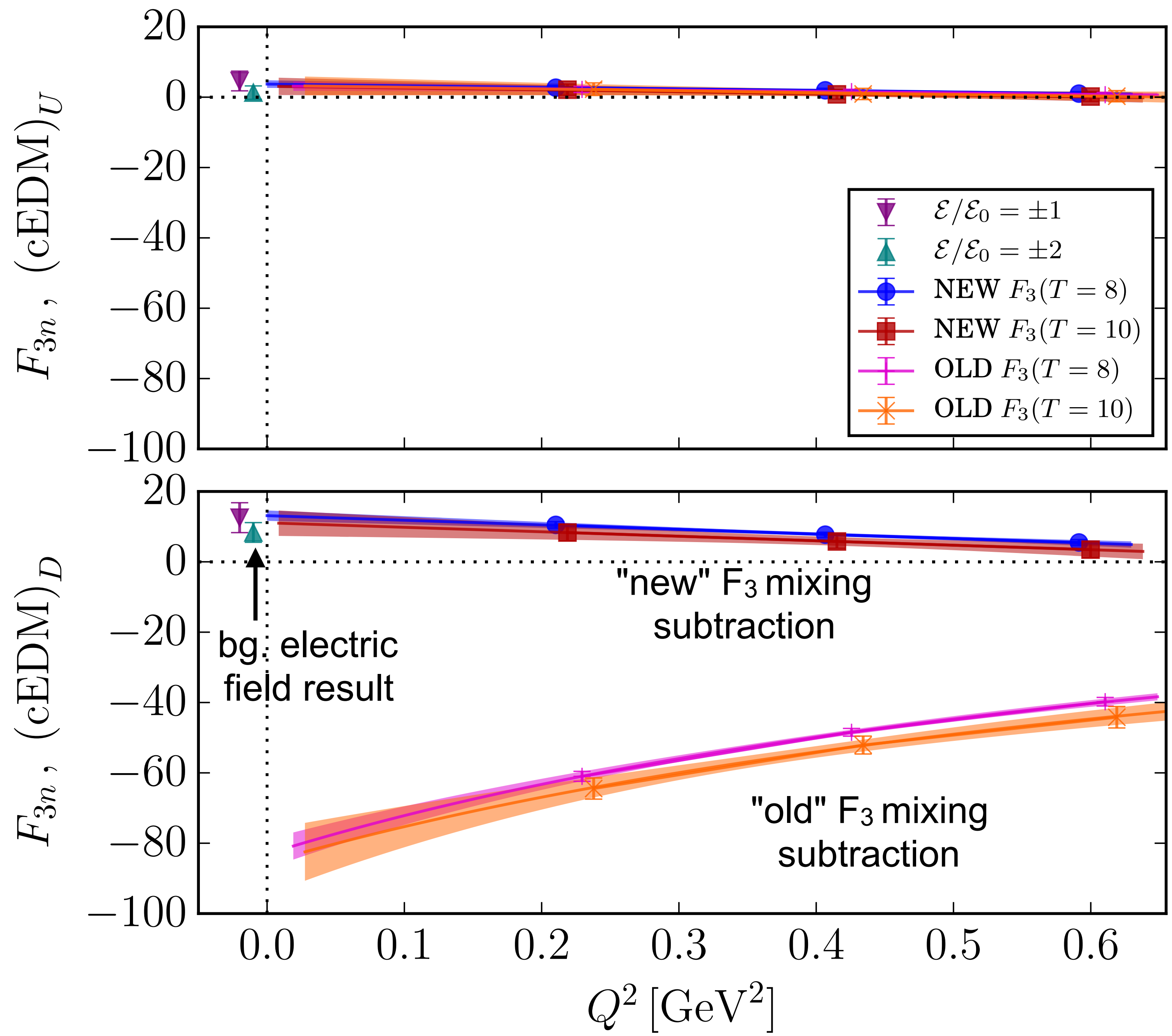
- Linearity in $d_N/\tilde{d}_q$, t , and $\mathcal{E}$
- No renormalization
- Connected-only

Electric field on a 24^3x64 lattice

$$\mathcal{E} = \frac{6\pi}{L_x L_t} \approx 0.037\ \mathrm{GeV}^2$$

$$\approx 186\ \mathrm{MV/fm}$$

Energy Shift vs. Form Factors (Neutron)



Mixing $\alpha_U \approx 0$

No F_2 contribution to F_3

$$\text{“}F_{3n}^U\text{”} \approx [F_{3n}^U]_{\mathrm{true}}$$

Mixing $\alpha_D \approx 30(0.2)$

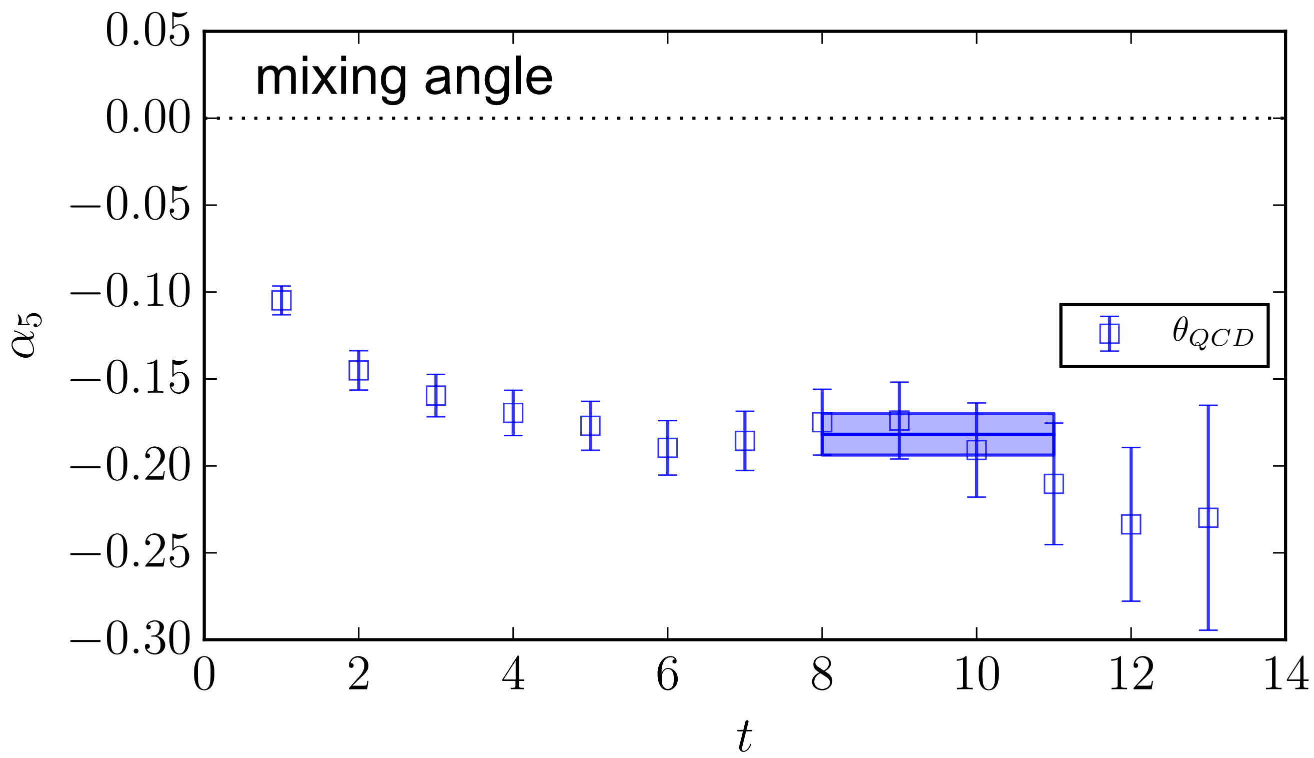
Large F_2 contribution to "F_3"

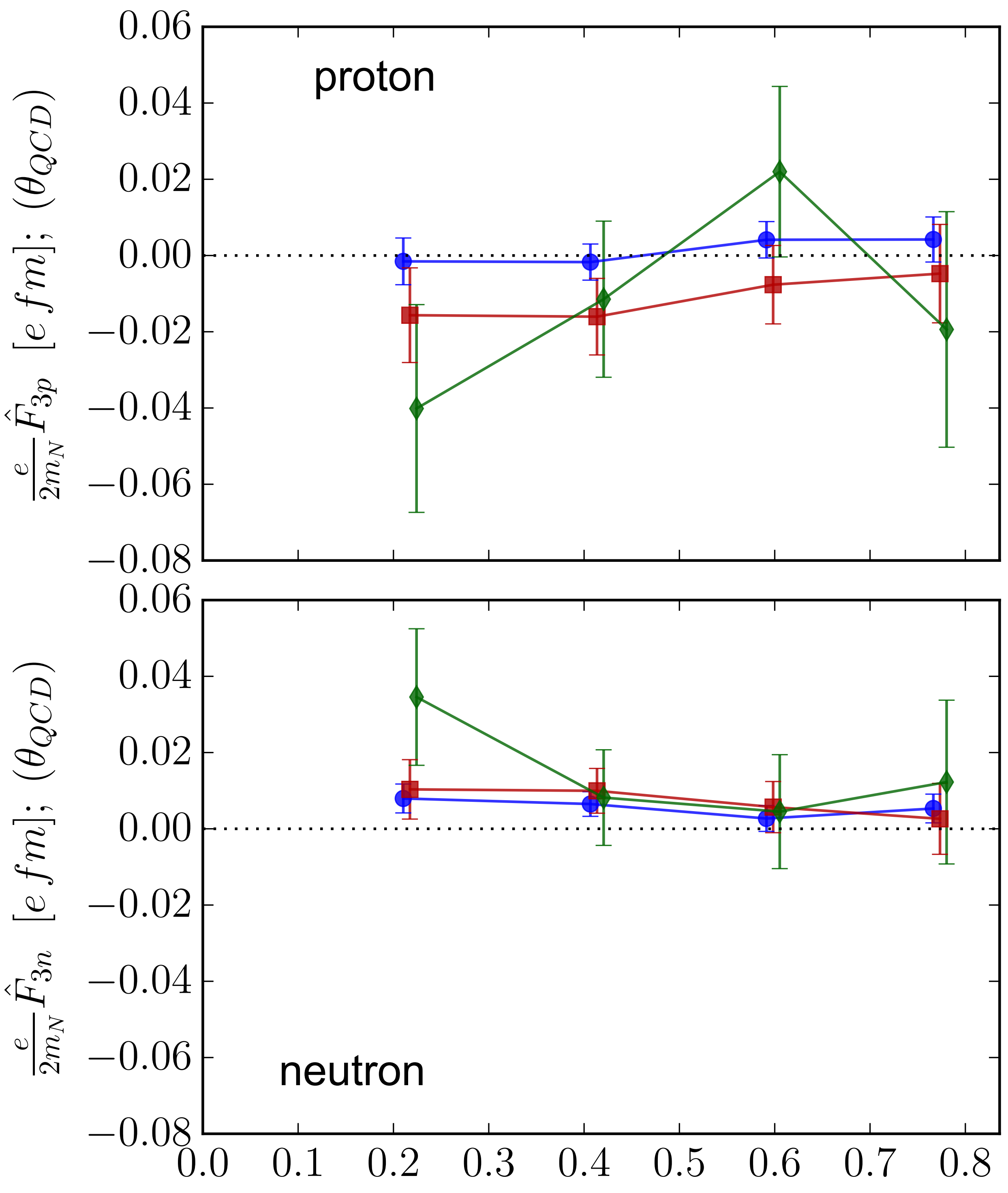
$$\text{“}F_{3n}^D\text{”} = [F_{3n}^D]_{\mathrm{true}} - 2\alpha_D F_{2n}$$

[S.Aoki, SNS, *et al* (2017) arXiv:1701.07792]

Agreement between the ***new*** *F_3 formula and the energy shift method*

θ_{QCD}-induced p,nEDM from F_3 (preliminary)





- 24^3x64 m_π=330 MeV, 10,500 samples
- Full-volume G*Gdual

$$|F_3| \lesssim 0.1 \quad \Rightarrow \quad |d_n|/\theta \lesssim 0.01\, e\cdot \mathrm{fm}$$

ChPT [Pospelov, Ritz, Ann.Phys.318:119]: $\hat{d}_n \sim e\frac{m_\pi^2}{\Lambda_\chi} \approx 0.002\, e\, fm$

QCDSR [Pospelov,Ritz, PRL83:2526]: $\hat{d}_n \approx 0.0025(13)\, e\, fm$

Summary

- Lattice QCD calculations of nEDM are important for the nuclear physics research program on fundamental symmetries
- Existing lattice calculations of θ-induced nEDM contain spurious contributions from mixing with the anomalous mag.moment
  - *most precise recent lattice results are consistent with zero after correction*
  - *⇒ high-statistics calculations of θ-induced nEDM are urgently needed!*
- Promising initial results for *quark-connected* cEDM-induced EDFF
  - *renormalization & mixing subtractions are underway*
- Calculations with background electric field show excellent agreement with the form factor method
  - *does not require momentum extrapolation, may be the method of choice*
- Outlook & Challenges for computing nEDM from cEDM
  - *disconnected diagrams*
  - *nEDM from θ-term*