

η and η' masses and decay constants

Jakob Simeth, Sara Collins, Gunnar Bali

Universität Regensburg – Institute for Theoretical Physics

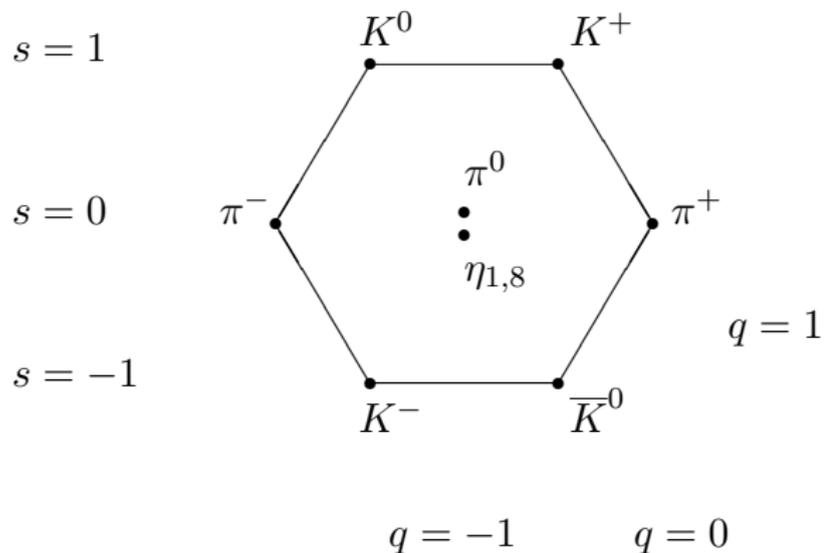
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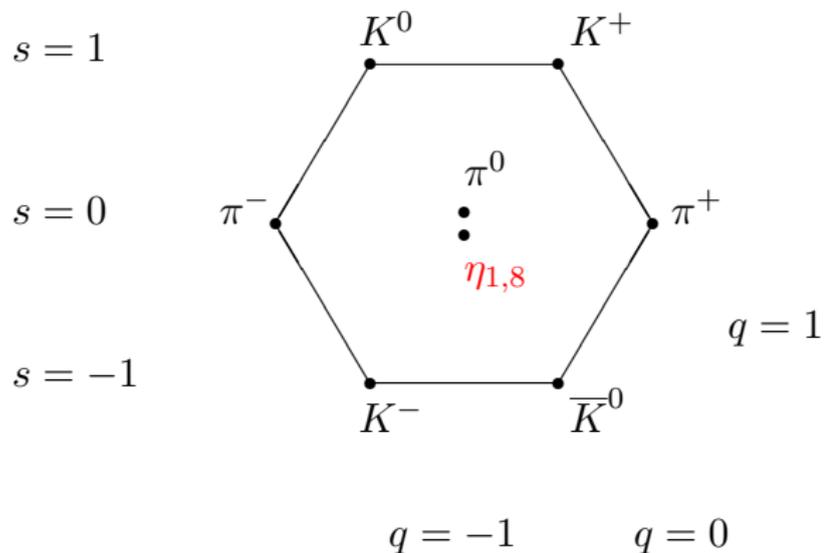
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 - masses
 - decay constants
- 4 Conclusions



$$SU(3) : 3 \otimes \bar{3} = 8 \oplus 1$$



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- Simple $SU(3)$ flavour symmetry fails: Predicted singlet and octet particles

$$\eta_1 = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}}, \quad \eta_8 = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}} \quad (1)$$

- In reality, η and η' are not flavour eigenstates:

$$U(\theta, \theta') \begin{pmatrix} \mathcal{O}_{88} & \mathcal{O}_{81} \\ \mathcal{O}_{18} & \mathcal{O}_{11} \end{pmatrix} U^T(\theta, \theta') = \begin{pmatrix} \mathcal{O}_{\eta\eta} & 0 \\ 0 & \mathcal{O}_{\eta'\eta'} \end{pmatrix} \quad (2)$$

with mixing angles θ and θ' ,

$$\eta = \cos \theta \eta_8 - \sin \theta \eta_1, \quad \eta' = \sin \theta' \eta_8 + \cos \theta' \eta_1 \quad (3)$$

- In a flavour symmetric world: $\eta = \eta_8 = \pi^0$

- η' becomes heavy compared to the octet mesons due to anomalous breaking of $U_A(1)$ axial symmetry.
- Witten and Veneziano could first explain the large mass of the η' by relating it to the topological susceptibility χ :

$$\frac{f_\pi^2 m_{\eta'}^2}{N_f} = \chi \Big|_{N_f=0} \quad (4)$$

(in the t'Hooft limit of $N_c \rightarrow \infty$)

- Sensitive to topology of the gluonic vacuum
 \Rightarrow long autocorrelation times

- When performing the Wick contractions for \mathcal{O}_{88} , \mathcal{O}_{18} , \mathcal{O}_{81} and \mathcal{O}_{11} , disconnected correlators arise:

$$D_{f_1 f_2}(\delta t) = \frac{1}{N_t} \sum_t L_{f_1}(t) L_{f_2}(t + \delta t). \quad (5)$$

- Disconnected loops are "all-to-all": $L(t) = \text{tr } \gamma_5 D_{xx}^{-1}$
 \Rightarrow stochastic estimation
- Noise reduction:
 - time dilution: Put random sources at every 4th time slice
 - Hopping parameter expansion: use locality of the Wilson Dirac operator and expand in small κ
 \Rightarrow using two and four applications for the pseudoscalar and axialvector loops, respectively

- no mixing, trivial matrix diagonalization:

$$C_{\eta}(t) = C_{\pi}(t) = C_{con}(t), \quad C_{\eta'}(t) = C_{con}(t) - N_f D(t) \quad (6)$$

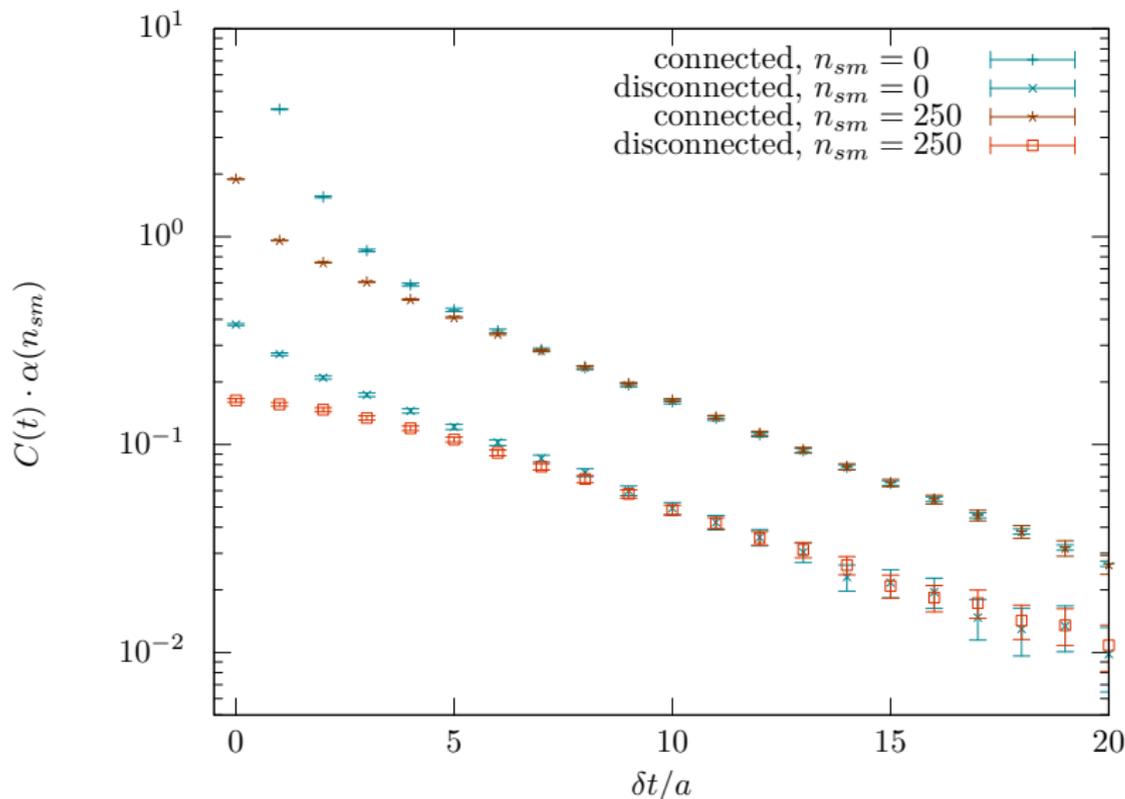
- Disconnected correlator asymptotically goes with $\exp(-m_{\pi}t)$:

$$D(t) = \frac{1}{N_f} (C_{con}(t) - C_{\eta'}(t)) \quad (7)$$

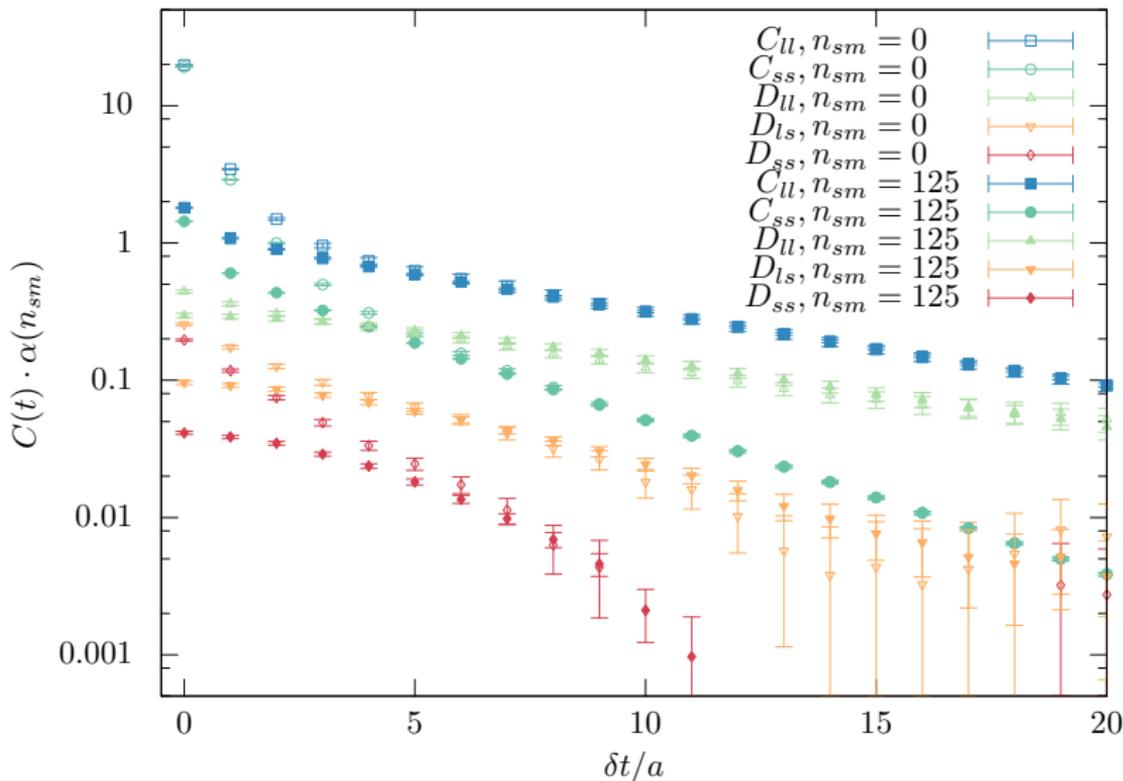
⇒ fitting the η' amounts to fitting a double-exponential

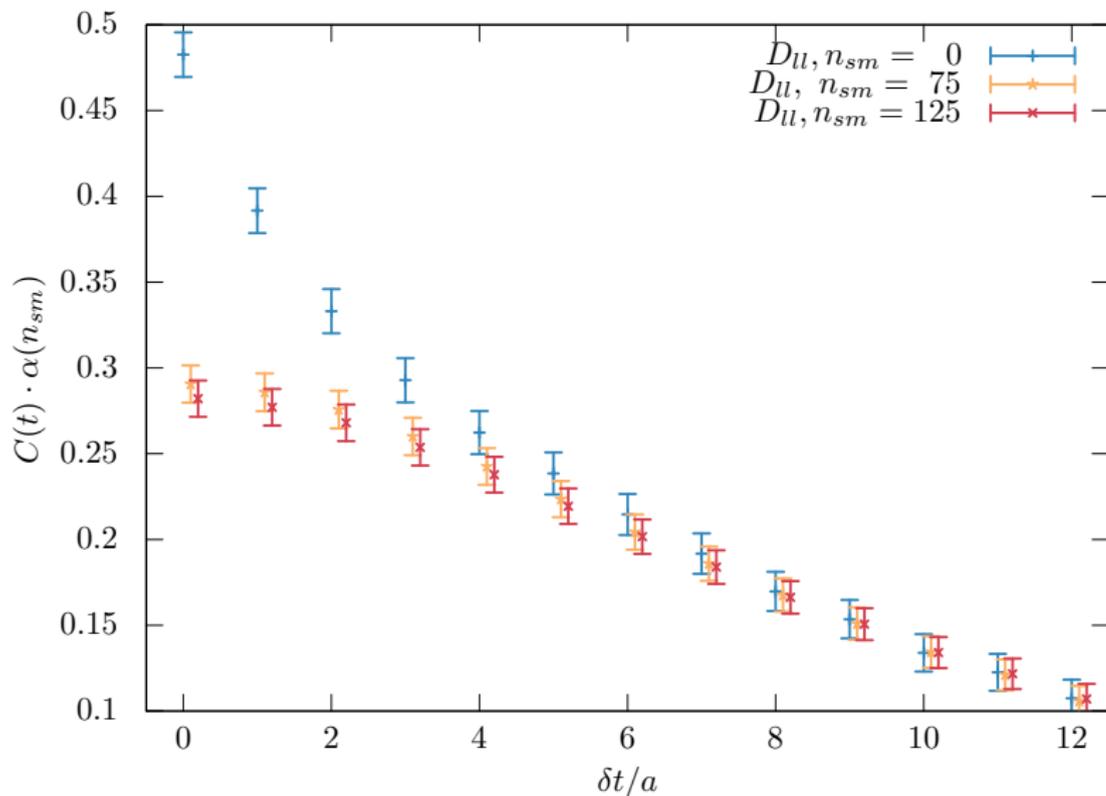
⇒ small time behaviour encodes η' physics!

- excited states???



⇒ Controlling excited states is crucial for reliable analysis





- Usual analysis solves the GEVP

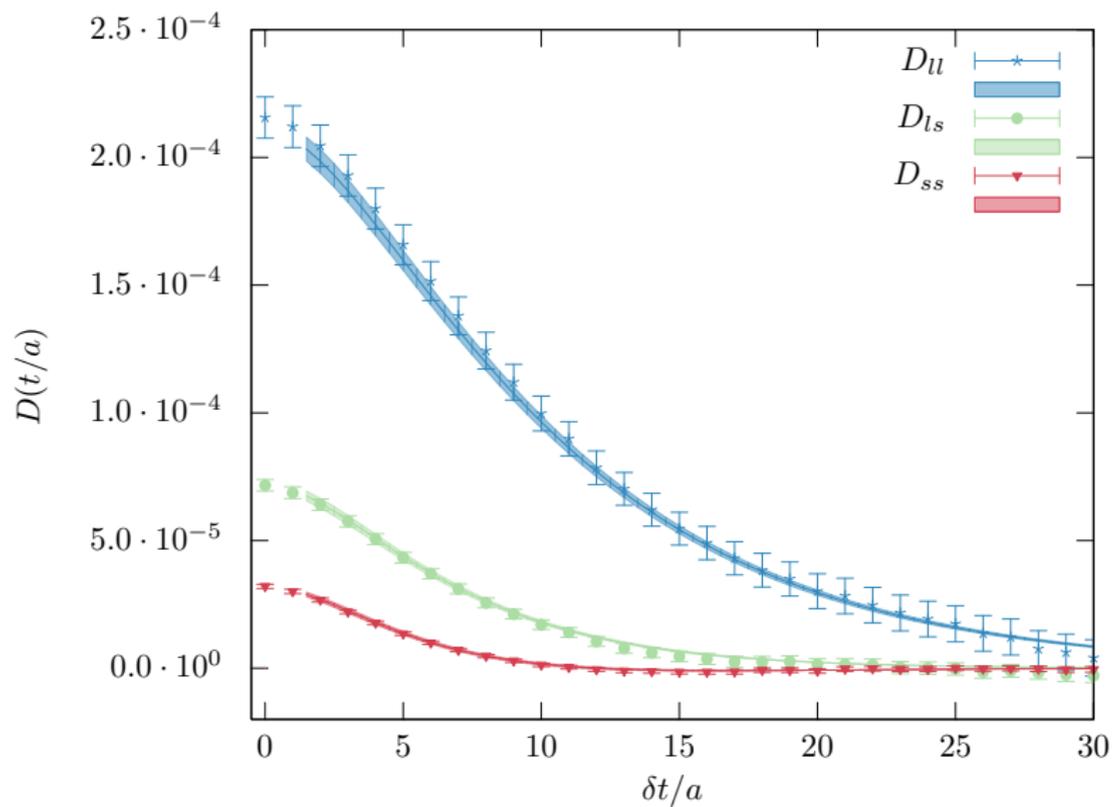
$$U(\theta, \theta') \begin{pmatrix} \mathcal{O}_{88} & \mathcal{O}_{81} \\ \mathcal{O}_{18} & \mathcal{O}_{11} \end{pmatrix} U^T(\theta, \theta') = \begin{pmatrix} \mathcal{O}_{\eta\eta} & 0 \\ 0 & \mathcal{O}_{\eta'\eta'} \end{pmatrix}. \quad (8)$$

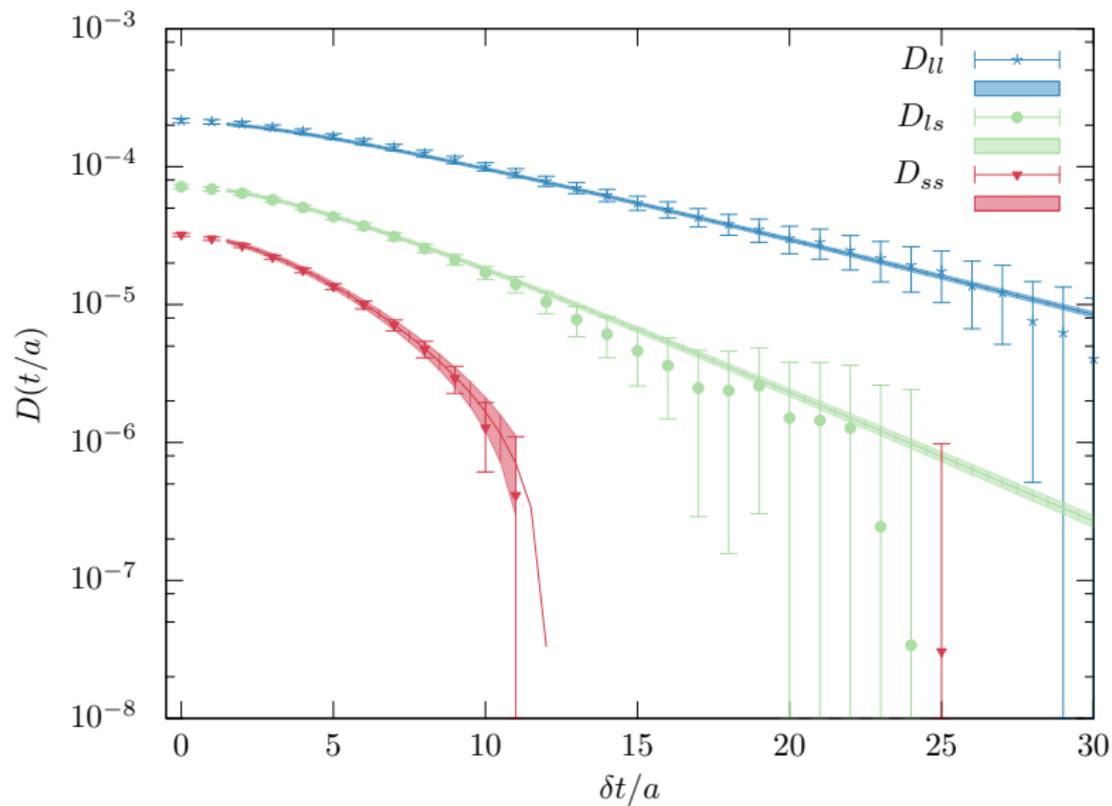
- Alternatively: Solve for the disconnected correlators $D_{\ell\bar{\ell}}$, $D_{s\bar{s}} = D_{s\bar{\ell}}$ and $D_{s\bar{s}}$

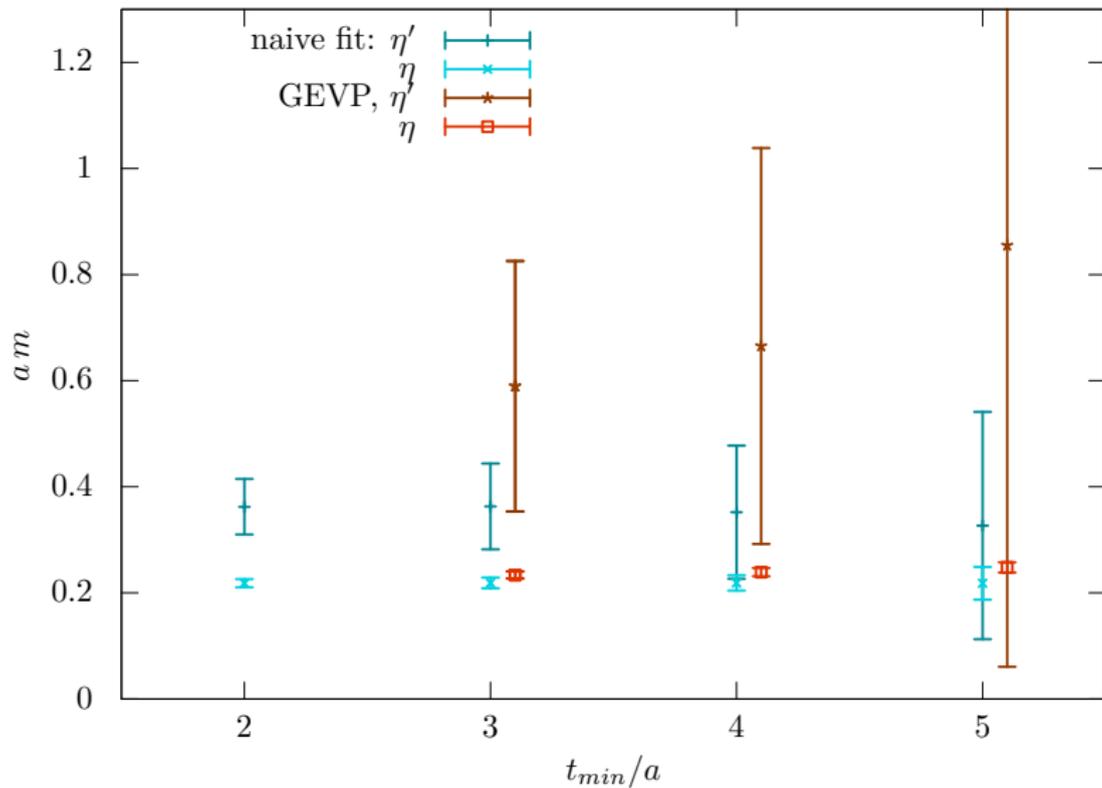
\Rightarrow obtain (complicated) fit forms, depending on ten parameters:

$A_{\eta^{(\prime)}}, m_{\eta^{(\prime)}}, \theta^{(\prime)}$ and the connected correlators, $C_{\pi}(t)$ and $C_{s\bar{s}}(t)$

- Combined fit to all disconnected correlators
- Connected correlators usually determined much more precisely: can be fitted separately and reduces parameter space to six.







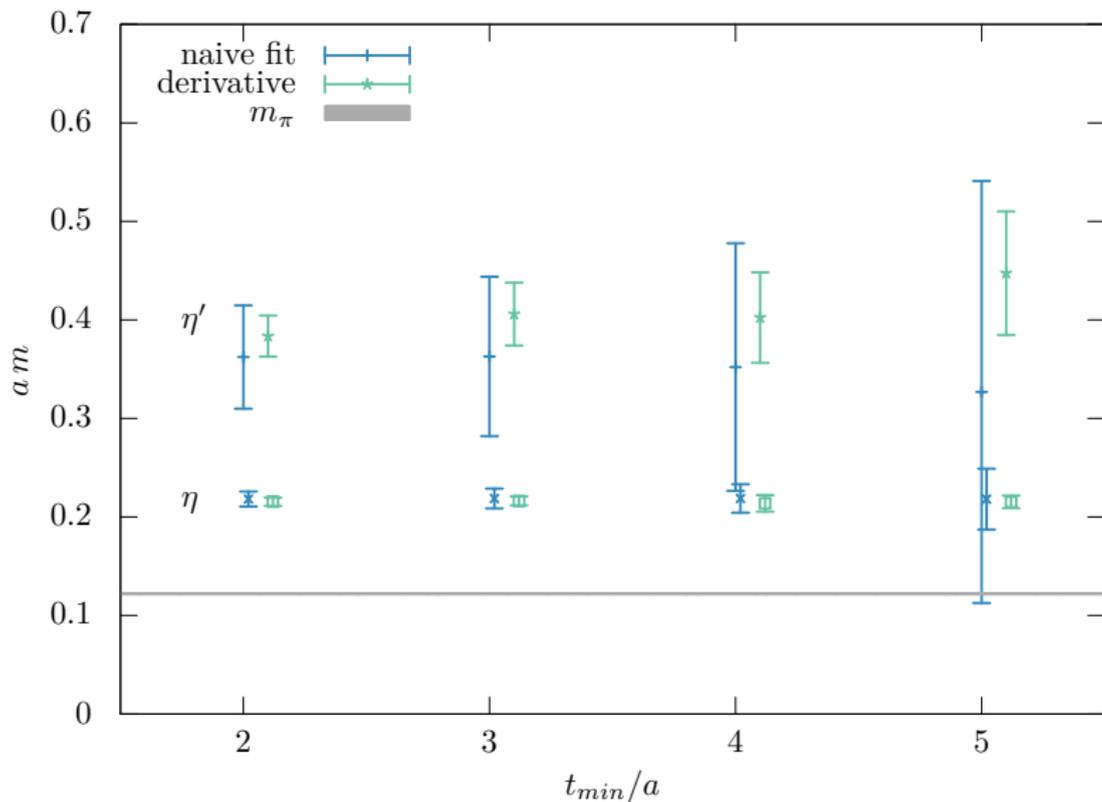
- Derivative trick: Fit to

[hep-lat/0701005, arxiv:0909.3255](#)

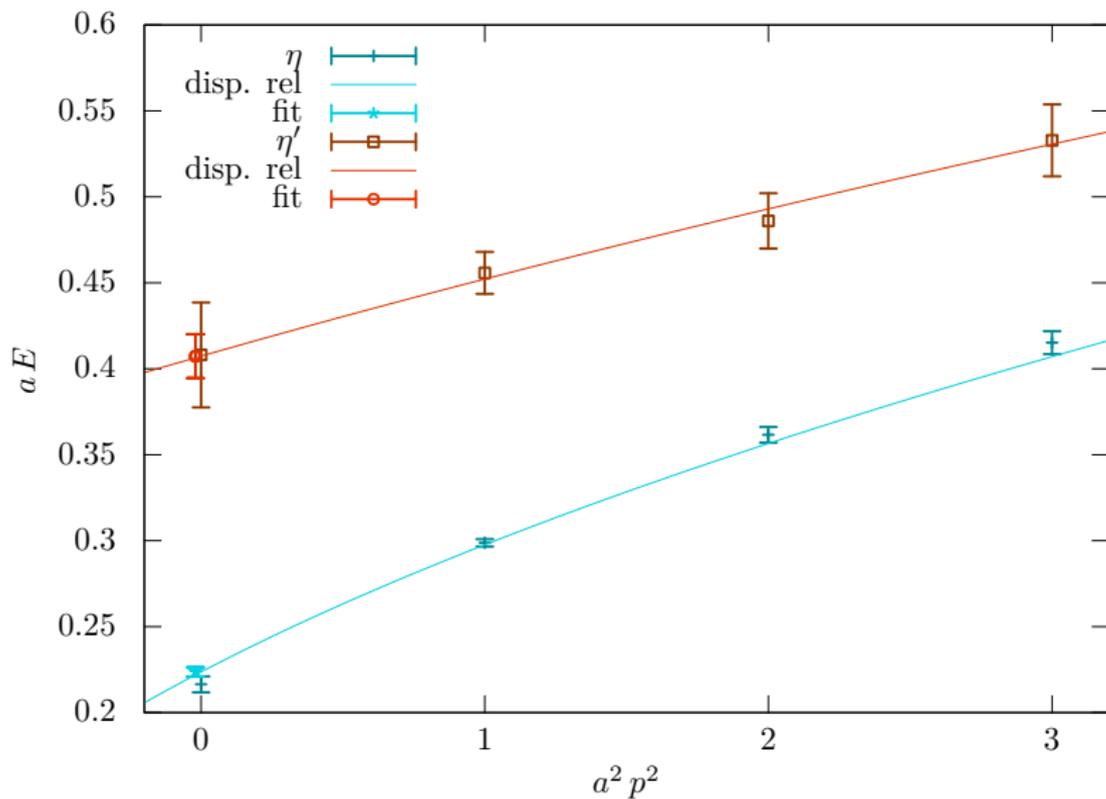
$$-a\hat{\partial}_4 C(\delta t) \approx a (C(\delta t) - C(\delta t + a)) \propto a \exp(-m\delta t) \quad (9)$$

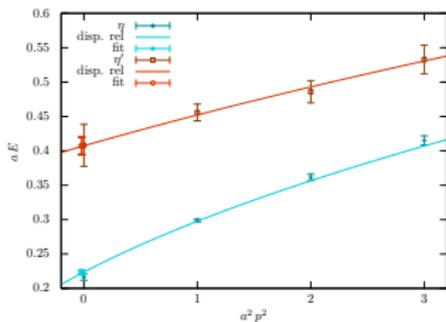
removes constant shifts (e.g., from topological objects) and correlations.

- for the masses: use dispersion relation to gain statistics



- Started with two CLS $N_f = 2 + 1$ ensembles with non-perturbatively improved Wilson fermions Symanzik gauge action:
 - **U103** at the flavour symmetric point, $m_\pi = m_K = 415$ MeV: ≈ 1200 configurations, separated by 16 MDU, 48 stochastic estimates per config
 - **H105** $m_\pi = 282$ MeV, $m_K = 467$ MeV: ≈ 700 configurations, separated by 16 MDU, 96 stochastic estimates per config
- $\beta = 3.4$, $a \approx 0.086$ fm
- errors obtained via a binned jackknife analysis





- **U103** (symmetric point)

$$m_\eta = 412(2) \text{ MeV} \quad (10)$$

$$m_{\eta'} = 833(46) \text{ MeV}$$

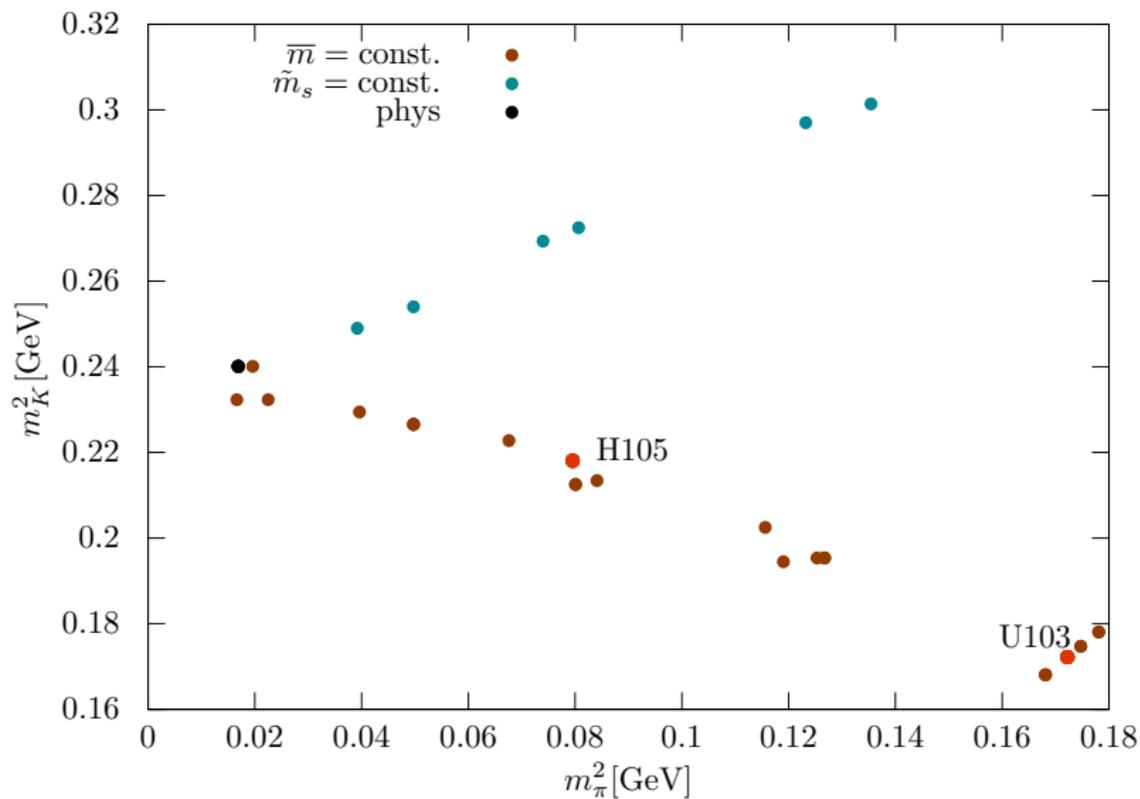
- **H105**

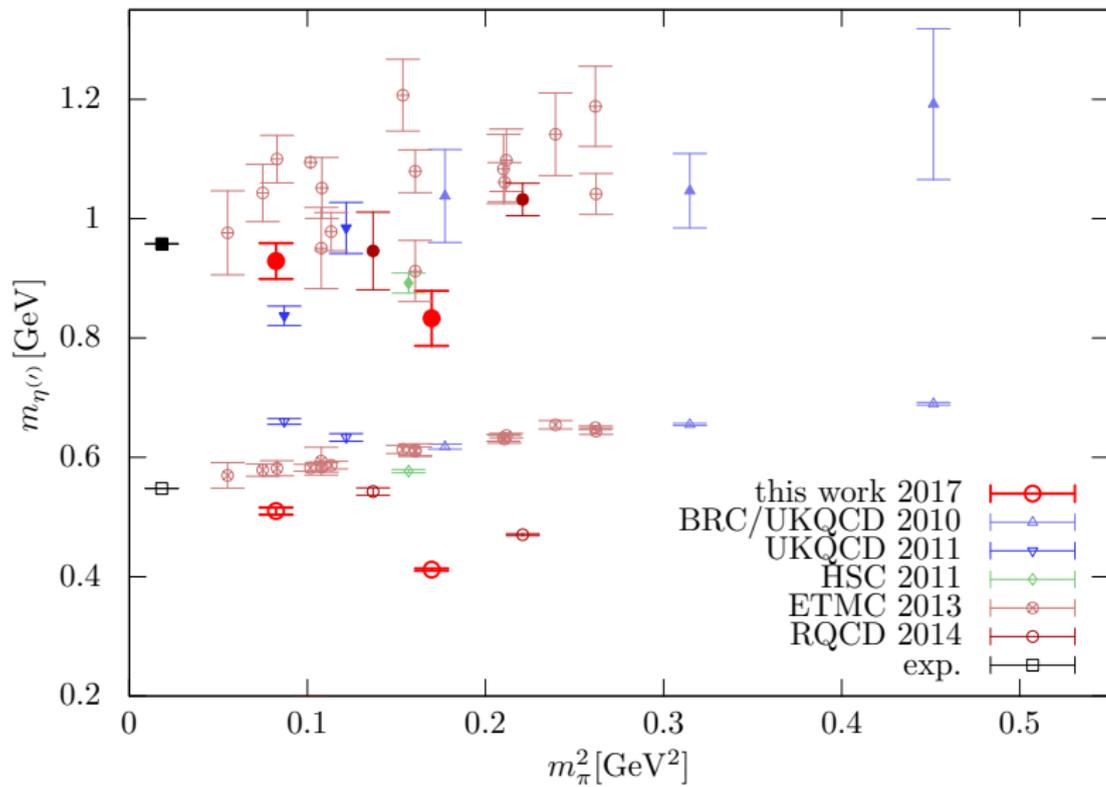
$$(m_\pi = 287 \text{ MeV}, m_K = 487 \text{ MeV})$$

$$m_\eta = 510(6) \text{ MeV} \quad (11)$$

$$m_{\eta'} = 929(30) \text{ MeV}$$

Results preliminary and errors are purely statistical.





- Need pseudoscalar-axialvector matrix element to annihilate with the vacuum:

$$\langle 0 | \partial_\mu A_\mu^{1,8} | \eta^{(\prime)}(0) \rangle = f_{\eta^{(\prime)}}^{1,8} m_{\eta^{(\prime)}}^2 \quad (12)$$

- decay constants usually parameterized by f_1 , f_8 and two angles θ_8 and θ_1

$$\begin{pmatrix} f_\eta^8 & f_\eta^1 \\ f_{\eta'}^8 & f_{\eta'}^1 \end{pmatrix} = \begin{pmatrix} f_8 \cos \theta_8 & -f_1 \sin \theta_1 \\ f_8 \sin \theta_8 & f_1 \cos \theta_1 \end{pmatrix} \quad (13)$$

- singlet renormalization: difference known perturbatively to two loops:

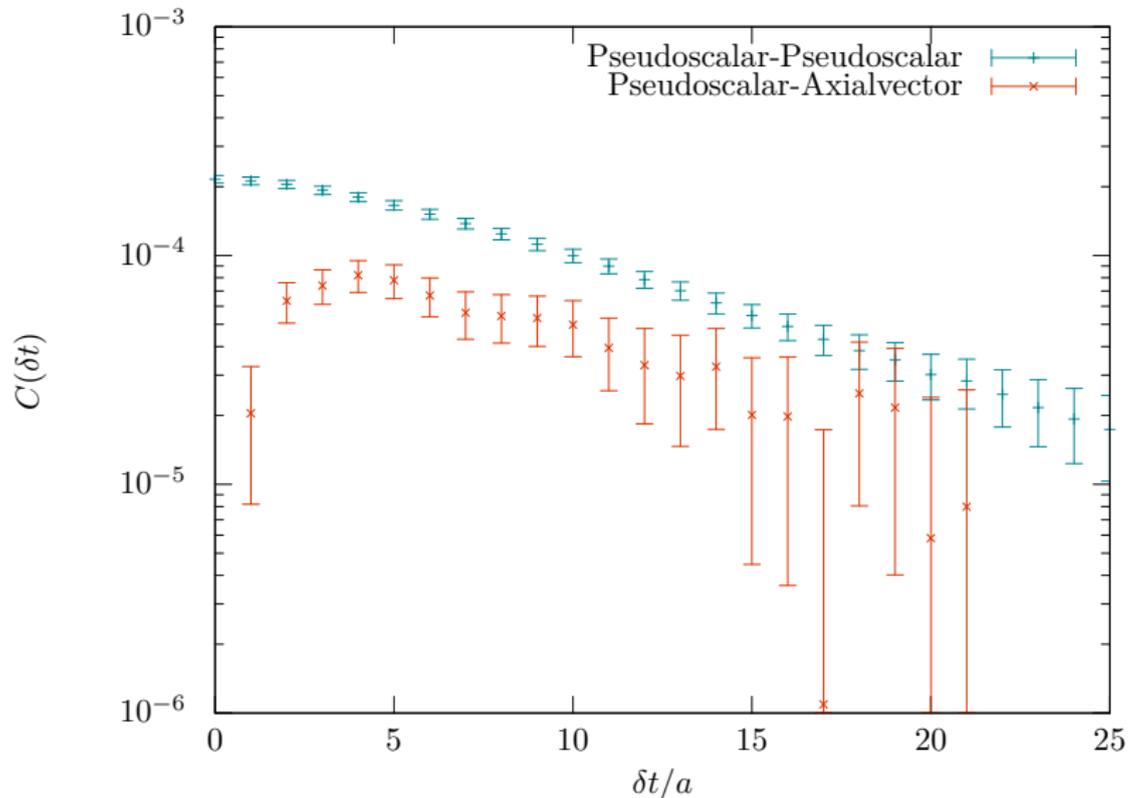
[arxiv:1604.05827](#), [arxiv:1610.06744](#)

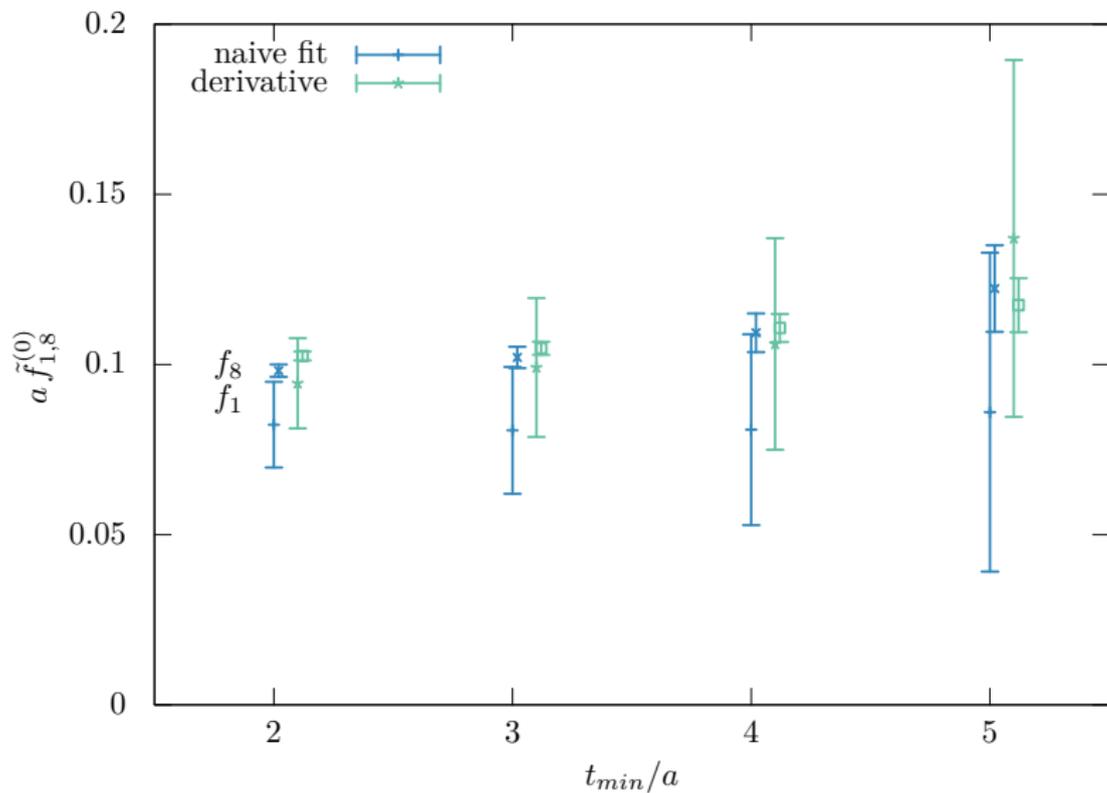
$$Z_A^1 = Z_A^{8,np} + g_0^4 z_A^{1,pt} + \mathcal{O}(g_0^6) \quad (14)$$

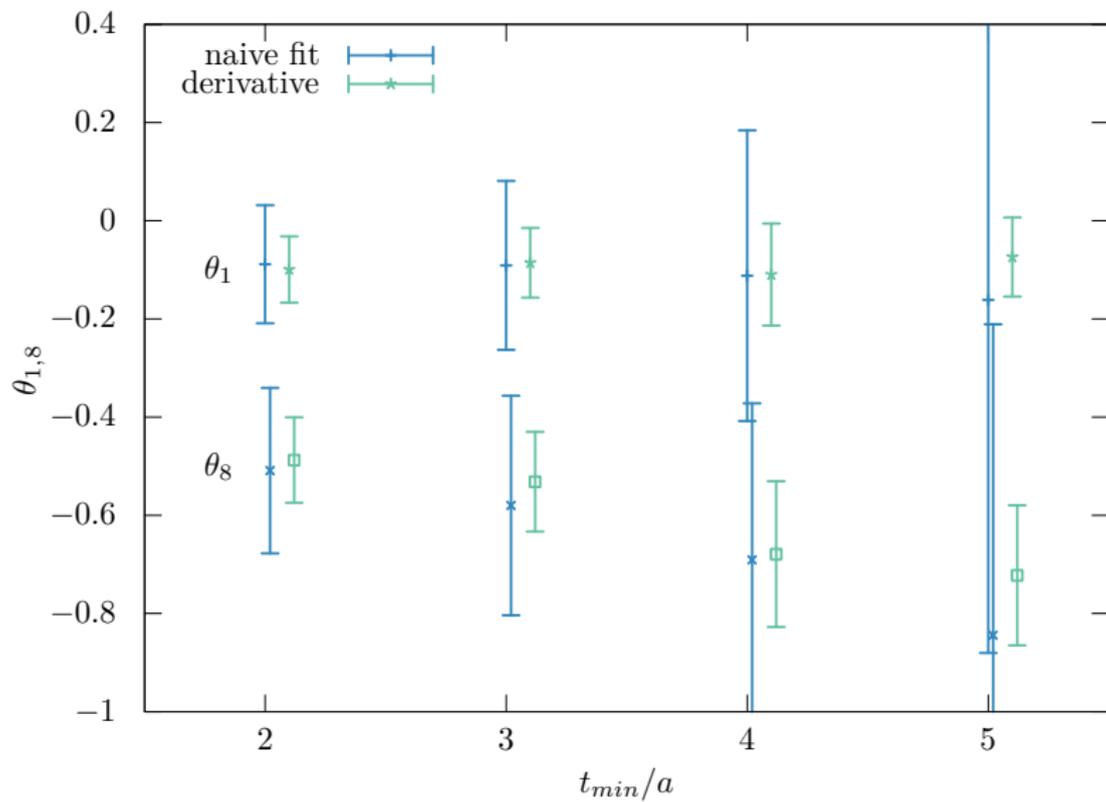
- $\mathcal{O}(a)$ improvement (neglecting small \tilde{b}_A coefficient):

[arxiv:1502.04999](#), [arxiv:1607.07090](#)

$$f_{1,8} = (1 + b_A a \overline{m}) Z_A^{1,8} (\tilde{f}_{1,8}^{(0)} + c_A a \tilde{f}_{1,8}^{(1)}) \quad (15)$$







- normalization: $f_{\pi}^{(exp)} = \sqrt{2}F_{\pi}^{(exp)} = 130 \text{ MeV}$
- **U103**: $N_f = 3$, no mixing, $\theta = \theta' = 0$

$$f_8 = f_{\pi} = 146(2) \text{ MeV}, \quad f_1 = 148(11)(2)^{\text{ren}} \text{ MeV}$$

- **H105**: $N_f = 2 + 1$

$$f_8 = 180(3) \text{ MeV}, \quad f_1 = 162(29)(2)^{\text{ren}} \text{ MeV}$$

$$\theta_8 = -0.54(8) = -31(5)^{\circ}, \quad \theta_1 = -0.10(8) = -6(5)^{\circ}$$

Results preliminary and errors are purely statistical.

- Noise reduction techniques allow accessing the η/η' system, including decay constants.
- Combined fit allows for a direct and controlled way of extracting physical quantities
- However, a understanding of excited states crucial for reliable analysis.
- Going towards finer lattice spacings will alleviate the “window problem”
- CLS ensembles along two different mass trajectories will allow for reliable chiral extrapolation.