Exploratory study of pion light-cone wavefunction using OPE on the lattice

William Detmold*, C.-J. David Lin** and *Santanu Mondal***

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Outline

- Calculation of light cone hadron structure directly on lattice is difficult because,
  - Euclidean space.
  - Reduced symmetry $(O(4) -> H(4)) \longrightarrow$ operator mixing and the associated power divergences.
- Recently proposed Ji's method is a nice alternative to it.
- A yet unexplored technique is to study hadron structure using lattice QCD using a fictitious heavy quark (Detmold and Lin, Phys.Rev. D73 (2006)) with primary goal to calculate parton distribution function.
- We extend and explore this method in the context of pion distribution amplitude.

OPE in the Euclidean space with a heavy quark

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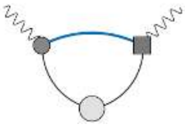

$$S^{\mu\nu}(p,q) = \int d^4x\ e^{iqx}\ \langle \pi^+(p)|\,T[V^{\mu}_{\Psi,\psi} A^{\nu}_{\Psi,\psi}]|0\rangle$$

Heavy light currents :

$$\begin{aligned} V^{\mu}_{\Psi,\psi} &= \overline{\Psi}\gamma^{\mu}\psi + \overline{\psi}\gamma^{\mu}\Psi \\ A^{\mu}_{\Psi,\psi} &= \overline{\Psi}\gamma^{\mu}\gamma^5\psi + \overline{\psi}\gamma^{\mu}\gamma^5\Psi \end{aligned}$$

$$s^{\mu\nu} = \overline{\Psi}\frac{-i(i\not{D}+\not{q})+m_\Psi}{(iD+q)^2+m_\Psi^2}\Psi + \overline{\Psi}\frac{-i(i\not{D}-\not{p}-\not{q})+m_\Psi}{(iD-p-q)^2+m_\Psi^2}\Psi$$

OPE of the heavy quark propagator:

$$\overline{\Psi}\frac{-i(i\not{D}+\not{q})+m_\Psi}{(iD+q)^2+m_\Psi^2}\Psi = -\overline{\Psi}\frac{-i(i\not{D}+\not{q})+m_\Psi}{D^2+q^2-m_\Psi^2}\sum_{n=0}^{\infty}\left(\frac{-2iq.D}{\tilde{Q}^2}\right)^n\Psi$$

$$\overline{\Psi}\frac{-i(i\not{D}+\not{q}+\not{p})+m_\Psi}{(iD+q+p)^2+m_\Psi^2}\Psi = -\overline{\Psi}\frac{-i(i\not{D}+\not{q}+\not{p})+m_\Psi}{D^2+q^2-m_\Psi^2}\sum_{n=0}^{\infty}\left(\frac{-2iq.D}{\tilde{Q}^2}\right)^n\Psi$$

$$\tilde{Q} = Q^2 - m_\Psi^2 + \alpha m_\Psi + \beta$$

Scale hierarchy: $\Lambda_{QCD}, p << |Q|, m_\Psi << a^{-1}$

Higher twist terms? Taylor expanding with respect to $(\frac{-2iq.D+D^2}{Q^2-m_\Psi^2}) \Longrightarrow$ scales as $(\frac{\Lambda_{QCD}^2}{q^2+m_\Psi^2})^n$.
Usual twist two operators and the twist three pseudo-scalar operators will contribute:

$$O^{\mu_1..\mu_n} = \overline{\Psi}\gamma^{\{\mu_1}\gamma^5(iD^{\mu_2})\ldots(iD^{\mu_n\}})\Psi - Traces \quad (1)$$

$$O^{\mu_1..\mu_n} = \overline{\Psi}\gamma^5(iD^{\{\mu_1})\ldots(iD^{\mu_n\}})\Psi - Traces \quad (2)$$

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Define moments of pion light cone distribution amplitude:

$$\langle \pi^+(p)| O^{\mu_1 .. \mu_n} |0\rangle = a_{n-1}\ f_\pi\ [p^{\mu_1} \cdots p^{\mu_n} - Traces]$$

Consider $\mu \neq \nu$

$$S^{\mu\nu} = 2i \sum_{n=0}^{\infty} \frac{(-\xi)^{n+1}}{n+1} a_n f_\pi \times \{4\eta C_n^2(\eta) \frac{(p^\mu p^\nu)}{p.q} + [-2\eta C_n^2(\eta) + (n+1) C_{n+1}^1(\eta)] \frac{q^{\{\mu} p^{\nu\}}}{q^2}\}$$

$$\xi = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2},\ \eta = \frac{p.q}{\sqrt{p^2 q^2}}$$

In a more compact form,

$$S^{\mu\nu} = \sum_{n=0}^{\infty} a_n f(n)$$

Where f(n) is known as a function of kinematic variables and the Wilson parameters.

a_n, α and β can be found by fitting lattice data.

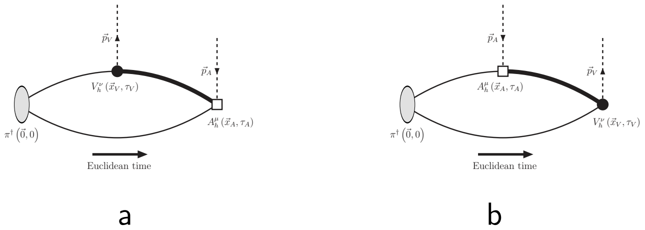


Figure: The three-point function, $C_3^{\mu\nu}(\tau_A,\tau_V;\vec{p}_A,\vec{p}_V)$, with the time orderings $\tau_A>\tau_V>0$ (a) and $\tau_V>\tau_A>0$ (b). The arrows on the dashed lines indicate the momentum flow. The thin (thick) lines represent propagators of light (heavy) quarks.

$$
\begin{aligned}
C_\pi(\tau_\pi;\vec{p}_\pi) &= \int d^3x e^{i\vec{p}_\pi\cdot\vec{x}} \left\langle 0 \middle| \mathscr{O}_\pi(\vec{x},\tau)\mathscr{O}_\pi^\dagger(\vec{0},0) \middle| 0 \right\rangle \\
C_3^{\mu\nu}(\tau_A,\tau_V;\vec{p}_A,\vec{p}_V) &= \int d^3x_A \int d^3x_V \\
&\quad e^{i\vec{p}_A\cdot\vec{x}_A} \mathrm{e}^{-i\vec{p}_V\cdot\vec{x}_V} \langle 0|\mathrm{T}[A_h^\mu(\vec{x}_A,\tau_A)V_h^\nu(\vec{x}_V,\tau_V)\mathscr{O}_\pi^\dagger(\vec{0},0)]|0\rangle
\end{aligned}
$$

$$
\begin{aligned}
C^{\mu\nu}_{3;\tau_A>\tau_V}(\tau_A,\tau_V;\vec{p}_A,\vec{p}_V) &= \sum_n \frac{1}{2E_n}\int \mathrm{d}^3x_A \int \mathrm{d}^3x_V e^{i\vec{p}_A\cdot\vec{x}_A} e^{-i\vec{p}_V\cdot\vec{x}_V} \\
&\times \left\langle 0 \left| A^{\mu}_h(\vec{x}_A-\vec{x}_V,\tau_A-\tau_V)\, V^{\nu}_h(\vec{0},0)\; e^{-i\hat{p}\cdot\vec{x}_V} e^{-\hat{H}\tau_V} \right| n \right\rangle \left\langle n \left| \mathscr{O}^{\dagger}_{\pi}(\vec{0},0) \right| 0 \right\rangle \\
&= \frac{1}{2E_\pi}\delta_{\vec{n}_\pi,\vec{n}_A-\vec{n}_V} \times e^{-E_\pi\tau_V}\langle \pi(\vec{p}_\pi) \left| \mathscr{O}^{\dagger}_{\pi}(\vec{0},0) \right| 0\rangle \\
&\quad \int \mathrm{d}^3x\; e^{i\vec{p}_A\cdot\vec{x}}\langle 0 \left| A^{\mu}_h(\vec{x},\tau_A-\tau_V)\, V^{\nu}_h(\vec{0},0) \right| \pi(\vec{p}_\pi)\rangle .
\end{aligned}
$$

Therefore, we can form the ratio

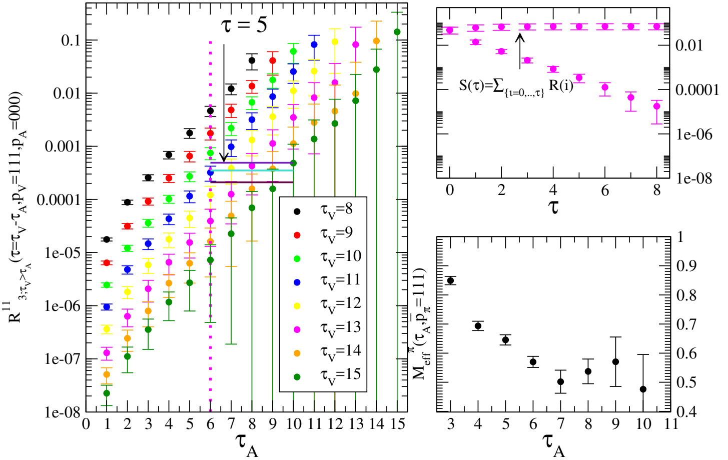
$$
\begin{aligned}
R^{\mu\nu}_{3;\tau_A>\tau_V}(\tau_A-\tau_V;\vec{p}_A,\vec{p}_\pi) &= \frac{C^{\mu\nu}_{3;\tau_A>\tau_V}(\tau_A,\tau_V;\vec{p}_A,\vec{p}_A-\vec{p}_\pi)}{C_\pi(\tau_V;\vec{p}_\pi)} \times \left\langle \pi(\vec{p}_\pi) \left| \mathscr{O}^{\dagger}_{\pi}(\vec{0},0) \right| 0 \right\rangle \\
&= \int \mathrm{d}^3x\; e^{i\vec{p}_A\cdot\vec{x}} \left\langle 0 \left| A^{\mu}_h(\vec{x},\tau_A-\tau_V)\, V^{\nu}_h(\vec{0},0) \right| \pi(\vec{p}_\pi) \right\rangle_{\tau_A>\tau_V} .
\end{aligned}
$$

After some algebra,

$$\int d\tau \; R^{\mu\nu}_{3;\tau_A>\tau_V}(\tau_A-\tau_V;\vec{p}_A,\vec{p}_\pi)\, e^{iq_4\tau} = S^{\mu\nu}(\vec{p}_\pi,E_\pi;-\vec{p}_A,-q_4)$$

From the back-side of the correlator:

$$\int d\tau \; R^{\mu\nu}_{3;\tau_V>\tau_A}(\tau_V-\tau_A;\vec{p}_A,\vec{p}_\pi)\, e^{iq_4\tau} = S^{\mu\nu}(\vec{p}_\pi,E_\pi;-\vec{p}_V,q_4)$$


τ = 5
τ_V=8
τ_V=9
τ_V=10
τ_V=11
τ_V=12
τ_V=13
τ_V=14
τ_V=15
τ_A
S(τ)=Σ_{ι=0,...,τ} R(i)
τ
τ_A
M_eff^π(τ_A, p̄_π=111)
R^11_3;τ_V>τ_A(τ=τ_V-τ_A,p_V=111,p_A=000)

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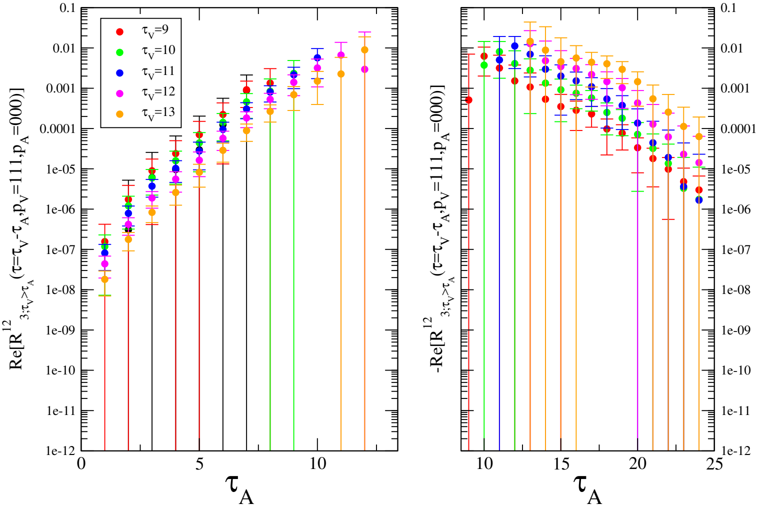

Summary

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- To maintain the scale hierarchy simulation at very fine lattice spacing is needed.
- Because of the heavy quark simulation is not that expensive, specially with quenched approximation.
- Careful continuum extrapolation needed to fit the data with the OPE formula.

Thanks for your attention!