Electromagnetic form factors and axial charge of the nucleon from $N_\mathrm{f} = 2 + 1$ Wilson fermions

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Lattice2017

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Introduction

- Charges and form factors of the nucleon encode important information about nucleon structure:
  - Electromagnetism: most important probe
  - Axial charge: crucial benchmark for precision

- Difficult observables:
  - Exponentially decaying signal-to-noise ratio
  - Large excited-state contamination

- Here: Update on Mainz effort to measure EM form factors and g_A on $N_\mathrm{f} = 2 + 1$ CLS ensembles

Observables

- Isovector nucleon matrix elements of vector and axial vector currents $V_\mu(x) = \overline{\psi}(x)\gamma_\mu\psi(x)$ and $A_\mu(x) = \overline{\psi}(x)\gamma_5\gamma_\mu\psi(x)$ decompose into form factors as

$$\left\langle N, \mathbf{p}', s' \middle| V_\mu(0) \middle| N, \mathbf{p}, s \right\rangle = \bar{u}(\mathbf{p}', s') \left[\gamma_\mu F_1(Q^2) + \frac{\sigma_{\mu\nu} Q_\nu}{2m_\mathrm{N}} F_2(Q^2) \right] u(\mathbf{p}, s),$$

$$\left\langle N, \mathbf{p}', s' \middle| A_\mu(0) \middle| N, \mathbf{p}, s \right\rangle = \bar{u}(\mathbf{p}', s') \left[\gamma_5 \gamma_\mu G_\mathrm{A}(Q^2) - i\gamma_5 \frac{Q_\mu}{2m_\mathrm{N}} G_\mathrm{P}(Q^2) \right] u(\mathbf{p}, s)$$

- Here we study the Sachs electromagnetic form factors

$$G_\mathrm{E}(Q^2) = F_1(Q^2) - \frac{Q^2}{2m_\mathrm{N}^2} F_2(Q^2)$$

$$G_\mathrm{M}(Q^2) = F_1(Q^2) + F_2(Q^2)$$

and the axial charge $g_\mathrm{A} = G_\mathrm{A}(0)$.

Observables

- Form ratios

$$R_J(t, t_s; \mathbf{q}) = \frac{C_{3,J}(t, t_s; \mathbf{q})}{C_2(t_s; \mathbf{q})} \sqrt{\frac{C_2(t_s - t; \mathbf{q}) C_2(t; \mathbf{0}) C_2(t_s; \mathbf{0})}{C_2(t_s - t; \mathbf{0}) C_2(t; \mathbf{q}) C_2(t_s; \mathbf{q})}}$$

[Alexandrou et al., 2008]

to determine effective form factors via

$$\mathrm{Re}\, R_{V_0}(t, t_s; \mathbf{q}) = \sqrt{\frac{m_\mathrm{N} + E_\mathbf{q}}{2E_\mathbf{q}}} G_\mathrm{E}^\mathrm{eff}(t, t_s; Q^2)$$

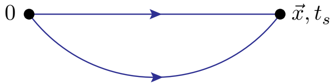
$$\mathrm{Re}\, R_{V_i}(t, t_s; \mathbf{q}) = \frac{\epsilon_{ij3} q_j}{\sqrt{2E_\mathbf{q}(E_\mathbf{q} + m_\mathrm{N})}} G_\mathrm{M}^\mathrm{eff}(t, t_s; Q^2)$$

$$\mathrm{Im}\, R_{A_3}(t, t_s; \mathbf{0}) = g_\mathrm{A}^\mathrm{eff}(t, t_s)$$

for baryons polarized using $\Gamma = \frac{1}{2}(1 + \gamma_0)(1 + i\gamma_5\gamma_3)$.

Observables

- Compute three-point functions $C_{3,J}((t, t_s; \mathbf{q})$ using sequential propagators



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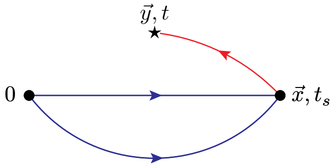

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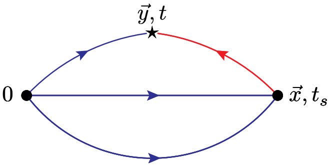

Techniques

- Compute three-point functions $C_{3,J}((t, t_s; \mathbf{q})$ using sequential propagators

- Truncated solver (AMA) to gain large statistics cheaply

[Bali et al., 2010; Shintani et al., 2014]

$$\mathcal{O}^{\mathrm{AMA}} = \frac{1}{N_G} \sum_{g \in G}^{N_G} \underbrace{\mathcal{O}^{(\mathrm{appx})\, g}}_{\mathrm{cheap}} + \frac{1}{N_{\mathrm{org}}} \sum_{f \in G}^{N_{\mathrm{org}}} \underbrace{\left[\mathcal{O}^{f} - \mathcal{O}^{(\mathrm{appx})\, f} \right]}_{\mathrm{expensive}}$$

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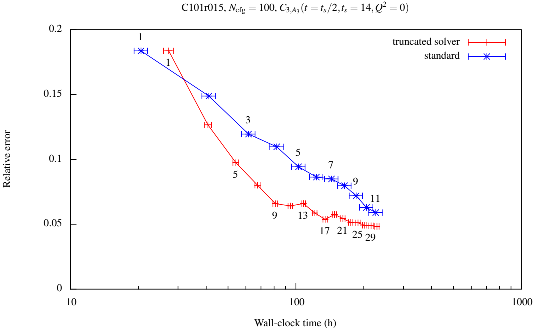

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- Use improved currents [Lüscher, 1997; Guagnelli, Sommer, 1998]

$$A_\mu^{\mathrm{I}} = A_\mu + a c_{\mathrm{A}} \partial_\mu P$$

$$V_\mu^{\mathrm{I}} = V_\mu + a c_{\mathrm{V}} \partial_\nu T_{\mu\nu}$$

and mass corrections b_{A}, $\tilde{b}_{\mathrm{A}}$ [Korcyl, Bali, 1607.07090]

Techniques

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$$A_\mu^{\mathrm{I}} = A_\mu + ac_{\mathrm{A}}\partial_\mu P$$
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- Nucleon sources and sinks are Wuppertal smeared using APE smeared links to improve overlap with ground state [Güsken et al, 1989; Albanese et al., 1987]

Excited states

- Excited states contribute at short time separations:

$$G_X^{\mathrm{eff}}(t,t_s) = G_X + \sum_{n>1}\left(a_n \mathrm{e}^{-(E_n-E_1)t} + b_n \mathrm{e}^{-(E_n'-E_1')(t_s-t)} + \ldots\right)$$

- For $t_s \gtrsim 1.0$ fm, corrections at $t = t_s/2$ non-negligible (no plateaux!)

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  - **Many-state fit**: for g_{A}, exploit that $a_n = b_n$ are slowly varying and $E_n' = E_n$ close to free to enable an explicit fit incorporating contributions from many excited states [Hansen, Meyer, 1610.03843]

CLS ensembles

Ensemble	a [fm]	m_π [MeV]	L/a	$m_\pi L$	$N_{\rm meas}$ per t_s	t_s/a
H102	0.086	350	32	4.9	7988	$\{12, 14, 16\}$
H105		280		3.9	55348	
C101		220	48	4.7	33344	
N200	0.064	280	48	4.4	20412	$\{12, 14, 16, 18, 20, 22\}$
D200		200	64	4.2	32672	$\{16, 18, 20, 22\}$
J303	0.05	280	64	4.1	5840	$\{20, 22, 24, 26\}$

- $N_{\rm f} = 2 + 1$ dynamical flavours of $\mathcal{O}(a)$-improved Wilson fermions, tree-level Symanzik gauge action [Bruno et al. 2014]
- Open boundary conditions in time to combat large autocorrelations in topological charge when approaching continuum limit [Lüscher, Schäfer 2011]
- Twisted-mass regulator to prevent problems with (near-)exceptional configurations [Lüscher, Palombi 2008]

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- additional finer lattice spacing
- increased statistics on most ensembles
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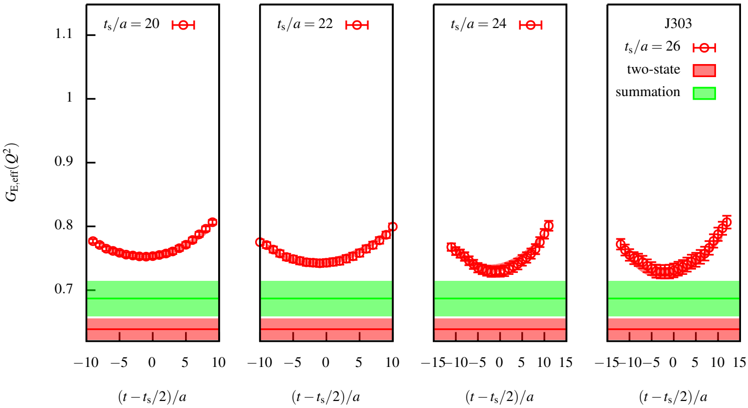

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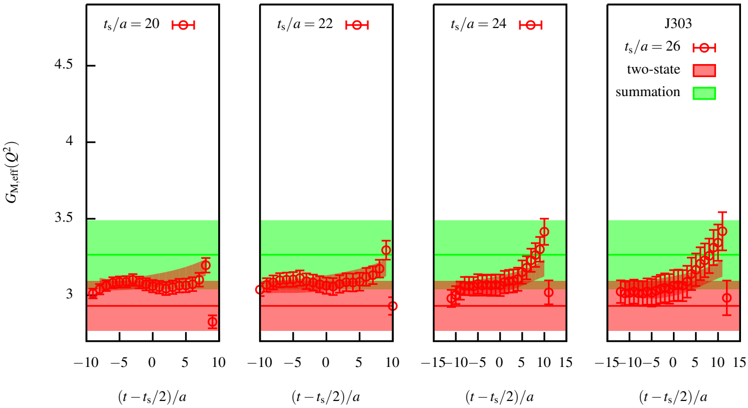
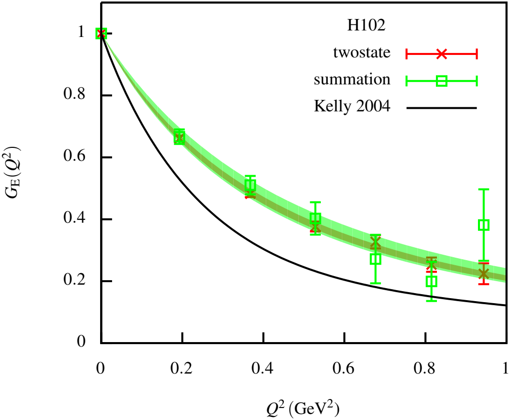


Results for EM form factors – Chiral trend for G_{E}



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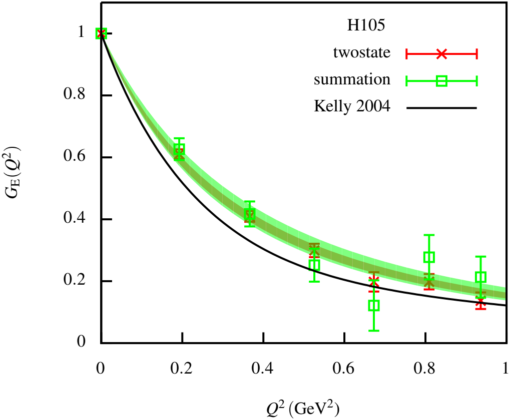

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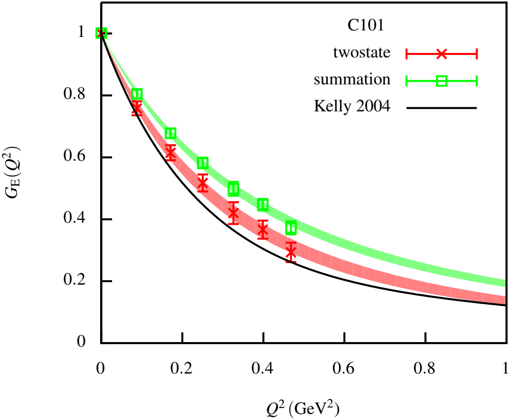
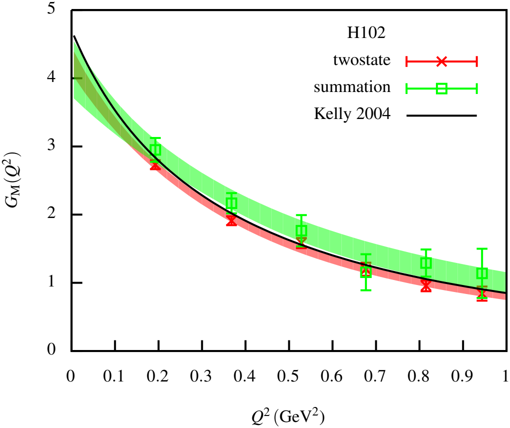


Results for EM form factors – Chiral trend for G_{M}



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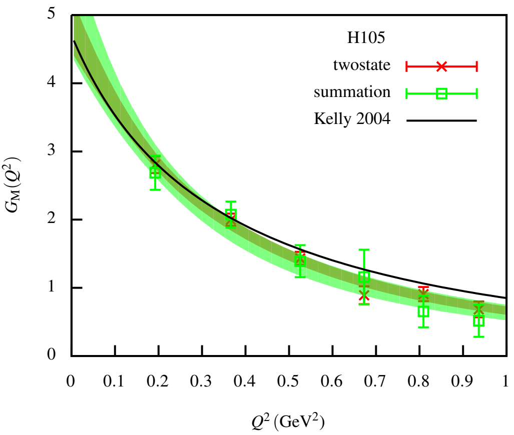

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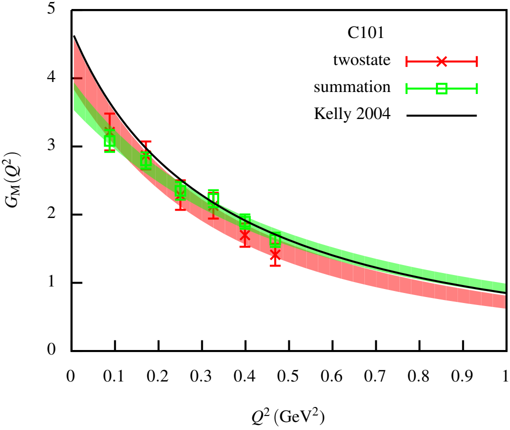
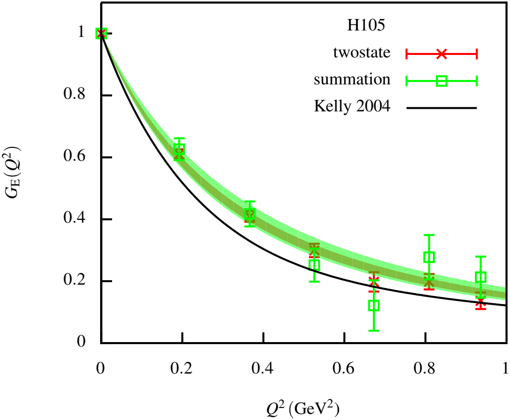


Results for EM form factors – Continuum trend for G_E



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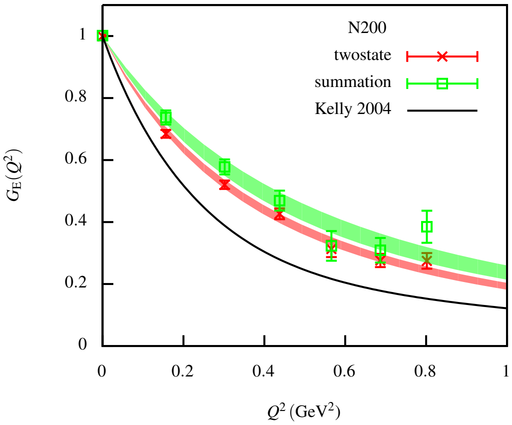

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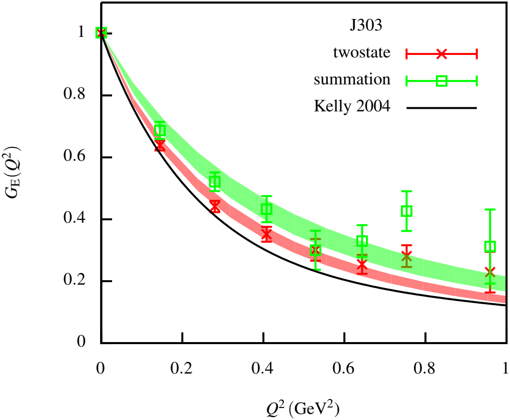

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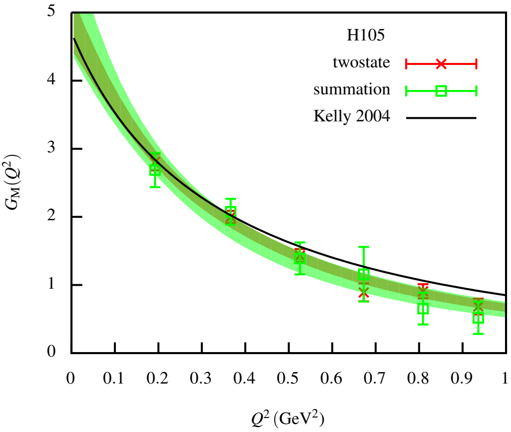

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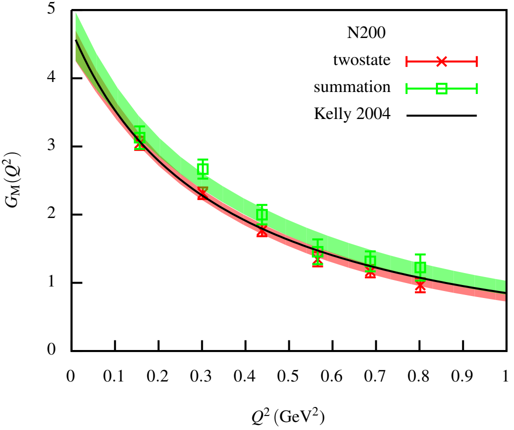

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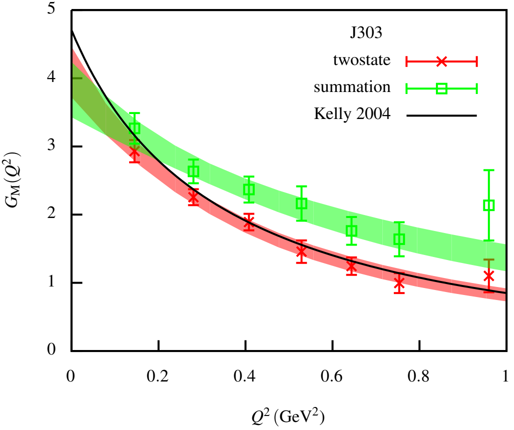

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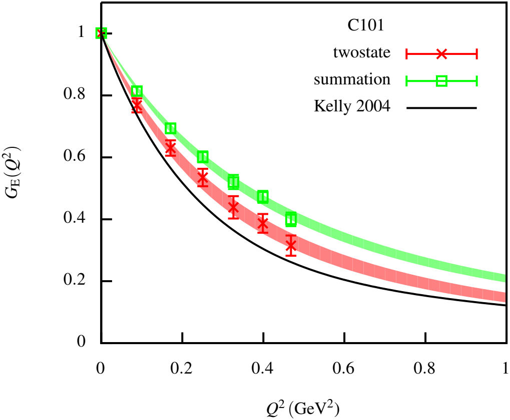

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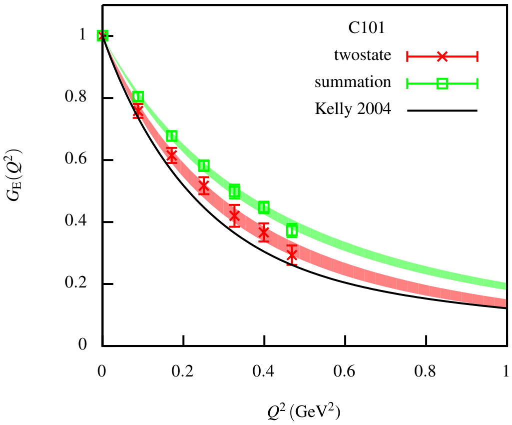

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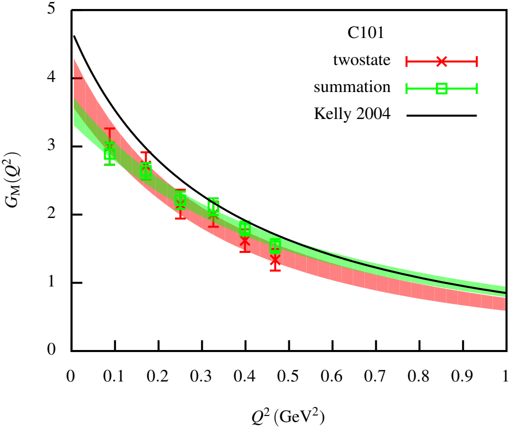

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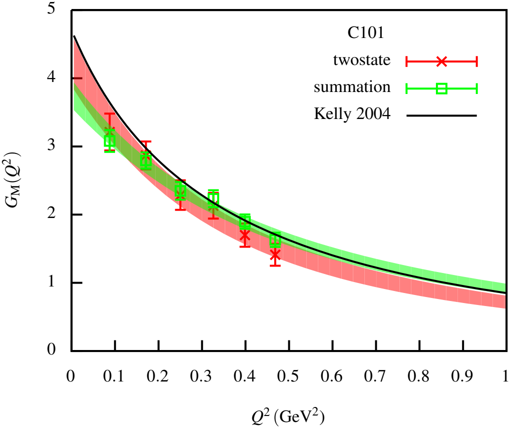

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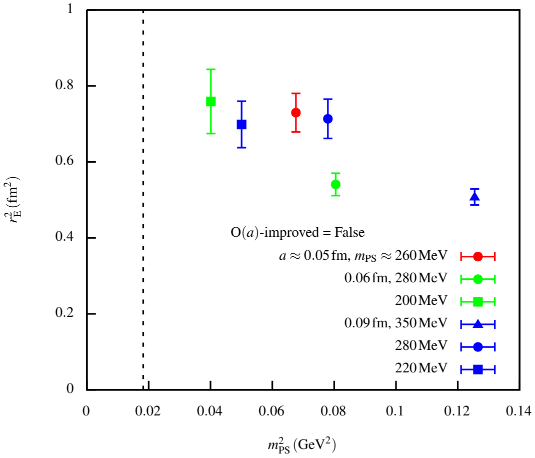

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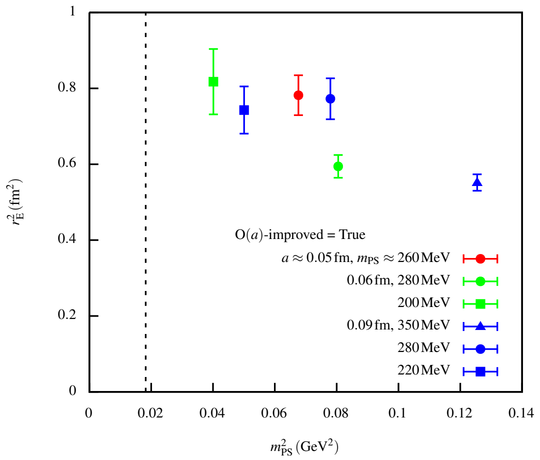

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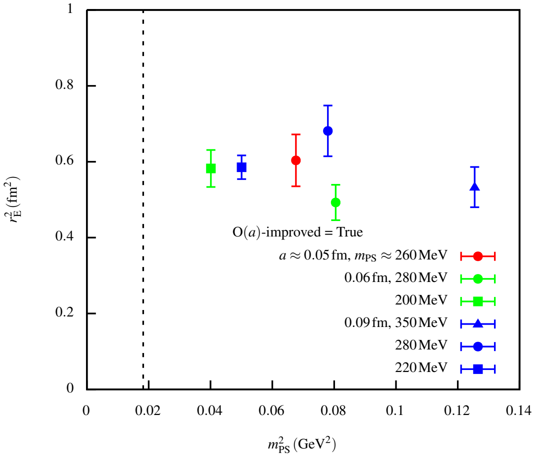

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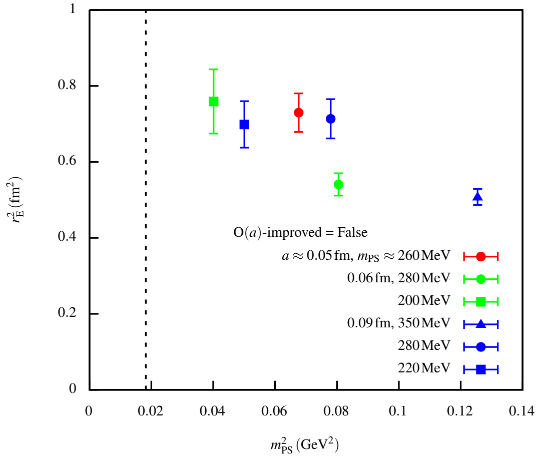

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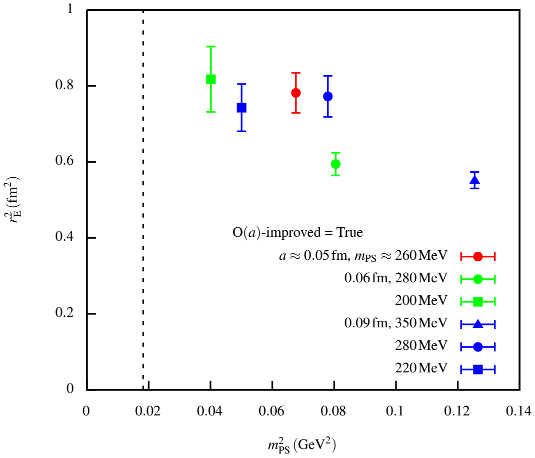

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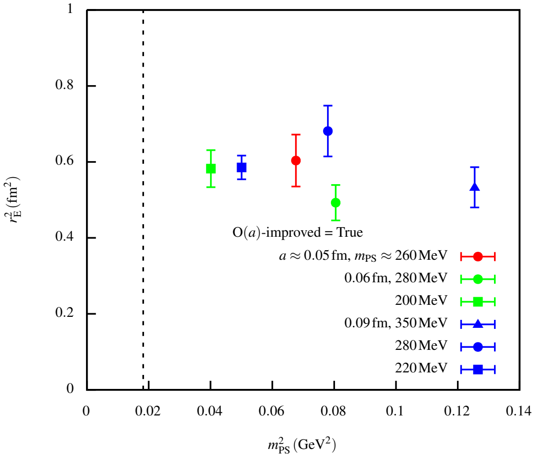

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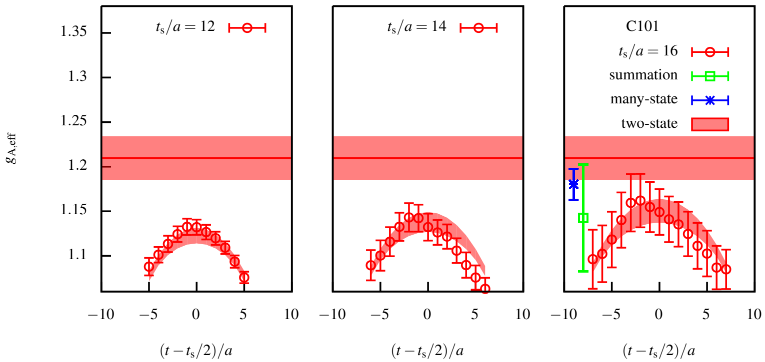

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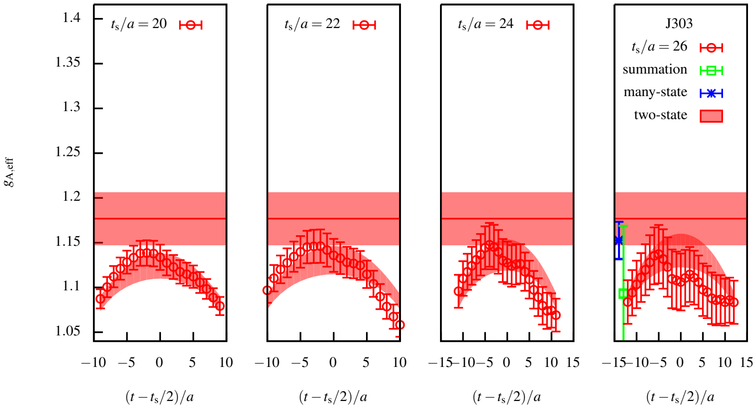

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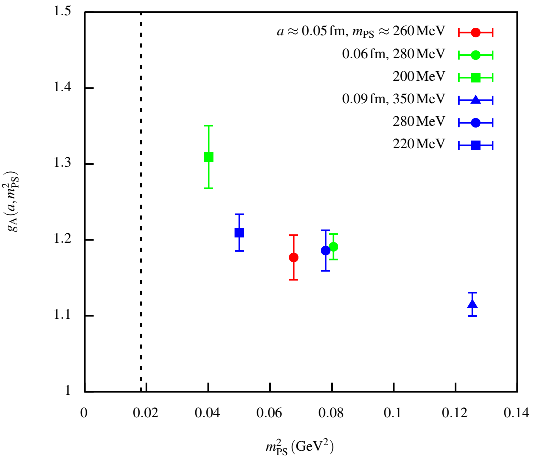

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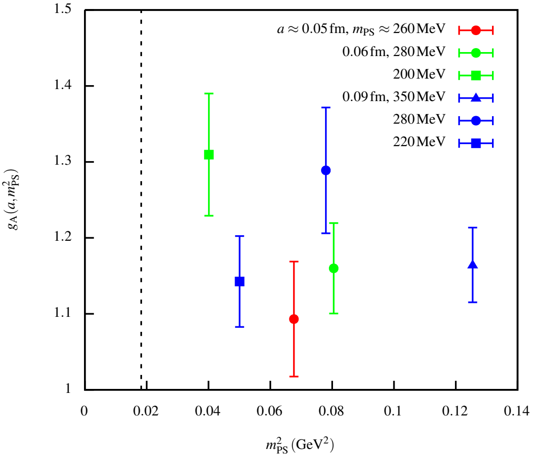


Outlook

- Not shown here:
  - Axial form factors $G_\mathrm{A}(Q^2)$, $G_\mathrm{P}(Q^2)$
  - Scalar and tensor charges g_S, g_T
  - Additional structure observables: $\langle x \rangle$, ... $\rightarrow$ Ottnad, Wed 12:50
- Work in progress:
  - Additional excited-state treatment (GPOF)
  - Additional pion masses at finest lattice spacings
  - Chiral and continuum extrapolation
- To be done:
  - Add point at physical pion mass $\rightarrow$ Mohler, Tue 18:10
  - Include isospin breaking effects $\rightarrow$ Poster by Risch
  - Also measure isoscalar quantities

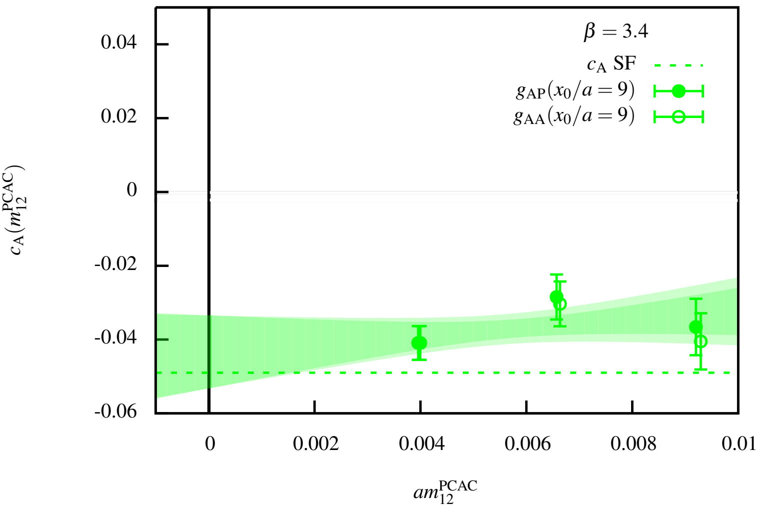
The End

Thank you for your attention

Backup Slides

– **BACKUP** –

Improvement coefficients



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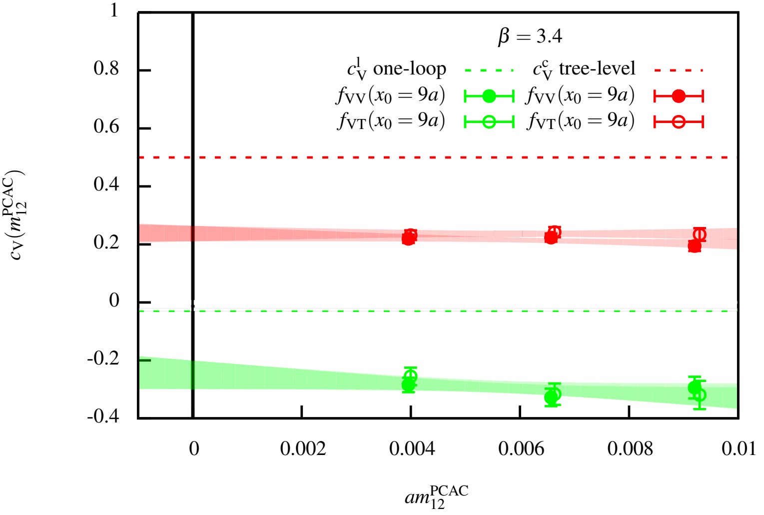

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