



Continuum extrapolation of critical point for finite temperature QCD with $N_f=3$

arXiv:1706.01178

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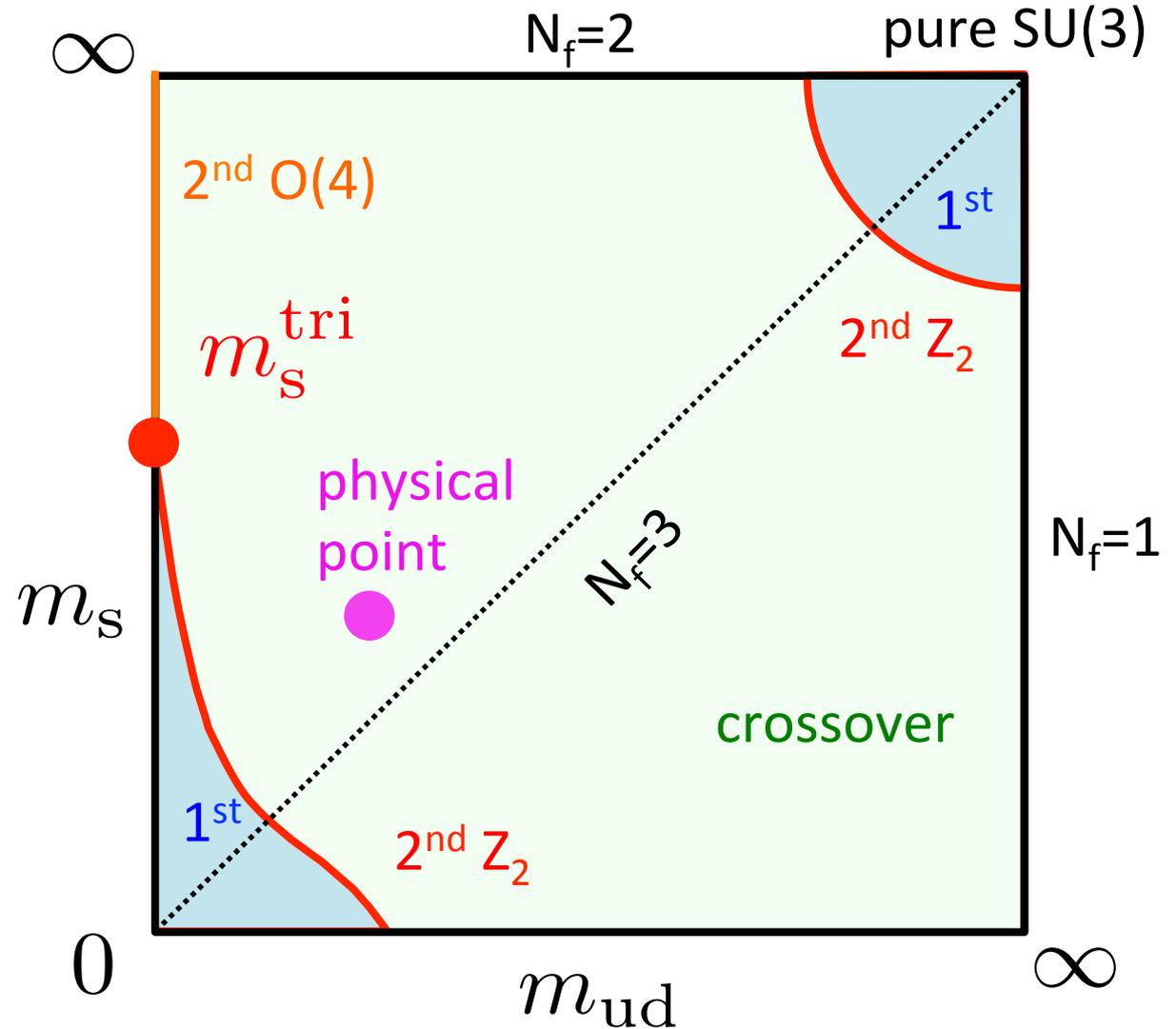
(Kanazawa University)

in collaboration with

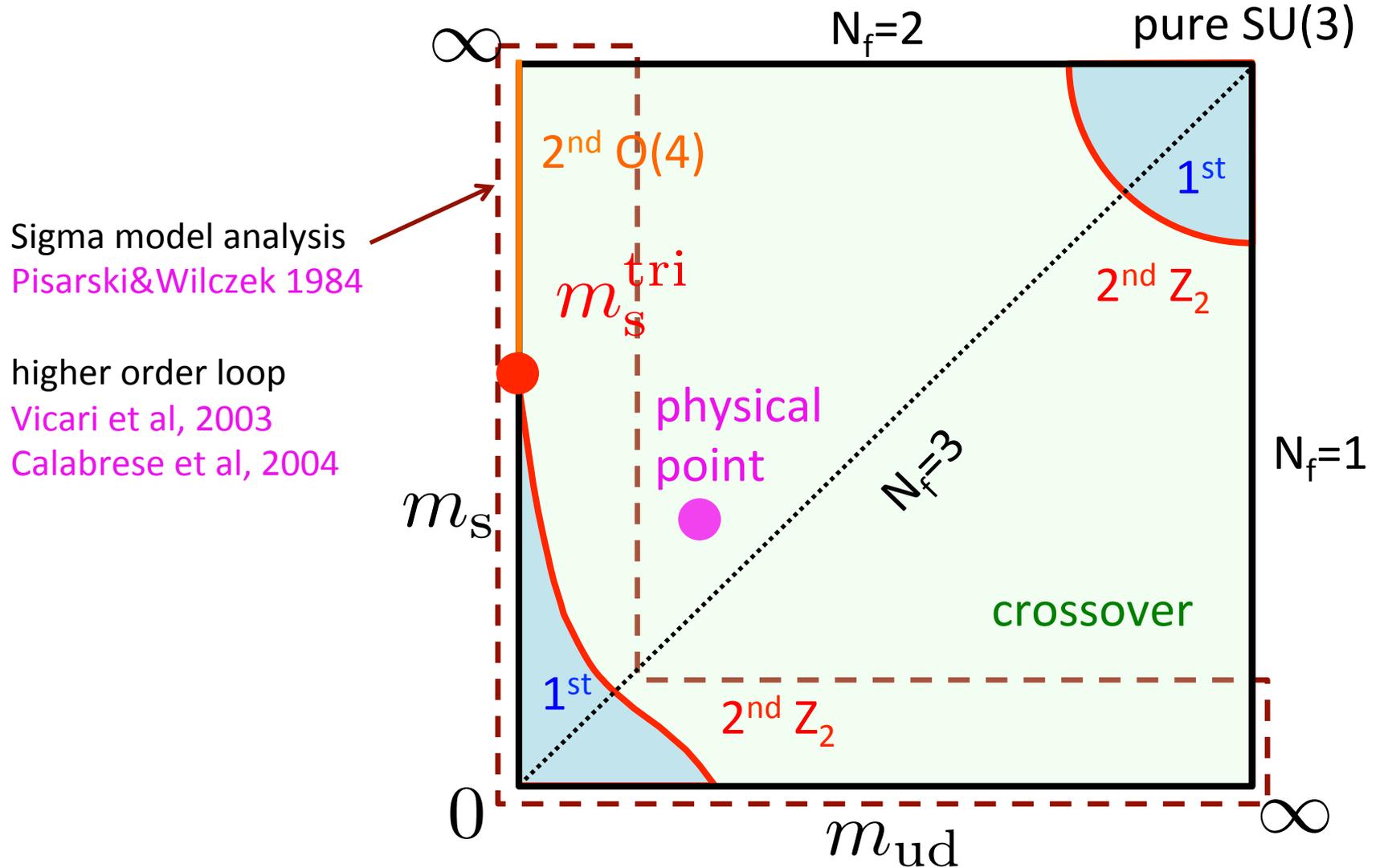
X-Y. Jin, Y. Kuramashi, Y. Nakamura & A. Ukawa

Lattice 2017 in Granada

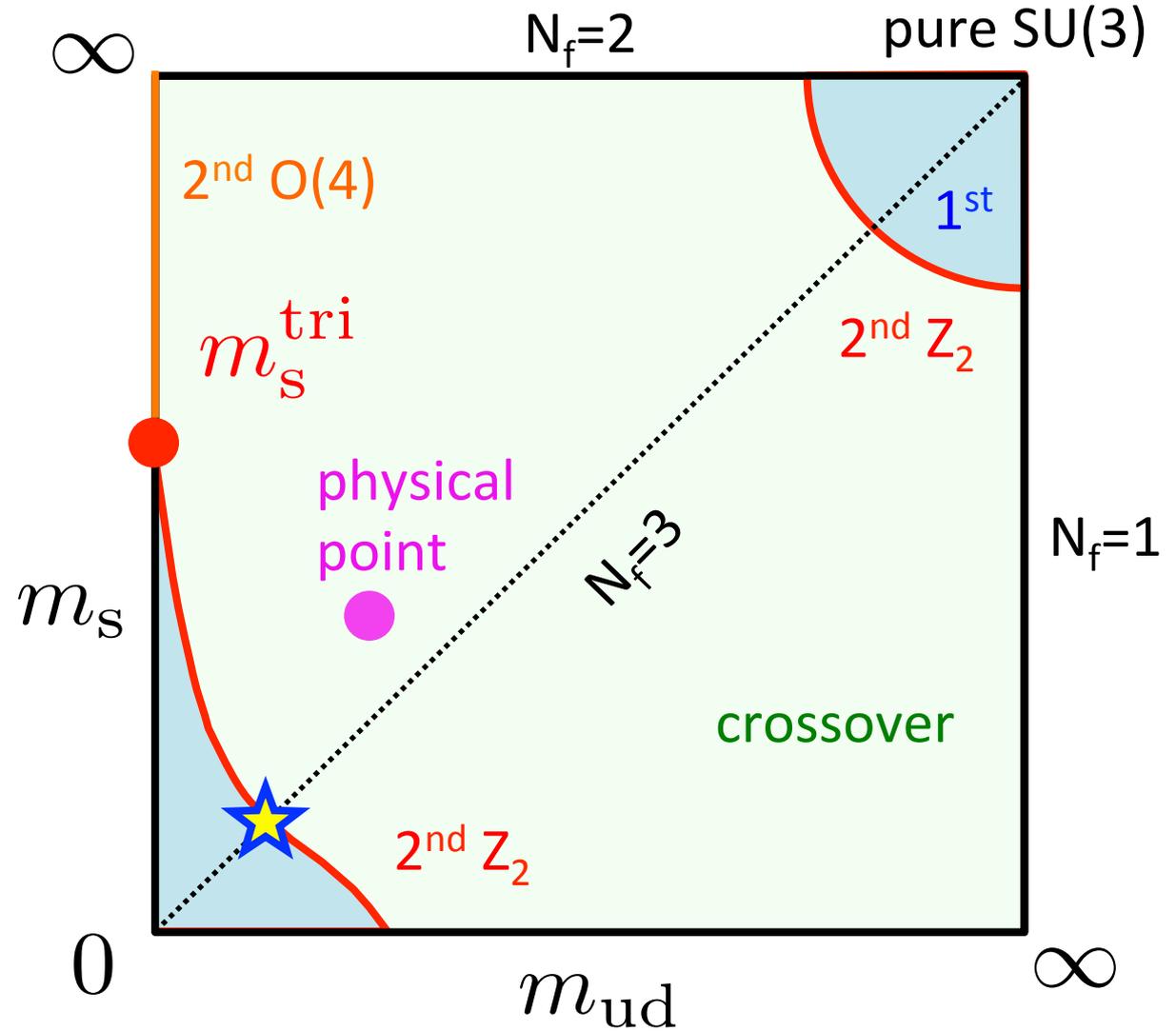
Columbia plot



Columbia plot



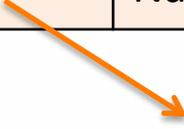
Columbia plot



History for $N_f=3$ QCD critical point

Action	N_t	$m_{\pi, \text{critical}}$	Ref.
KS, standard	4	290MeV	Karsch et al '01, Liao '01
KS, p4	4	67MeV	Karsch et al '04
KS, standard	6	150MeV	de Forcrand et al '07
KS, HISQ	6	$\lesssim 50\text{MeV}$	Ding et al '17
KS, stout	4-6	could be 0	Varnhorst '14

Wilson, standard	4	$\lesssim 670\text{MeV}$	Iwasaki et al, '96
Wilson, $O(a)$ impr.	6-8	300MeV	Nakamura et al, '14

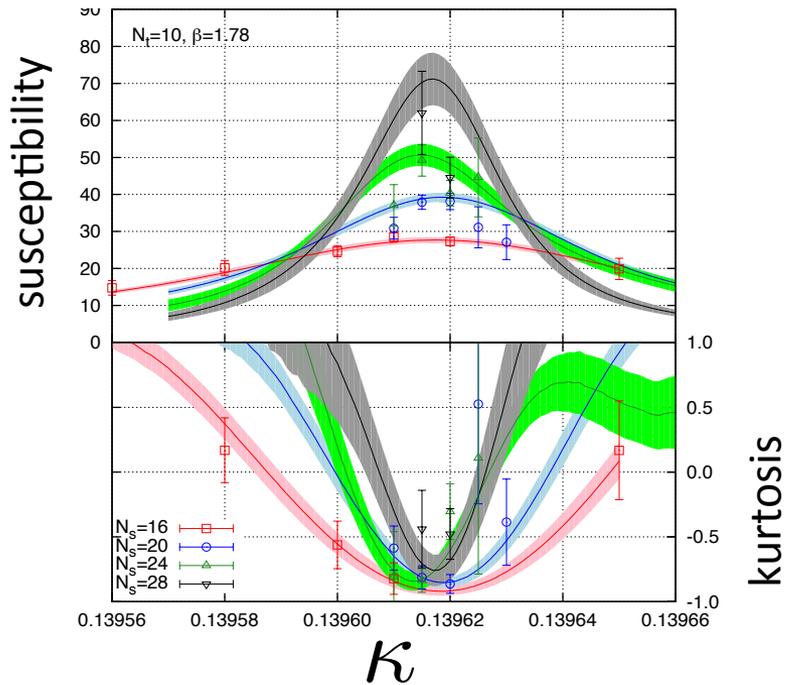
 We will update it!

de Forcrand et al '16, Varnhost '14, Szabo '13

Our simulation details

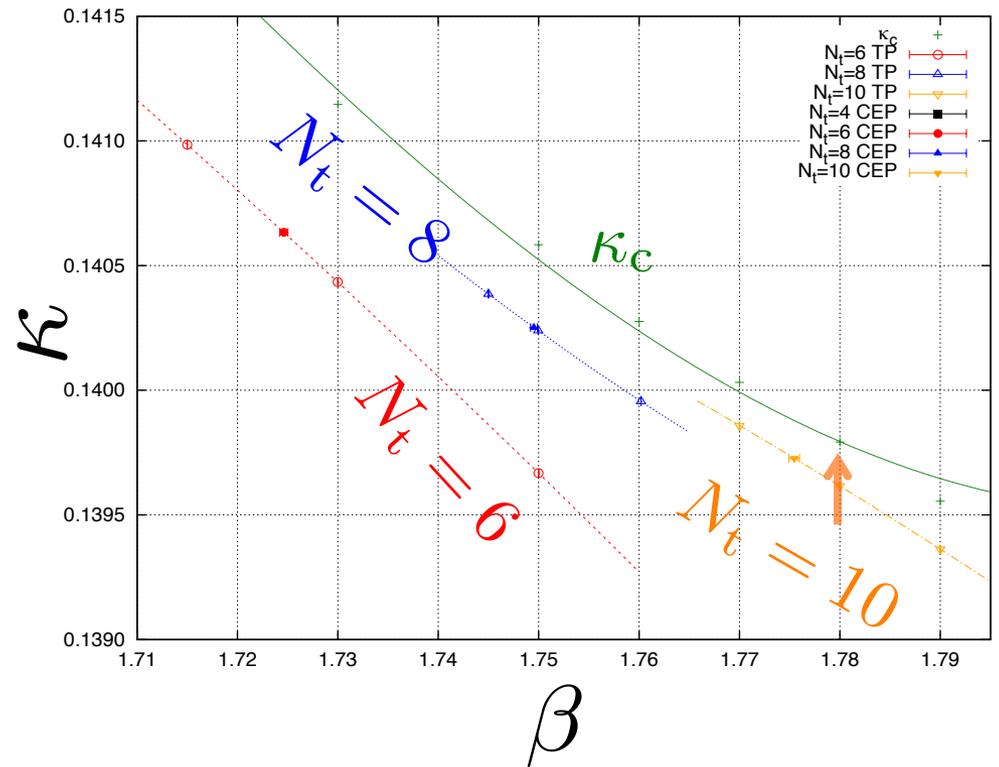
- Iwasaki gauge + NP $O(a)$ improved Wilson
- $N_t=4, 6, 8$ and 10
- Kurtosis intersection to determine critical point
- Chiral condensate as “order parameter”

Moments for chiral condensate



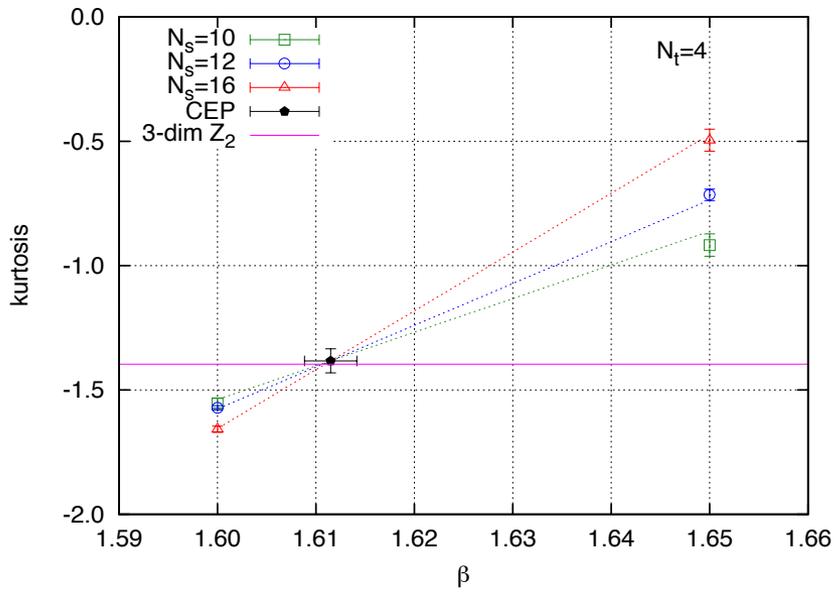
$$N_t = 10, \beta = 1.78$$

$$N_s = 16 - 28$$

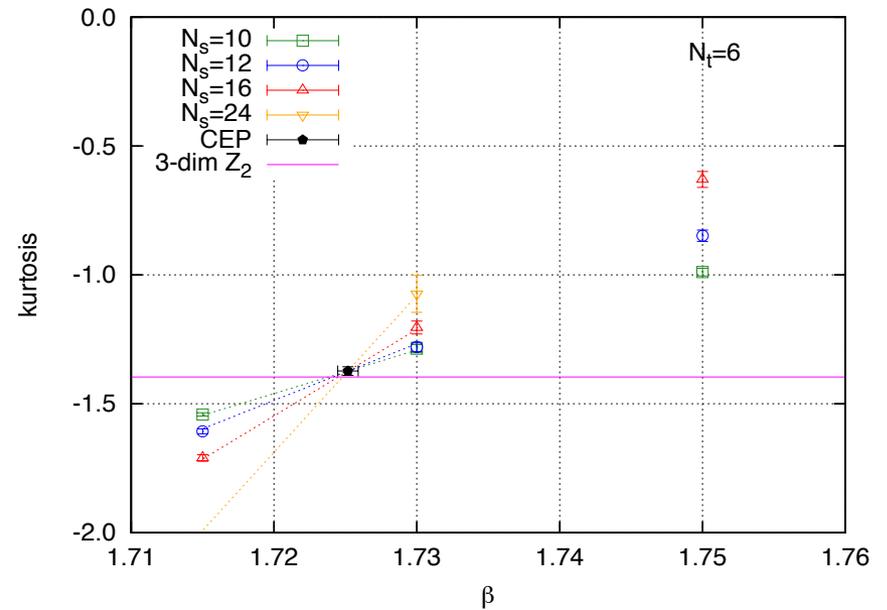


Kurtosis intersection

$N_t = 4$



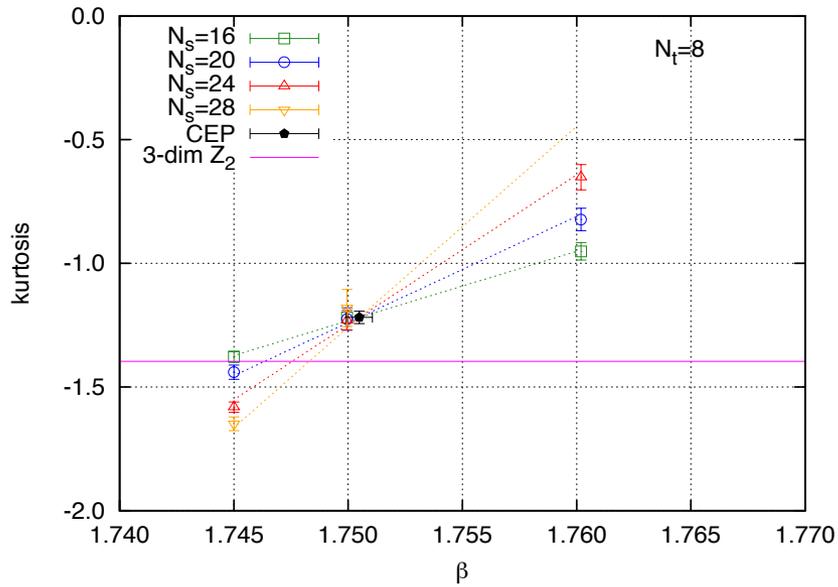
$N_t = 6$



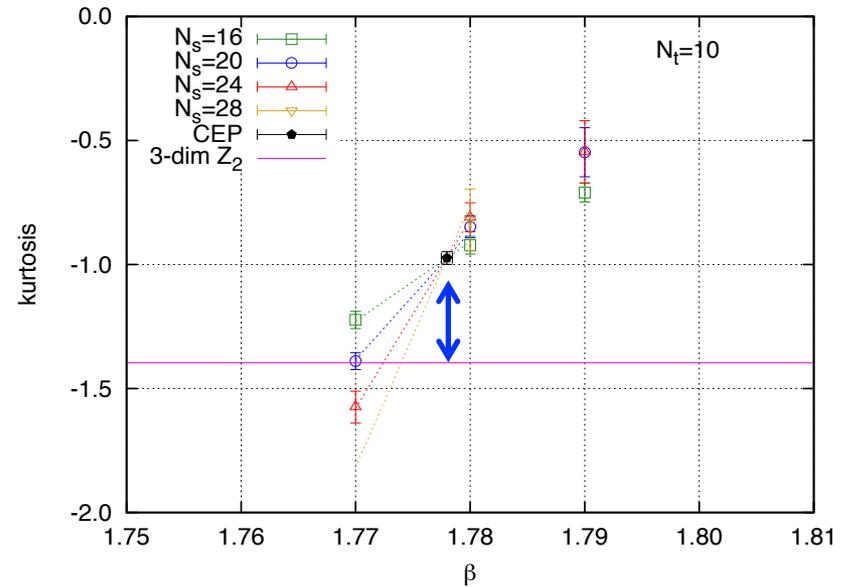
$$K = K_E + AN_s^{1/\nu} (\beta - \beta_E)$$

Kurtosis intersection

$$N_t = 8$$



$$N_t = 10$$



Due to finite size effect?

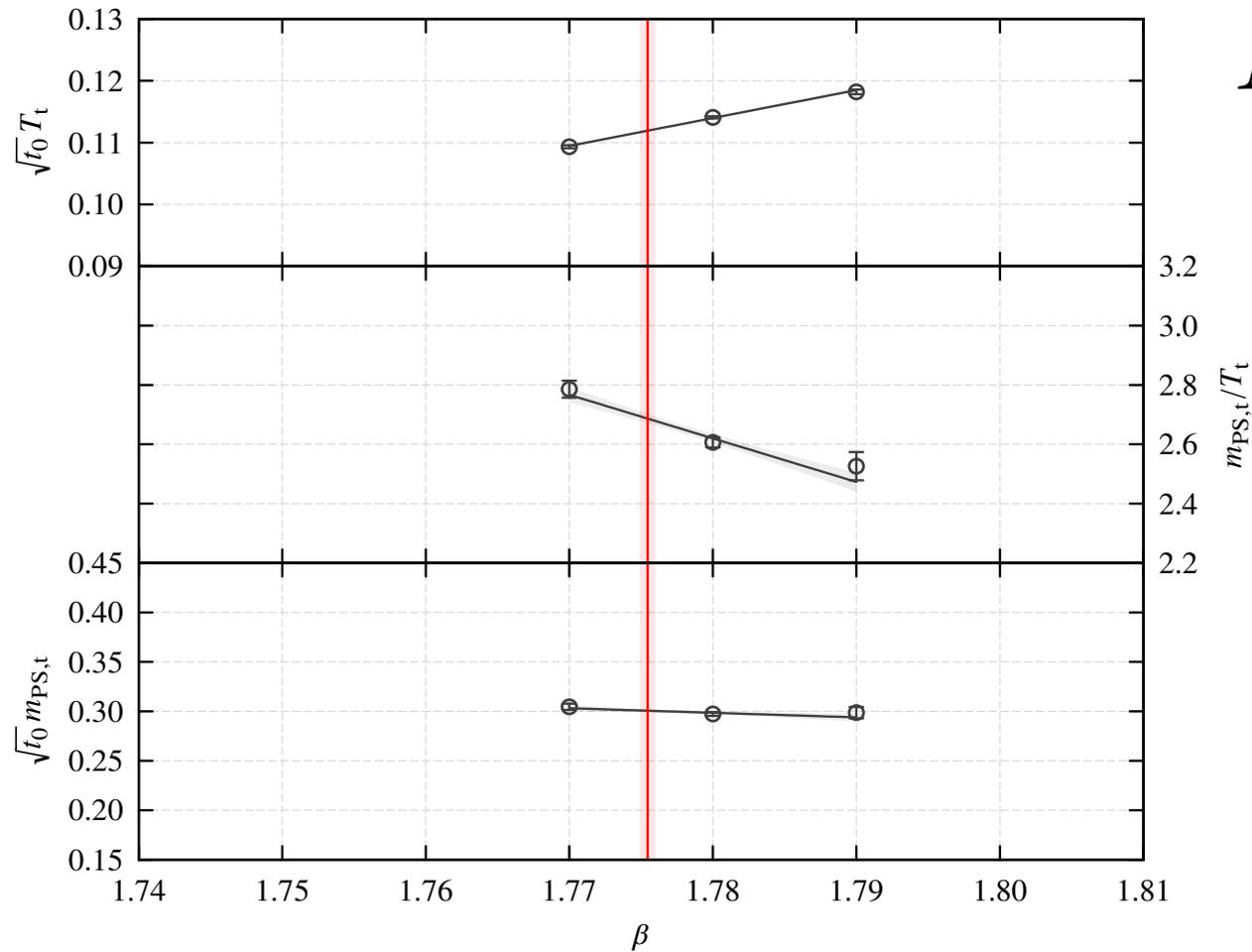
Fitting for kurtosis intersection

$$K = \left[K_E + AN_s^{1/\nu} (\beta - \beta_E) \right] \left(1 + \underline{BN_s^{-c}} \right)$$

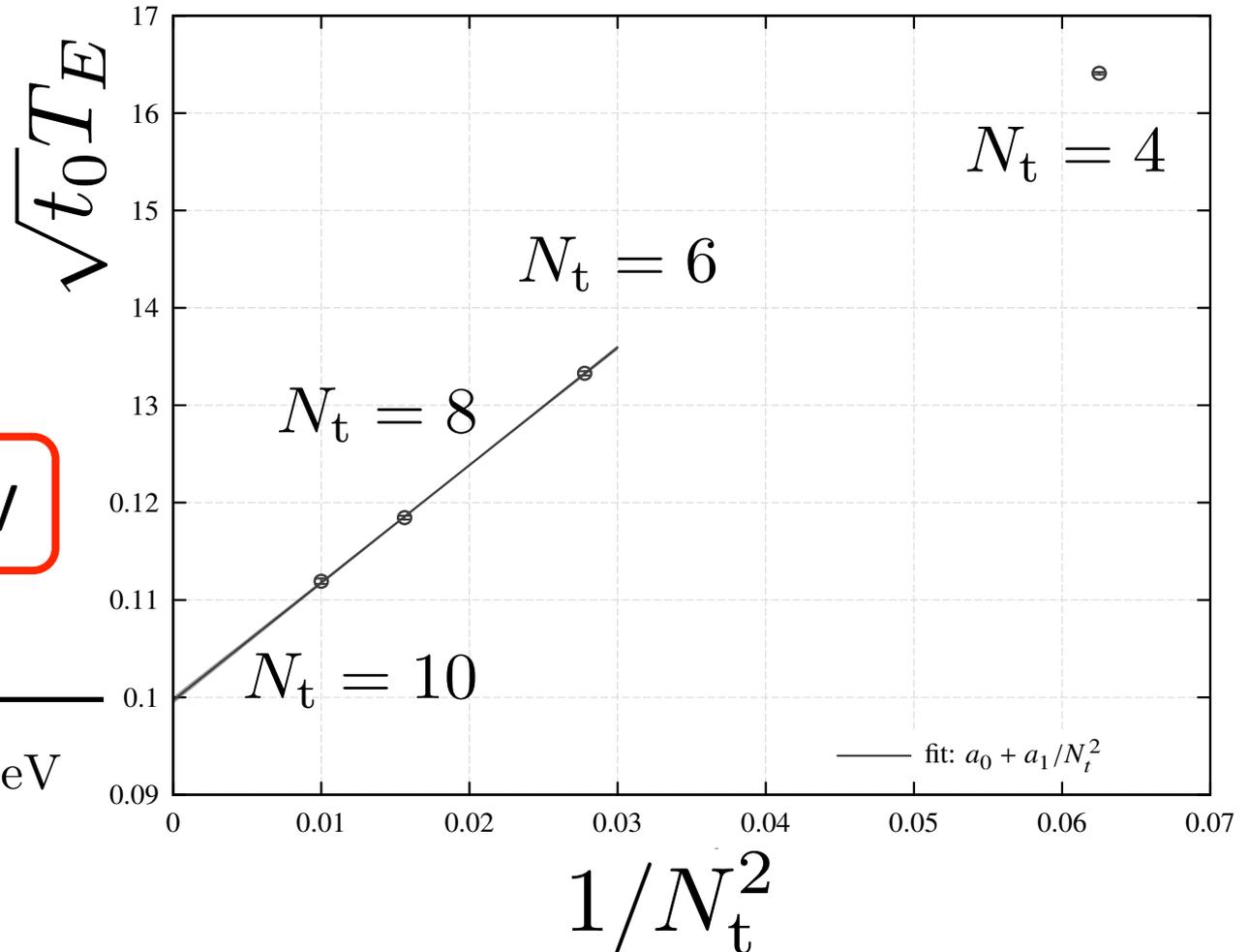
Correction term of finite size effect

N_t	Fit	β_E	κ_E	K_E	ν	A	B	$y_t - y_h$	$\chi^2/\text{d.o.f.}$
4	1	1.6115(26)	0.1429337(13)	-1.383(48)	0.84(13)	0.88(42)	×	×	1.75
	2	1.61065(61)	0.1429713(13)	-1.396	0.63	0.313(12)	×	×	3.05
	3	1.6099(17)	0.1430048(13)	-1.396	0.63	0.311(14)	0.10(21)	-0.894	3.77
6	1	1.72518(71)	0.1406129(14)	-1.373(17)	0.683(54)	0.58(17)	×	×	0.68
	2	1.72431(24)	0.1406451(14)	-1.396	0.63	0.418(11)	×	×	0.70
	3	1.72462(40)	0.1406334(14)	-1.396	0.63	0.422(12)	-0.052(52)	-0.894	0.70
8	1	1.75049(57)	0.1402234(11)	-1.219(25)	0.527(55)	0.146(88)	×	×	0.73
	2	1.74721(42)	0.14031921(76)	-1.396	0.63	0.404(36)	×	×	5.99
	3	1.74953(33)	0.1402512(10)	-1.396	0.63	0.414(13)	-1.33(15)	-0.894	0.73
10	1	1.77796(48)	0.1396661(17)	-0.974(25)	0.466(45)	0.084(52)	×	×	0.22
	2	1.7694(16)	0.1398724(22)	-1.396	0.63	0.421(95)	×	×	10.03
	3	1.77545(53)	0.1397274(17)	-1.396	0.63	0.559(29)	-2.97(25)	-0.894	0.43

Hadronic quantity at critical point



Continuum extrapolation

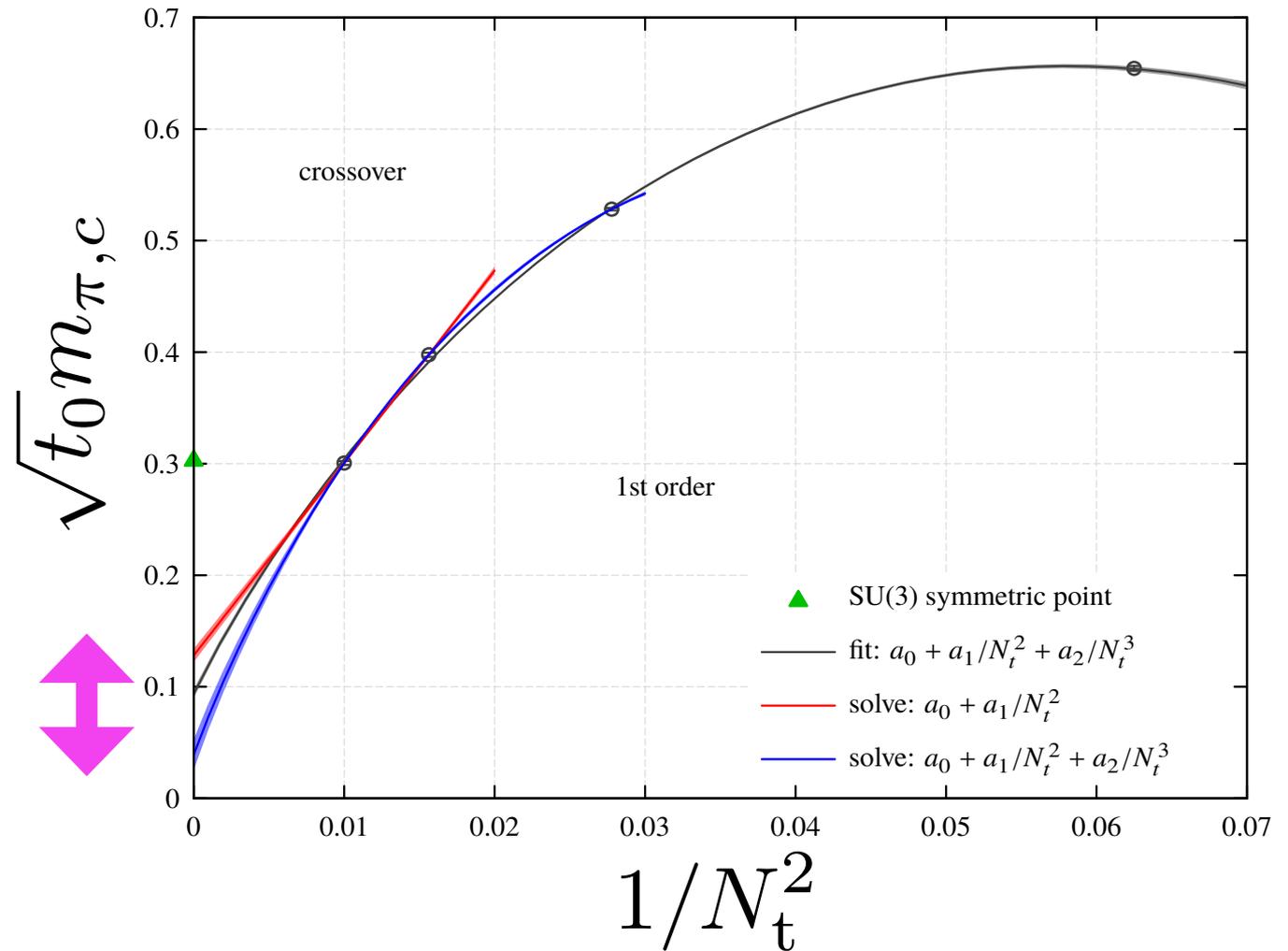


$T_E = 134(3) \text{ MeV}$

$1/\sqrt{t_0} = 1347(30) \text{ MeV}$

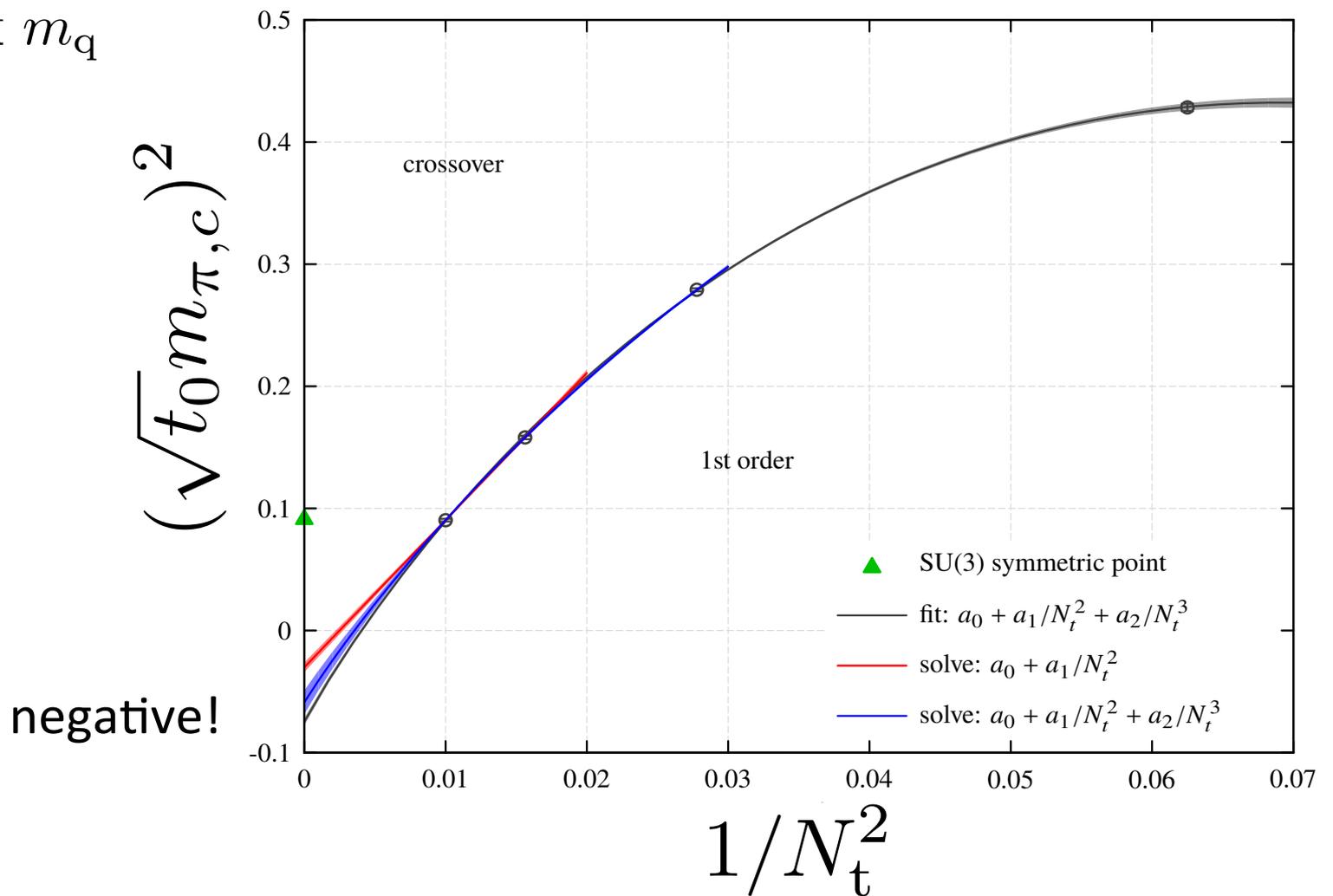
Borsanyi et al, 2012

Continuum extrapolation



Continuum extrapolation

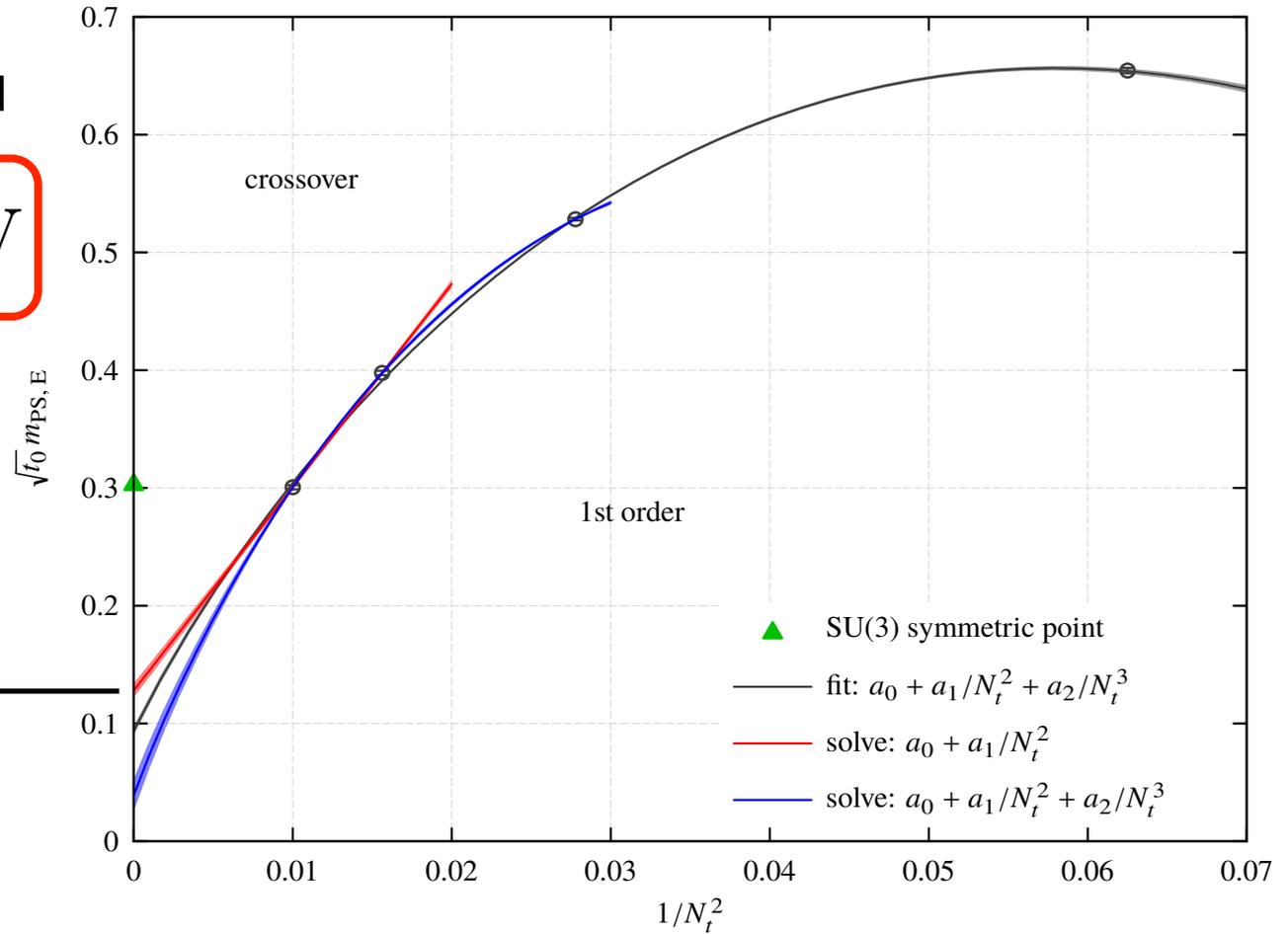
$$m_\pi^2 \propto m_q$$



Continuum extrapolation

upper bound

$$m_{\pi,c} \lesssim 170\text{MeV}$$

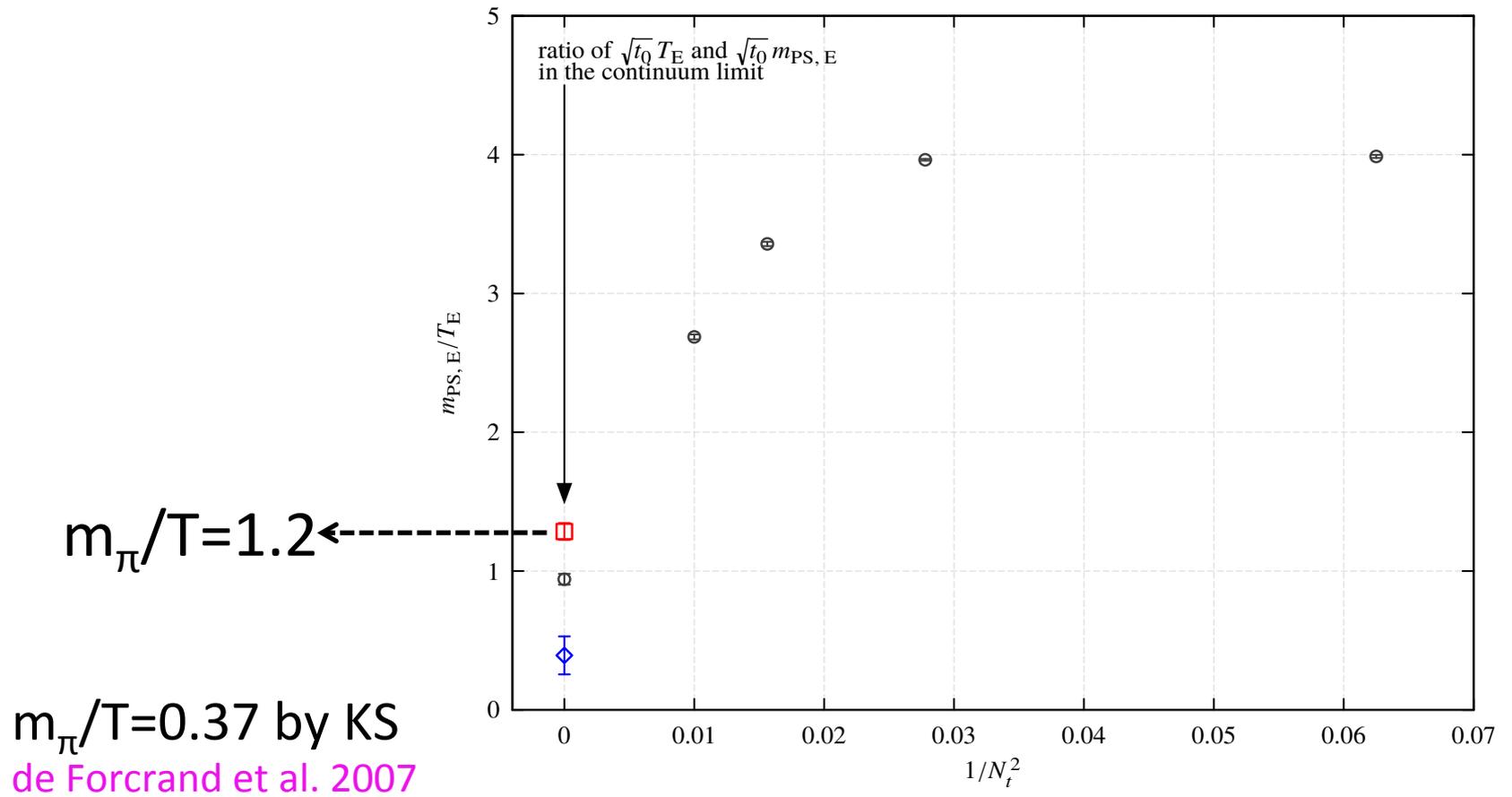


Summary and outlook

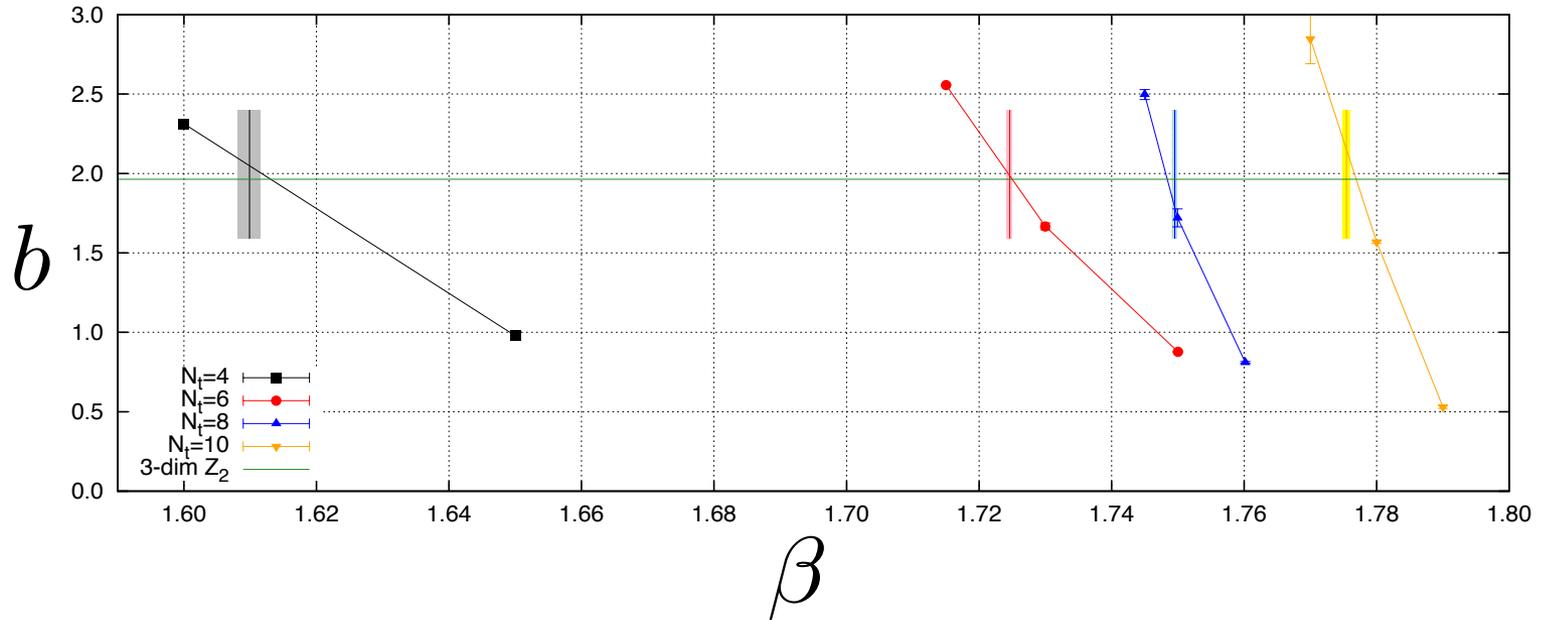
- Upper bound $m_\pi \lesssim 170 \text{ MeV}$ ($m_\pi \lesssim 50 \text{ MeV}$ by KS HISQ)
- Our estimate of the upper bound is derived from the existence of critical point as an edge of 1st order phase transition
- Further large N_t or some improvement are needed in future
- For critical line on Columbia plot, see the next talk by Nakamura
- Checking universality for $N_f=4$ QCD where no rooting issue, see Ohno's talk on 6/21

BACKUP SLIDES

Continuum extrapolation



$$\chi_{\max} \propto N_s^b$$



Exponent b along the transition line projected on β