Confinement-deconfinement transition in dense SU(2) QCD

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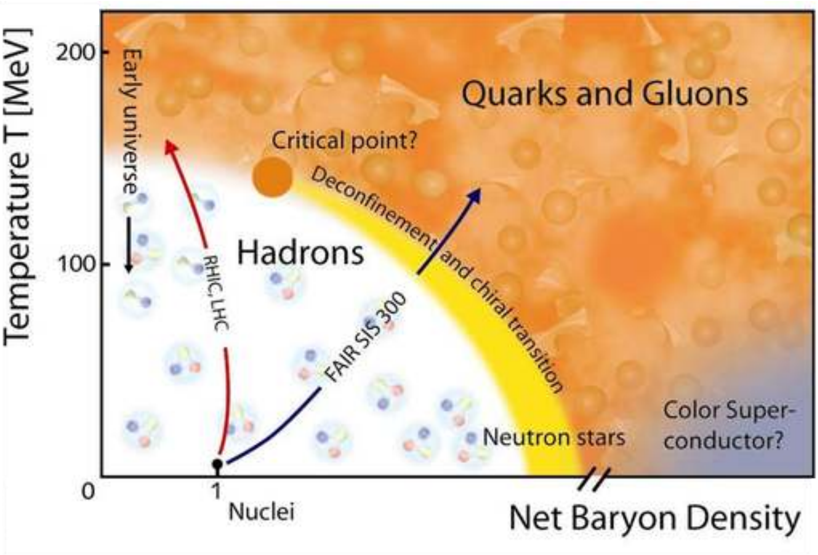
Outline

- Introduction
- Specifics of QC_2D
- Results at zero T
- Results at finite T
- Conclusions

QCD phase diagram



No sign problem in QC_2D

SU(3) QCD

- Eigenvalues of $\hat{D}$: $\pm i\lambda$, $\det(\hat{D}+m) = \prod_\lambda (\lambda^2 + m^2) > 0$
- But $\det(\hat{D} - \mu\gamma_4 + m)$ is complex

SU(2) QCD

- $$det\left[M(\mu_q)\right] = det\left[(\tau_2 C\gamma_5)^{-1} M(\mu_q)\,(\tau_2 C\gamma_5)\right] =$$
  $$= det\left[M(\mu_q^*)\right]^*, \text{ where } C = \gamma_2\gamma_4$$
- In LQC$_2$D with fundamental quarks $det\left[M(\mu_q)\right]$ is positive definite at real μ_q [see S. Hands, I. Montvay, S. Morrison, M. Oevers, L. Scorzato, J.-I. Skullerud, EPJ **C17**, 285 (2000)]

At real μ_q in QC$_2$D
$det\left[M(\mu_q)\right]$ is real, $det\left[M^\dagger(\mu_q)M(\mu_q)\right] > 0$ at $m_q \neq 0$.

Global symmetries in SU(2) QCD

$$\mathcal{L} = \overline{\psi}\gamma_\mu D_\mu \psi = i \begin{pmatrix} \psi_L^* \\ \psi_R^* \end{pmatrix}^T \begin{pmatrix} \sigma_\mu D_\mu & 0 \\ 0 & -\sigma_\mu^\dagger D_\mu \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\sigma_\mu = (\sigma_k, -i), \quad \sigma_2 \sigma_\mu \sigma_2 = -\sigma_\mu^T$$

4-spinor may be defined as $\Psi = \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ \sigma_2 \tau_2 \psi_R^* \end{pmatrix}$ and

$$\mathcal{L} = i \begin{pmatrix} \psi_L^* \\ \tilde{\psi}_R^* \end{pmatrix}^T \begin{pmatrix} \sigma_\mu D_\mu & 0 \\ 0 & \sigma_\mu D_\mu \end{pmatrix} \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix} = i \Psi^\dagger \sigma_\mu D_\mu \Psi$$

- $SU(2N_f)$ symmetry instead of $SU_R(N_f) \times SU_L(N_f)$
- Symmetry breaking scenario is $SU(2N_f) \to Sp(2N_f)$, Goldstone bosons ($N_f = 2$): $\pi^+, \pi^-, \pi^0, d, \bar{d}$

Similarities between QC_2D and QCD

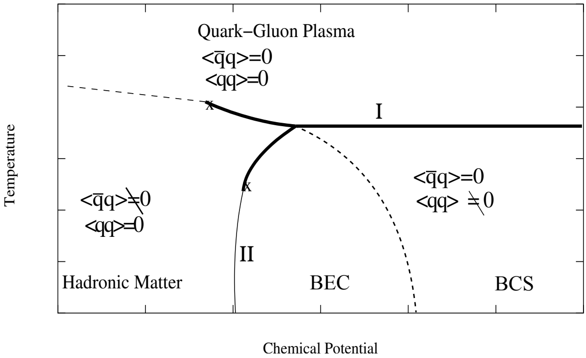
- Phase transitions: confinement/deconfinement, chiral symmetry restoration
- Some observables (normalized) are nearly equal in both theories:

  Topological susceptibility [B. Lucini *et. al.*, Nucl. Phys. B715 (2005) 461]:
  $\chi^{1/4}/\sqrt{\sigma} = 0.3928(40)\ (SU(2)), \quad \chi^{1/4}/\sqrt{\sigma} = 0.4001(35)\ (SU(3))$

  Critical temperature [B. Lucini *et. al.*, Phys. Lett. B712 (2012) 279]:
  $T_c/\sqrt{\sigma} = 0.7092(36)\ (SU(2)), \quad T_c/\sqrt{\sigma} = 0.6462(30)\ (SU(3))$

  Shear viscosity:
  $\eta/s = 0.134(57)\ (SU(2))$ [N.Yu. Astrakhantsev *et. al.*, JHEP 1509 (2015) 082]
  $\eta/s = 0.102(56)\ (SU(3))$ [H.B. Meyer, PRD 76 (2007) 101701]
- Mass spectrum (T. DeGrand, Y. Liu, PRD 94, 034506 (2016))
- Thermodynamical properties (M. Caselle *et. al.* JHEP 1205 (2012) 135)

Tentative phase diagram of QC_2D



- J.B. Kogut *et. al.*, Nucl. Phys. **B582** (2000) 477–513
- J.B. Kogut, D. Toublan, D.K. Sinclair, Nucl. Phys. **B642** (2002) 181–209
- S. Cotter, P. Giudice, S. Hands, J.-I. Skullerud, PRD **87**, 034507 (2013)
- T. Boz, S. Cotter, L. Fister, D. Mehta ,J.-I. Skullerud, EPJ **A49** (2013)
- V.V. Braguta *et. al.*, PRD **94**, 114510 (2016) (our previous study)

Diquark source

In QC_2D there is a possibility to add diquark source to the action to study spontaneous breakdown of $U(1)_V$:

$$S_F = \sum_{x,y} \left[\overline{\chi}_x M(\mu_q)_{xy} \chi_y + \frac{\lambda}{2} \delta_{xy} \left(\chi^T \tau_2 \chi + \overline{\chi} \tau_2 \overline{\chi}^T \right) \right],$$

which modifies partition function as follows:

$$Z = \int DU \, det \left[M^\dagger(\mu_q) M(\mu_q) + \lambda^2 \right]^{\frac{1}{2}} e^{-S_G[U]}$$

instead of

$$Z = \int DU \, det \, M(\mu_q) e^{-S_G[U]} .$$

$\langle qq \rangle$ is colorless, gauge invariant and thus may be measured.

Action and lattice set-up

We study $N_f = 2$ of rooted staggered fermions:

$$Z = \int DU\, det\left[M^\dagger(\mu_q)M(\mu_q) + \lambda^2\right]^{\frac{1}{4}} e^{-S_G^{impr.}[U]}\,,$$

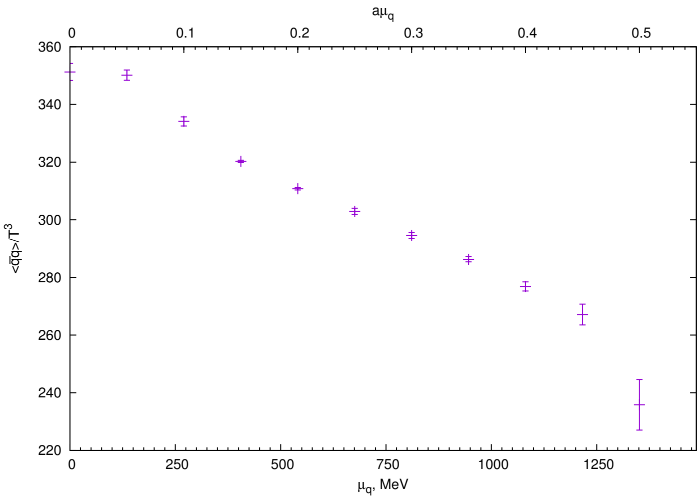
where $S_G^{impr.}[U]$ is the tree-level improved gauge action and

$$M_{xy}(\mu_q) = m_q a\delta_{xy} + \frac{1}{2}\sum_{\mu=1}^{4}\eta_\mu(x)\Big[U_{x,\mu}\delta_{x+\hat{\mu},y}e^{\mu_q a\delta_{\mu,4}} - \\ -U^\dagger_{x-\hat{\mu},\mu}\delta_{x-\hat{\mu},y}e^{-\mu_q a\delta_{\mu,4}}\Big]\,.$$

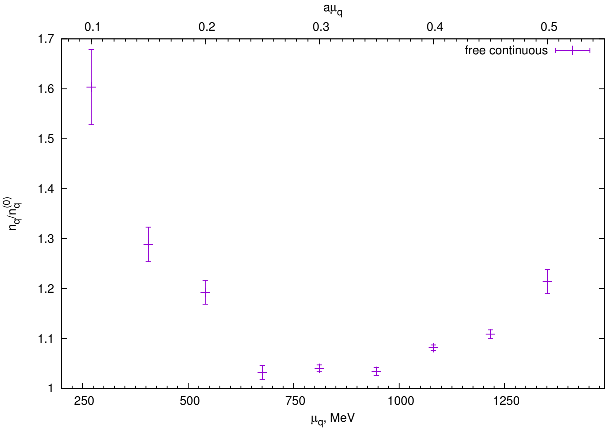
Simulation parameters

- Lattices: 32^4 ($T = 0$), $32^3 \times 24 \ldots 8$ (finite temperature)
- $\beta = 1.8$, $a = 0.073(1)$ fm (from $\sqrt{\sigma} = 440$ MeV), $L_s \approx 2.3$ fm
- $ma = 0.0075$, $M_\pi = 434(24)$ MeV; $M_\pi L_s \approx 5$
- Fixed $\lambda = 0.00075$, $\lambda^2 << (ma)^2$

Zero temperature results: chiral condensate

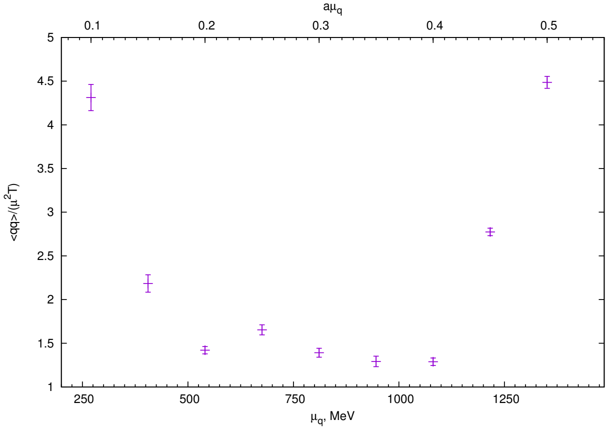


Zero temperature results: baryon density



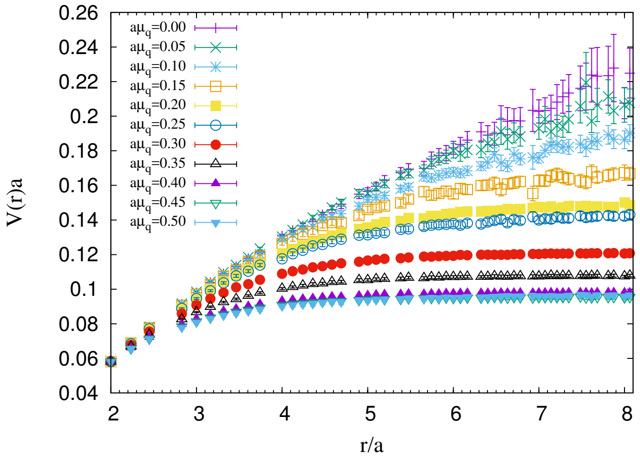
- $n_q^{(0)} = (4\mu_q^3)/(3\pi^2)$
- Plateau for $a\mu_q \in [0.25; 0.35]$, quarks inside the Fermi sphere

Zero temperature results: diquark condensate



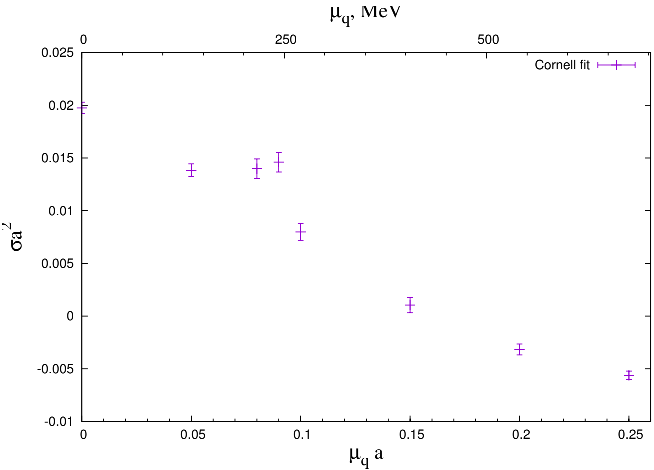
- $\langle qq \rangle \propto \mu_q^2$ for $a\mu_q \in [0.2; 0.4]$
- Baryons on the Fermi surface

Zero temperature results: $V_{\bar{Q}Q}$



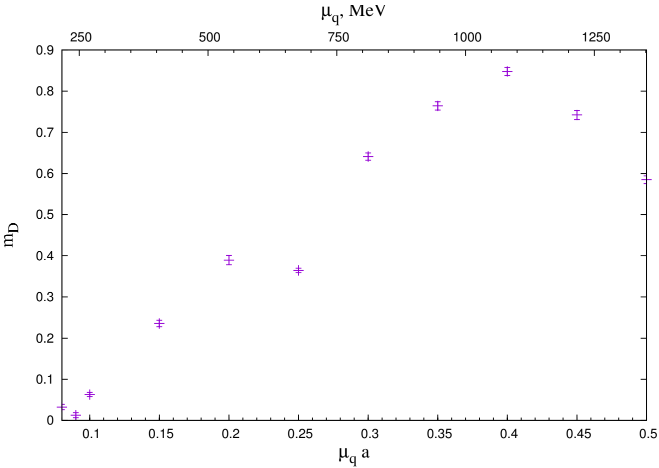
- Potentials were extracted from $\langle Tr[L(0)]\,Tr[L^{+}(r)]\rangle$
- HYP + APE smearing were employed (HYP2 par. set)

Zero temperature results: $V_{\bar{Q}Q}$ fitting (Cornell)



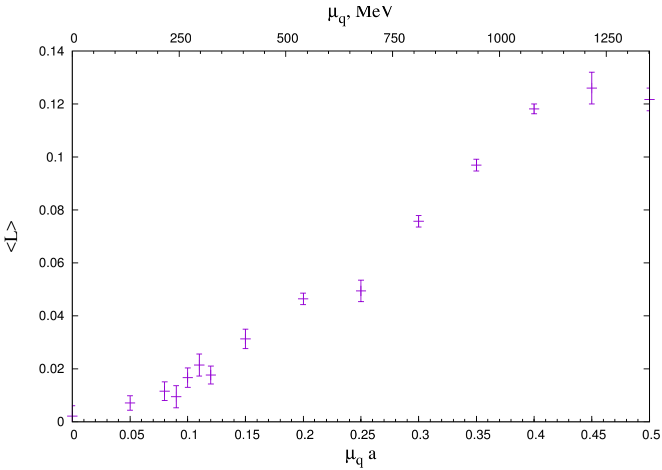
- Fit with $V(r) = A - B/r + \sigma r$, $r \in [3.0; 6.8]$
- Not suitable for $a\mu_q \geq 0.2$, σ becomes negative

Zero temperature results: $V_{\bar{Q}Q}$ fitting (Debye)



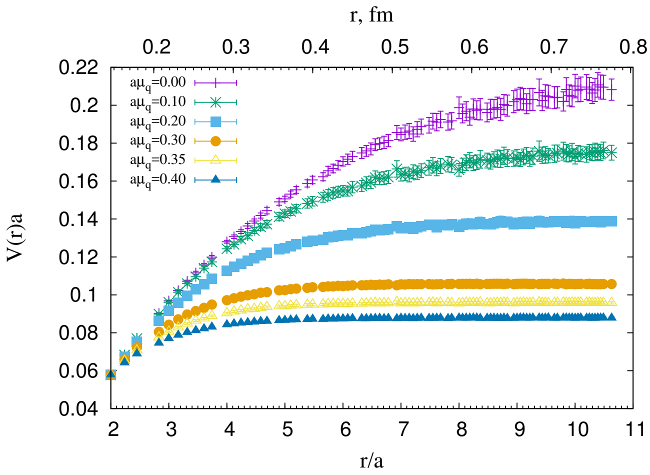
- Fit with $V(r) = V_0 - Be^{-m_D r}/r$, where $V_0 = -2T\ln(L)$, $r \in [3.0; 7.8]$
- $\chi^2/dof > 1$ for $a\mu_q \leq 0.2$, but for $a\mu_q > 0.2$ it drops below 0.35

Zero temperature results: Polyakov loop



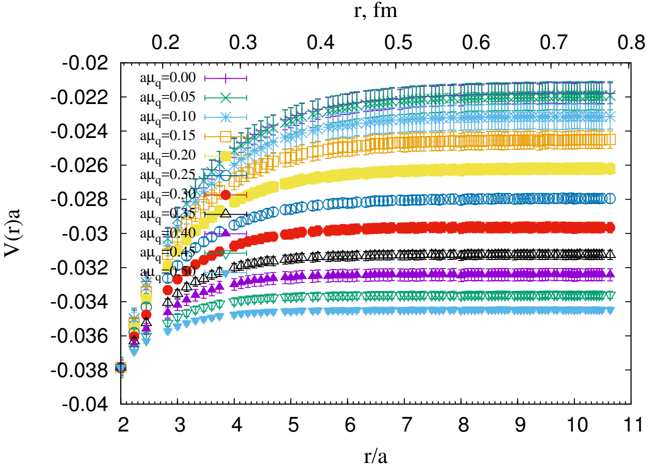
- HYP + APE smearing were employed (HYP2 par. set)

Finite temperature results: $V_{\bar{Q}Q}$



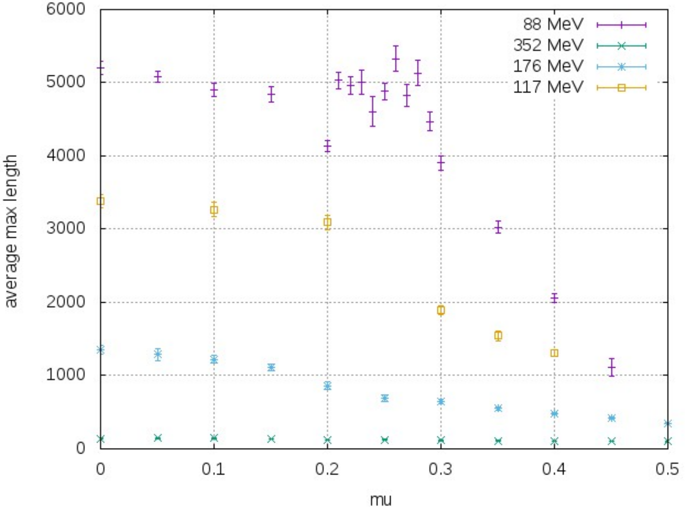
- $32^3 \times 24$ lattices, $T \approx 115$ MeV

Finite temperature results: $V_{\bar{Q}Q}$



- $32^3 \times 8$ lattices, $T \approx 350$ MeV

Average maximal length of the percolating cluster



Conclusions

- Crucial difference in the behaviour of $V_{\bar{Q}Q}(r)$ in the hadronic and BCS phases. At low temperatures $V_{\bar{Q}Q}(r)$ clearly saturates in the dense phase
- BEC-BCS transition is not easy to locate
- Debye potential seems to be reasonable fit at large enough μ_q

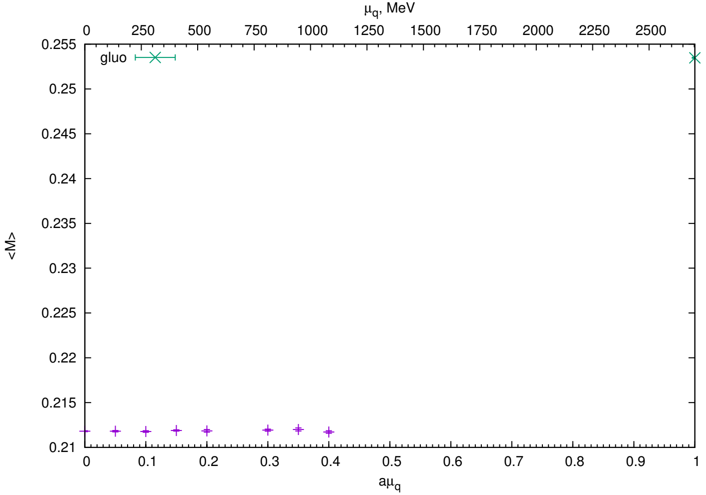
Ongoing study

- Gluon propagators
- Monopoles

The next talk by O. Hajizadeh is also devoted to QC_2D

Thank you for attention

Z_2 monopole density



$$M = 1 - \left(\sum_{cubes} \prod_{P\in\partial C} sign[TrU_P]\right)/N_{cubes}$$

Average plaquette