QCD at finite isospin chemical potential: Phase diagram and equation of state

Bastian Brandt

ITP Goethe Universität Frankfurt

In collaboration with
Gergely Endrődi and Sebastian Schmalzbauer

20.06.2017

Contents

1. Introduction and simulation setup

QCD at finite isospin chemical potential

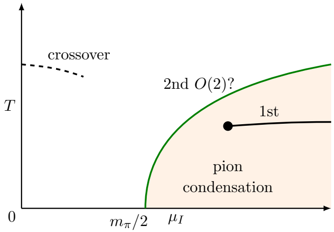
QCD at finite chemical potential ($N_f = 2$):
u quark: μ_u d quark: μ_d

- Can be decomposed in baryon and isospin chemical potentials:
  $\mu_B = 3(\mu_u + \mu_d)/2$ and $\mu_I = (\mu_u - \mu_d)/2$
  Here: consider $\mu_B = 0$!
- Non-zero μ_I introduces an asymmetry between isospin ± 1 particles
- Such situations occur regularly in nature
- Finite μ_I breaks $SU_V(2)$ explicitly to $U_{\tau_3}(1)$.
  For $\mu_I > m_\pi/2$: $U_{\tau_3}(1)$ spontaneously broken
  $\Rightarrow$ Pion condensation

See Sebastians talk!!!

Expected phase diagram

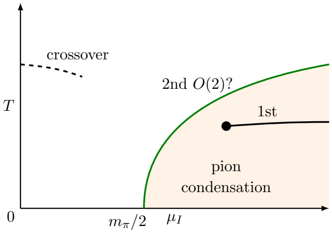
χPT at finite μ_I:



[Son, Stephanov, PRL86 (2001)]

Expected phase diagram

χPT at finite μ_I:



[Son, Stephanov, PRL86 (2001)]

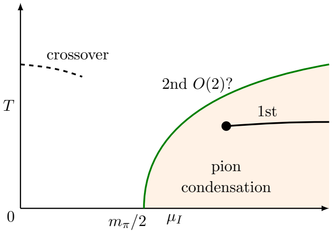
First results from lattice QCD:

$N_t = 4$, unphysical masses, unimproved:

N_f=2 [Kogut, Sinclair, PRD66(2002); PRD70(2004)]
N_f=8 [de Forcrand, *et al*, PoS LAT2007 (2007)]

Expected phase diagram

χPT at finite μ_I:



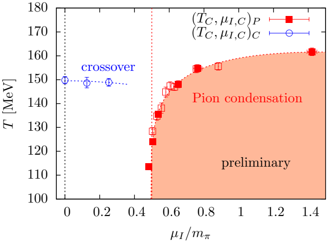
[Son, Stephanov, PRL86 (2001)]

First results from lattice QCD:

$N_t = 4$, unphysical masses, unimproved:

$N_f = 2$ [Kogut, Sinclair, PRD66(2002); PRD70(2004)]
$N_f = 8$ [de Forcrand, *et al*, PoS LAT2007 (2007)]

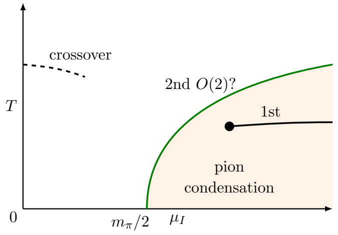
$N_f = 2 + 1$, physical quark masses,
$N_t = 6$, improved actions:



[BB, Endrődi, PoS LAT2016 (2016)]

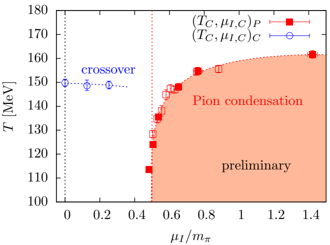
Expected phase diagram

χPT at finite μ_I:



[Son, Stephanov, PRL86 (2001)]

$N_f = 2+1$, physical quark masses,
$N_t = 6$, improved actions:



[BB, Endrődi, PoS LAT2016 (2016)]

First results from lattice QCD:

$N_t = 4$, unphysical masses, unimproved:

N_f=2 [Kogut, Sinclair, PRD66(2002); PRD70(2004)]
N_f=8 [de Forcrand, *et al*, PoS LAT2007 (2007)]

Here we will update these results!

Simulation setup

[G. Endrődi, PRD90 (2014)]

- Gauge action: Symanzik improved
- Mass-degenerate u/d quarks: [Kogut, Sinclair, PRD66 (2002); PRD70 (2004)]

  Fermion matrix: $M = \begin{pmatrix} D(\mu) & \lambda\gamma_5 \\ -\lambda\gamma_5 & D(-\mu) \end{pmatrix}$

  $D(\mu)$: staggered Dirac operator with 2×-stout smeared links

  λ: small explicit breaking of residual symmetry (unphysical)

  - Necessary to observe spontaneous symmetry breaking at finite V.
  - Serves as a regulator in the pion condensation phase.
- Strange quark: rooted staggered fermions (no chemical potential)
- Quark masses are tuned to their physical values.
- For physical results: extrapolate $\lambda \to 0$

  Use valence quark improvement and LO reweighting!
  $\Rightarrow$ Extrapolation well under control!
  (see Sebastians talk and [BB, Endrődi, PoS LAT2016 (2016)])

  From now on: all results at $\lambda = 0$!

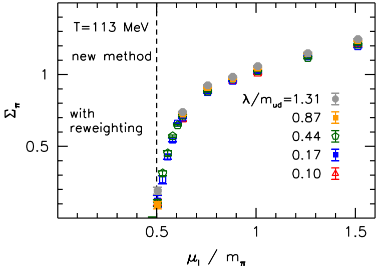
2. Phase diagram at finite μ_I

Pion condensation phase

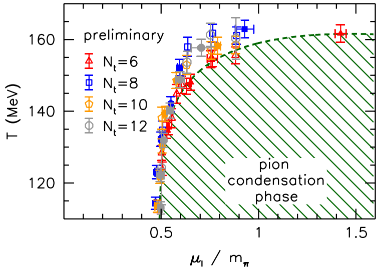
[BB, Endrődi, PoS LAT2016 (2016)]

Defined by non-vanishing

$$\langle \pi \rangle = \frac{T}{V} \frac{\partial \log Z}{\partial \lambda}$$



Approach to continuum:



Deconfinement transition

Investigate the finite temperature transition (crossover) for $\mu_I < \mu_I^C$.

Transition temperature T_C is defined by the behaviour of $\langle \bar{\psi}\psi \rangle$:

- Standard: Use the inflection point of the condensate.
- Easier alternative for $\mu_I < \mu_I^C$:

  Use the point where renormalised condensate reaches a certain value.
  only valid for $\mu_I < \mu_I^C(T = 0) = m_\pi/2$

Deconfinement transition

Investigate the finite temperature transition (crossover) for $\mu_I < \mu_I^C$.

Transition temperature T_C is defined by the behaviour of $\langle\bar{\psi}\psi\rangle$:

- Standard: Use the inflection point of the condensate.
- Easier alternative for $\mu_I < \mu_I^C$:

  Use the point where subtracted condensate reaches a certain value.
  only valid for $\mu_I < \mu_I^C(T=0) = m_\pi/2$

Deconfinement transition

Investigate the finite temperature transition (crossover) for $\mu_I < \mu_I^C$.

Transition temperature T_C is defined by the behaviour of $\langle \bar{\psi}\psi \rangle$:

- Standard: Use the inflection point of the condensate.
- Easier alternative for $\mu_I < \mu_I^C$:

  Use the point where subtracted condensate reaches a certain value.
  only valid for $\mu_I < \mu_I^C(T = 0) = m_\pi/2$

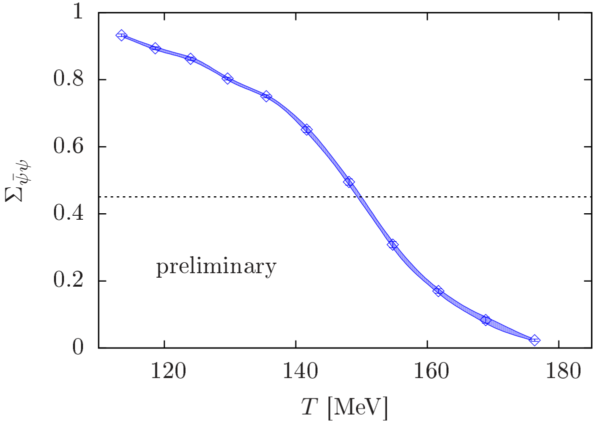
Renormalised light quark condensate:

$$\Sigma_{\bar{\psi}\psi} = \frac{m_{ud}}{m_\pi^2 f_\pi^2} \left[\langle \bar{\psi}\psi \rangle_{T,\mu_I} - \langle \bar{\psi}\psi \rangle_{0,0} \right] + 1$$

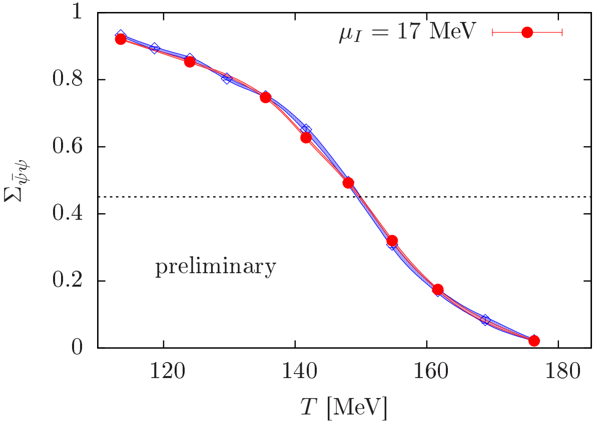
Value at the transition (in continuum): $\Sigma_{\bar{\psi}\psi} = 0.4505$

[BW: Borsanyi *et al*, JHEP1009 (2010)]

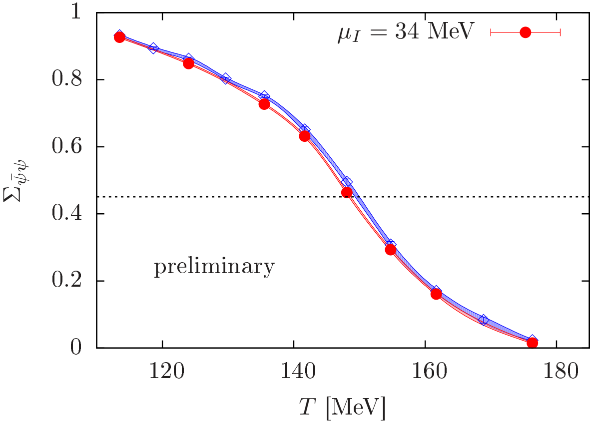
$\Sigma_{\bar{\psi}\psi}$ at fixed μ_I (6×24^3 lattice)



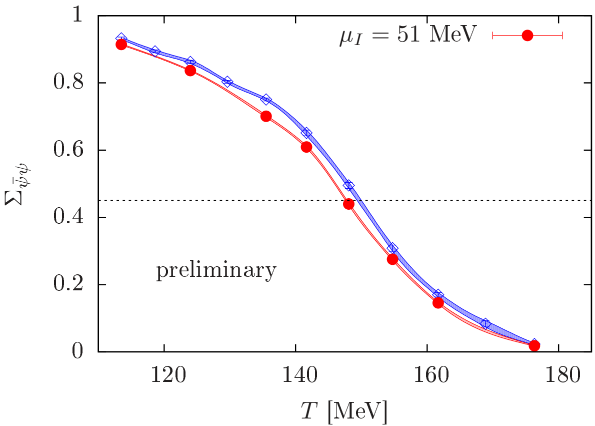
$\Sigma_{\bar{\psi}\psi}$ at fixed μ_I $(6 \times 24^3$ lattice$)$



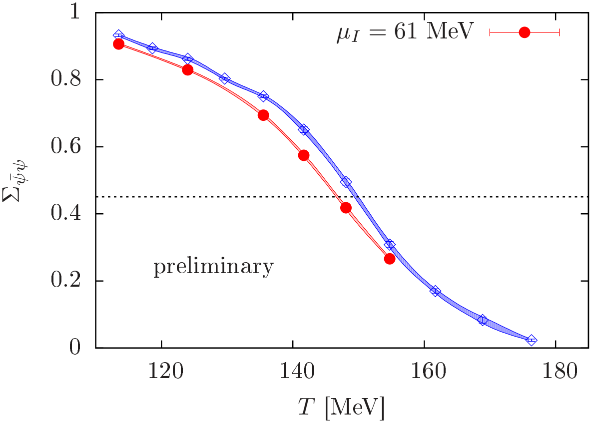
$\Sigma_{\bar{\psi}\psi}$ at fixed μ_I (6×24^3 lattice)



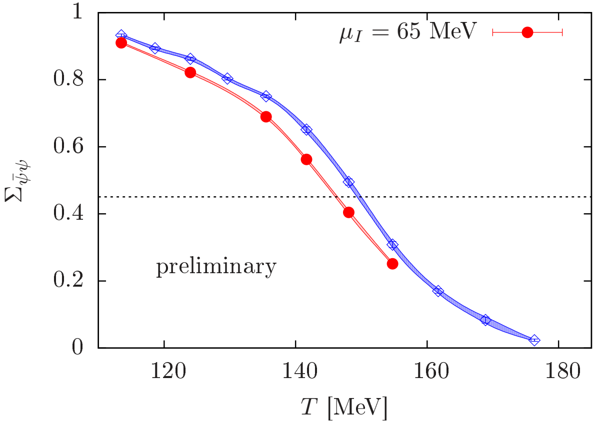
$\Sigma_{\bar{\psi}\psi}$ at fixed μ_I (6×24^3 lattice)



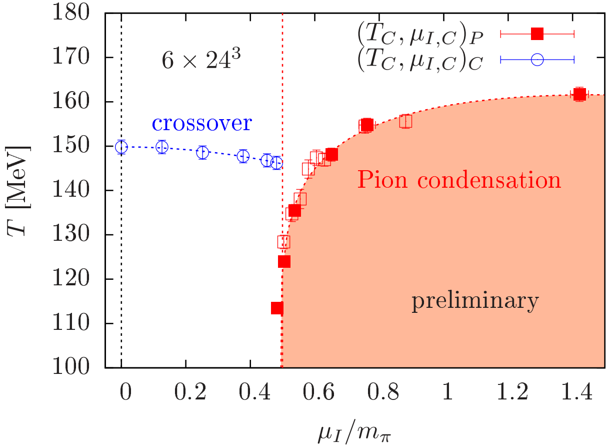
$\Sigma_{\bar\psi\psi}$ at fixed μ_I (6×24^3 lattice)



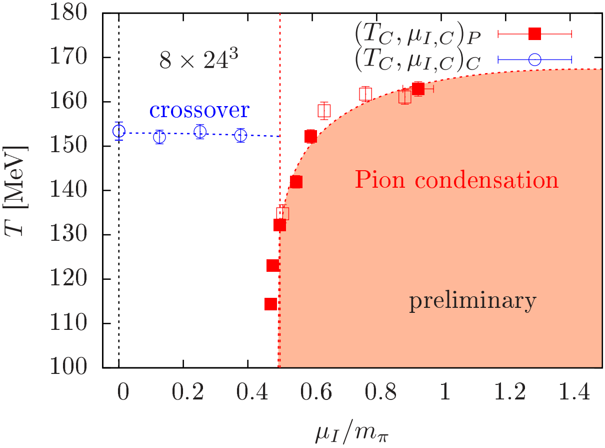
$\Sigma_{\bar{\psi}\psi}$ at fixed μ_I (6×24^3 lattice)



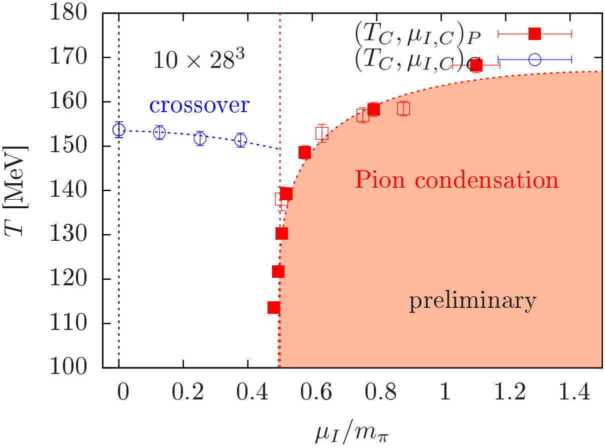
Phase diagram for 6×24^3



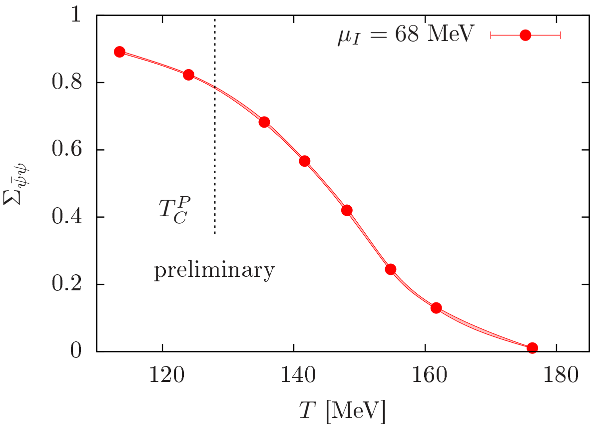
Phase diagram for 8×24^3



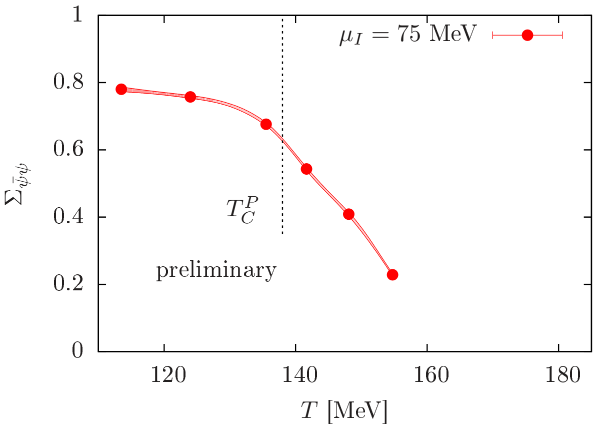
Phase diagram for 10×28^3



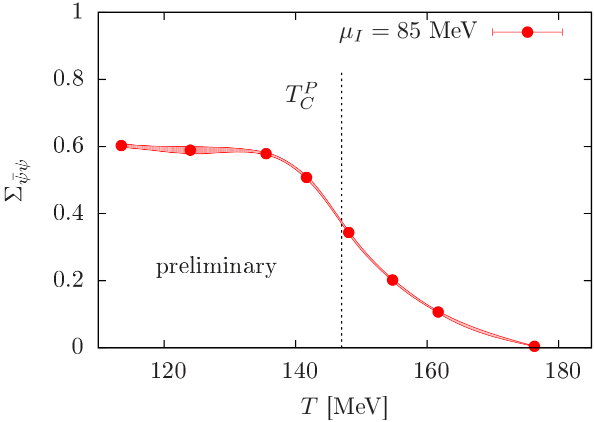
Deconfinement transition for $\mu_I > m_\pi/2$ (6×24^3 lattice)



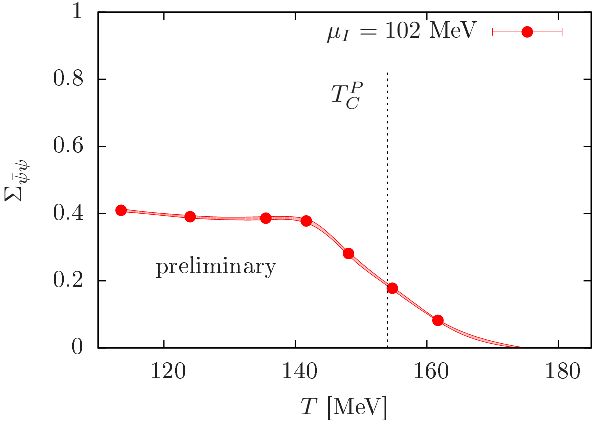
Deconfinement transition for $\mu_I > m_\pi/2$ (6×24^3 lattice)



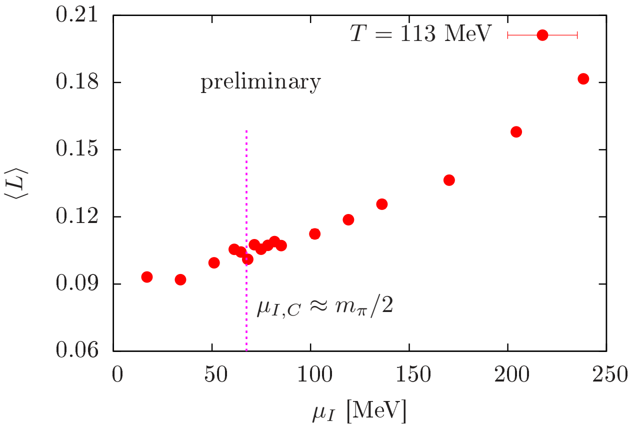
Deconfinement transition for $\mu_I > m_\pi/2$ (6×24^3 lattice)



Deconfinement transition for $\mu_I > m_\pi/2$ $(6 \times 24^3$ lattice)



Behaviour of $\langle L \rangle$ at fixed T (6×24^3 lattice)



Phase diagram: Open questions

- Location of meeting point between crossover and pion cond. boundary?

  (to investigate: need other definition of T_C)

- Are deconfinement transition and the boundary of the pion condensation phase equivalent?

  $\Rightarrow$ Deconfinement transition appears to be at the boundary.

- What is the order of the transition on the boundary?

  Presence of a tri-critical point?

  [Kogut, Sinclair, PRD66 (2002); PRD70 (2004)]

- What happens in the $\mu_I \to \infty$ limit?

3. Comparison to Taylor expansion around $\mu_I = 0$

Taylor expansion around $\mu_I = 0$

Simulations at finite μ_B suffer from a sign problem!

One of the most important tools to obtain information at finite μ_B:
Taylor expansion around $\mu_B = 0$.

However: Range of applicability at a given order is unknown!

Here: test Taylor expansion method using our data for $\mu_I \neq 0$

- As an observable we use the isospin density (analogue to Baryon density):

$$\langle n_I \rangle = \frac{T}{V} \frac{\partial \log Z}{\partial \mu_I}$$

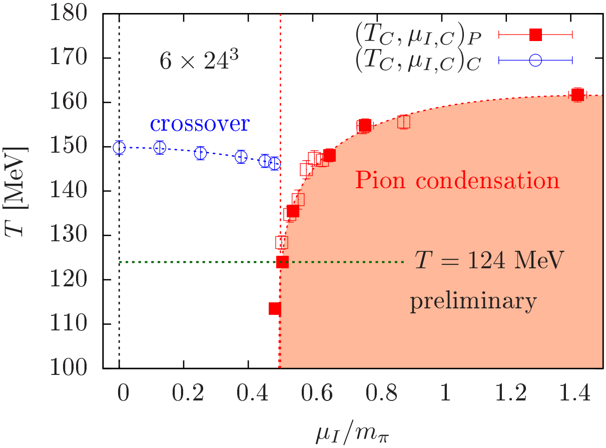
- Associated Taylor expansion (follows from expansion of pressure p/T^4):

$$\frac{\langle n_I \rangle}{T^3} = c_2 \left(\frac{\mu_I}{T} \right) + \frac{c_4}{6} \left(\frac{\mu_I}{T} \right)^3$$

Take values from Budapest-Wuppertal [BW: Borsanyi *et al*, JHEP1201 (2012)]

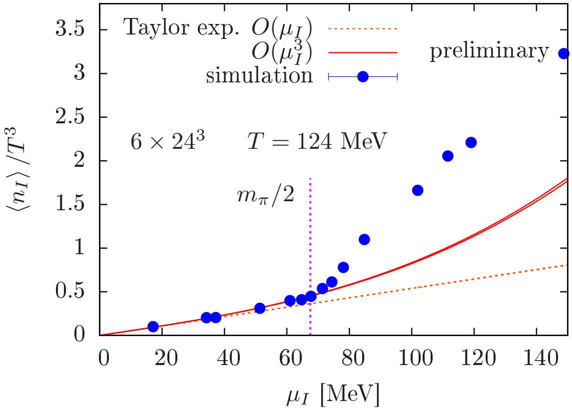
Comparison to data at finte μ_I

Compare data for 6×24^3 lattice:



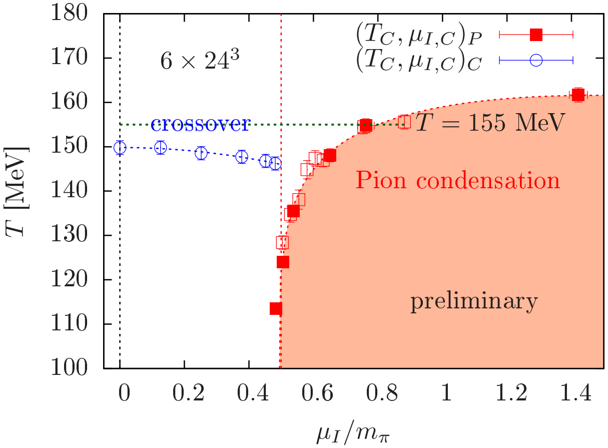
Comparison to data at finte μ_I

Compare data for 6×24^3 lattice, $T < T_C$:



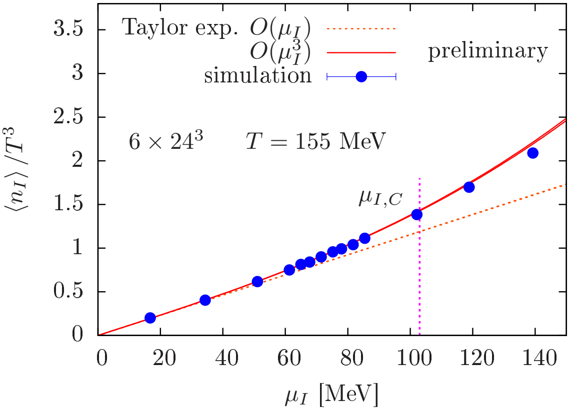
Comparison to data at finte μ_I

Compare data for 6×24^3 lattice, $T > T_C$:



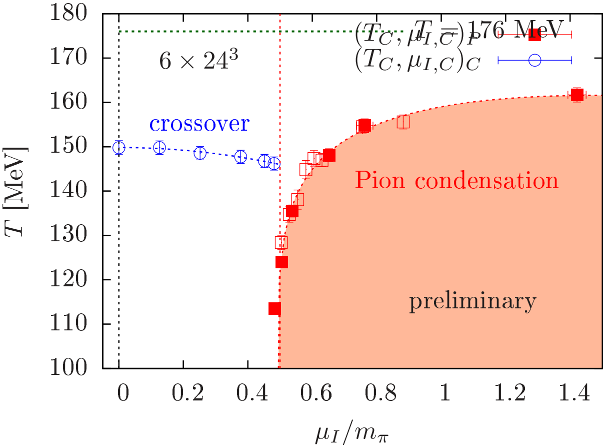
Comparison to data at finte μ_I

Compare data for 6×24^3 lattice, $T > T_C$:



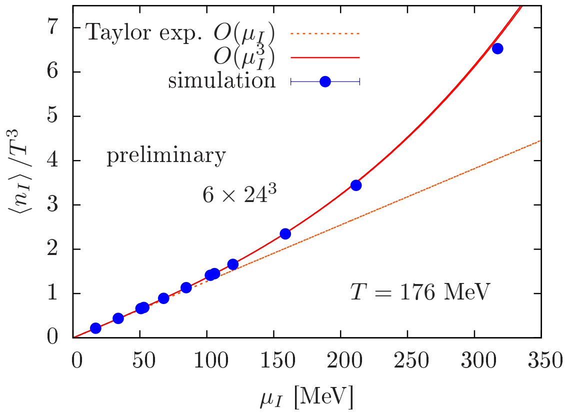
Comparison to data at finte μ_I

Compare data for 6×24^3 lattice, $T > T_C$:



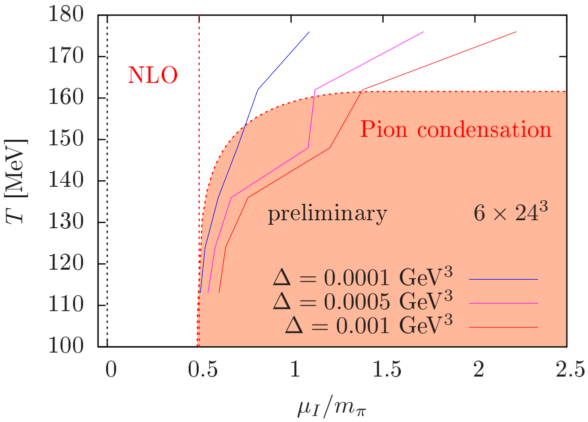
Comparison to data at finte μ_I

Compare data for 6×24^3 lattice, $T > T_C$:



Comparison to data at finte μ_I

Contour plot using $\Delta \equiv \left| \langle n_I \rangle - \langle n_I \rangle^{\text{Taylor}}_{\text{NLO}} \right|$:



4. Equation of state at finite μ_I

Pressure and trace anomaly

Most important quantities to study equation of state (EOM):

- Pressure: $$\frac{p}{T^4} = -\frac{1}{T^3 V} \log \mathcal{Z}$$
- Trace anomaly: $$\frac{I}{T^4} = \frac{\epsilon - 3p}{T^4} = T \frac{\partial}{\partial T} \frac{p}{T^4} + \frac{\mu_I n_I}{T^4}$$

$\Rightarrow$ All other quantities derive from those and the number densities!

Here: Consider these quantities at finite μ_I!

First: Focus on the pressure!

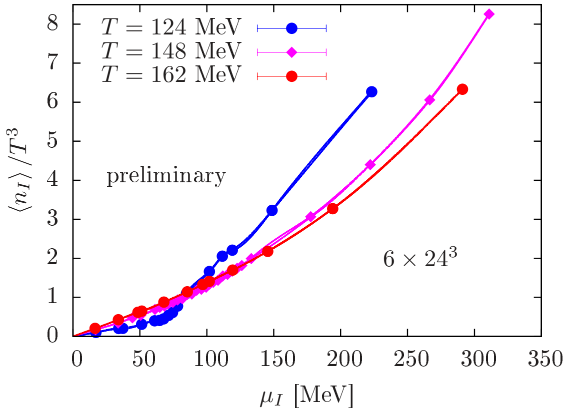
$$\Rightarrow \quad p(T, \mu_I) = p(T, 0) + \int_0^{\mu_I} d\mu_I' \, n_I(T, \mu_I') \equiv p(T, 0) + \Delta p(T, \mu_I)$$

(since $n_I = \frac{\partial p}{\partial \mu_I}$)

$p(T, 0)$ take results from [Borsanyi, *et al*, JHEP 1011 (2010)]

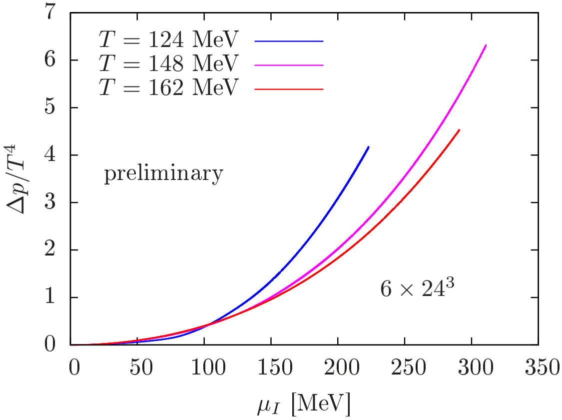
Pressure at finite μ_I

Interpolation of $\langle n_I \rangle$ for 6×24^3 lattice:



Pressure at finite μ_I

Pressure for 6×24^3 lattice:



5. An application for the EOS: Compact stars

Application: mass-radius relation of compact boson stars

Use $p(0, \mu_I)$ and $n_I(0, \mu_I)$ for the construction of a compact star!

Condensing particles: charged pions $\Rightarrow$ Obtain a boson star!

- hypothetical objects [Kaup, PR172 (1968)]
- have been considered in the literature
  [reviews: Jetzer, PR220 (1992); Liebling, Palenzuela, LRR15 (2012)]
- charged pions lead to positively charged star

  $\Rightarrow$ Need to include electrons!

Application: mass-radius relation of compact boson stars

Use $p(0, \mu_I)$ and $n_I(0, \mu_I)$ for the construction of a compact star!

Condensing particles: charged pions ⇒ Obtain a boson star!

- hypothetical objects [Kaup, PR172 (1968)]
- have been considered in the literature [reviews: Jetzer, PR220 (1992); Liebling, Palenzuela, LRR15 (2012)]
- charged pions lead to positively charged star

⇒ **Need to include electrons!** (work in progress)

Application: mass-radius relation of compact boson stars

Use $p(0, \mu_I)$ and $n_I(0, \mu_I)$ for the construction of a compact star!

Condensing particles: charged pions ⇒ Obtain a boson star!

- hypothetical objects [Kaup, PR172 (1968)]
- have been considered in the literature [reviews: Jetzer, PR220 (1992); Liebling, Palenzuela, LRR15 (2012)]
- charged pions lead to positively charged star

  ⇒ **Need to include electrons!** (work in progress)

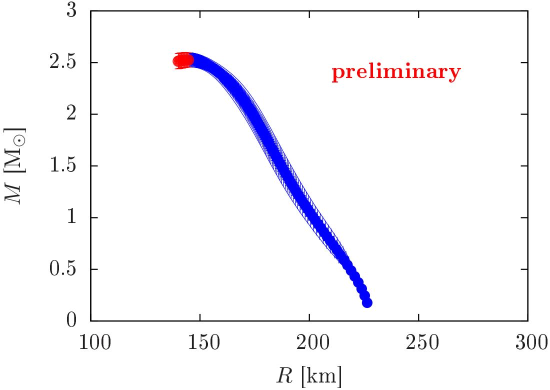
Plugging the relation $p(0, \mu_I)$ and $n_I(0, \mu_I)$ into the TOV equation: [Glendenning, Compact stars: . . . (1997)]

Obtain the allowed mass-radius relation!

(similar study in G_2 QCD [Hajizadeh, Maas, arXiv:1702.08724])

Application: mass-radius relation of compact boson stars

Data: unimproved stag. quarks + Wilson gauge, 8^4, $T = 0$: [S. Schmalzbauer]



(electrons not yet included)

Summary and Perspectives

- We have investigated the phase structure of QCD at finite isospin chemical potential μ_I.

  Remaining tasks:

  - investigate deconfinement transition at $\mu_I > m_\pi/2$
  - determine order of transition
    (presence of tricritical point on phase boundary for large μ_I?)
  - perform continuum limit

- Can use the theory to test Taylor expansion around $\mu_I = 0$.

- Started to measure the equation of state at finite μ_I.

- An interesting application: construction of compact boson stars.

- A lot of other interesting things to look at ...

Thank you for your attention!