Nucleon quark content with smeared clover fermions

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Budapest-Marseille-Wuppertal collaboration
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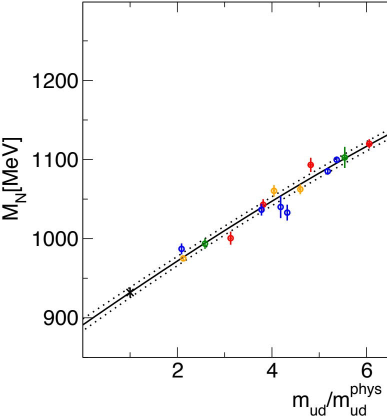
Scalar quark content of the nucleon



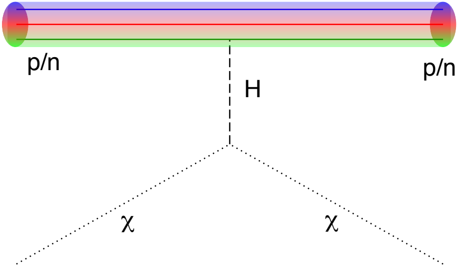
Appears in:

- Proton structure
- $N\pi$ and NK scattering
- Higgs coupling
- WIMP searches

Related to nucleon mass derivative by Feynman-Hellman theorem

$$f_q M_N = \sigma_q = m_q \langle N|\bar{q}q|N\rangle = \frac{\partial M_N}{\partial m_q} = \frac{\partial \langle N|m_q\bar{q}q|N\rangle}{\partial m_q}$$

Direct WIMP dark matter detection



- DM (χ) couples to quarks via Higgs
- Measured: nuclear recoil cross-section

☞ Need QCD to connect the two

Spin independent nuclear recoil cross-section for low energies:

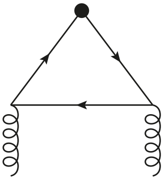
$$\frac{d\sigma_{\chi^A_Z X}}{dq^2} = \frac{1}{\pi V^2}\left(Z f_p + (A-Z) f_n\right)^2 \underbrace{|F_X(q^2)|^2}_{\xrightarrow{q^2\to 0} 1}$$

with the form factors from DM-quark couplings and quark content

$$f_N = \sum_q \lambda_q \langle N|\bar{q}q|N\rangle$$

Which quarks are most relevant?

Heavy quark relation: (Shifman et.al. 78)



$$m_Q\bar{Q}Q = -\frac{1}{3}\frac{\alpha_s}{4\pi}G^2 + O\left(\frac{\alpha_s^2}{m_Q^2}\right)$$

Since $M_N = \langle N|T_\mu^\mu|N\rangle$ and

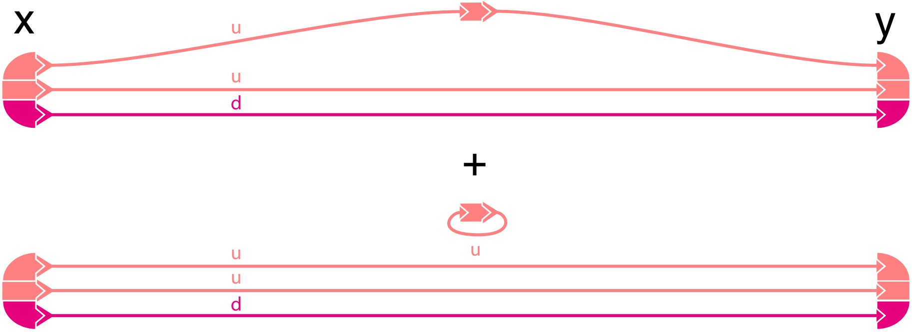
$$T_\mu^\mu = (1-\gamma_m(\alpha_s))\left(\sum_{q=u,d,s} m_q\bar{q}q + \sum_{Q=c,s,t} m_Q\bar{Q}Q\right) + \frac{\beta(\alpha_s)}{2\alpha_s}G^2$$

we can express the heavy in terms of the light quark content

$$\sigma_Q = m_Q\langle N|\bar{Q}Q|N\rangle = \frac{2M_N}{27}\left(1 - \sum_{q=u,d,s} m_q\langle N|\bar{q}q|N\rangle\right)(1+O(\alpha_s))$$

Nucleon quark content

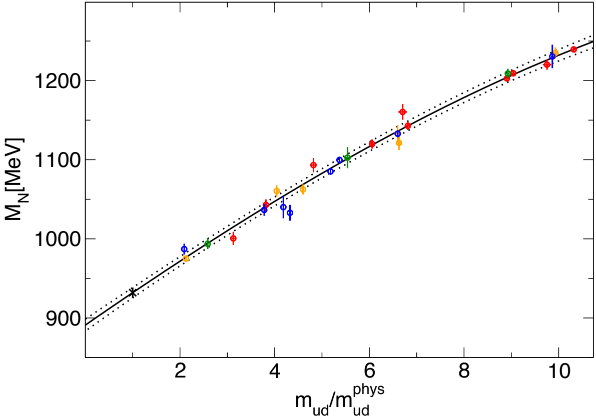
Compute matrix element $m_q \langle N | \bar{q} q | N \rangle$ directly



Or via Feynman-Hellman theorem

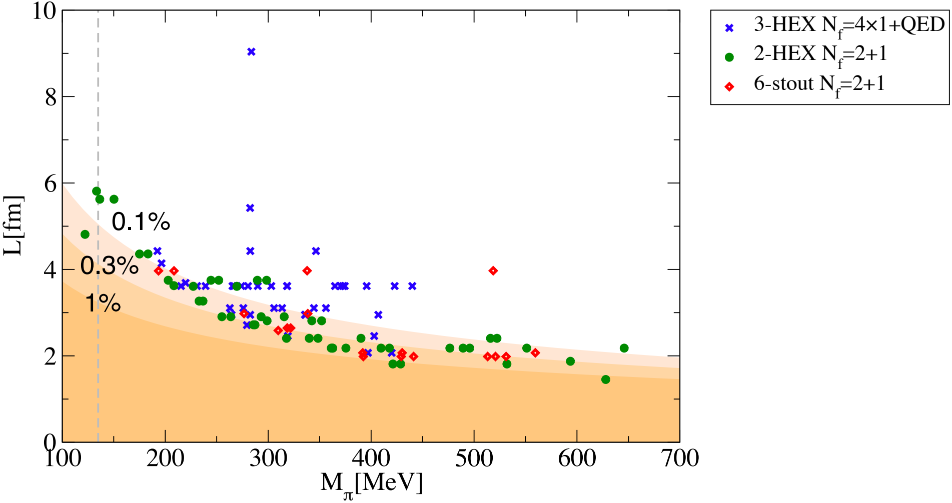
$$\frac{\partial M_N}{\partial m_q} = \frac{\partial \langle N | m_q \bar{q} q | N \rangle}{\partial m_q} = \langle N | \bar{q} q | N \rangle$$

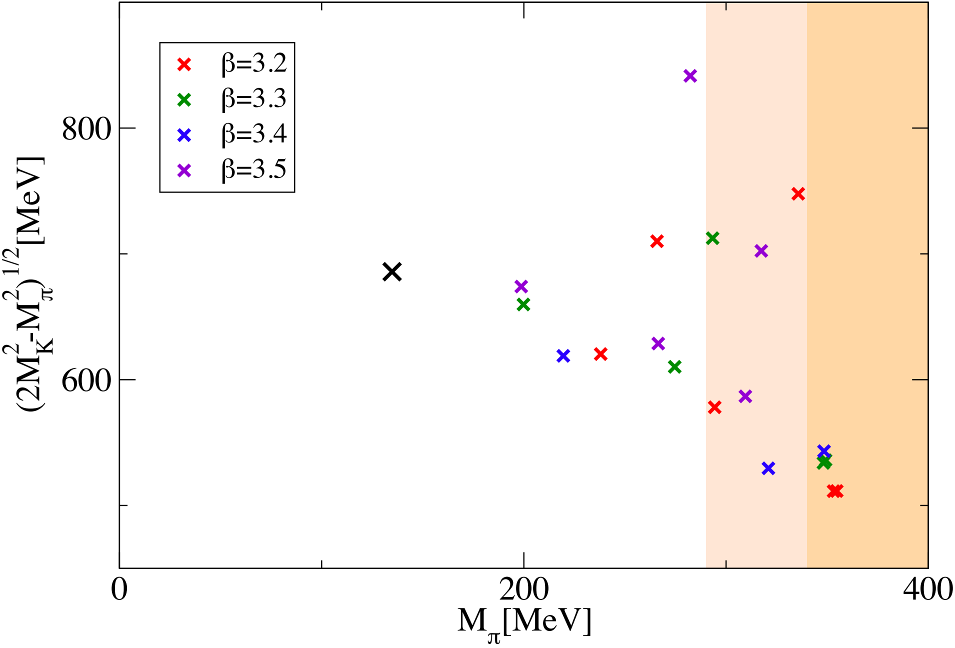
FH method



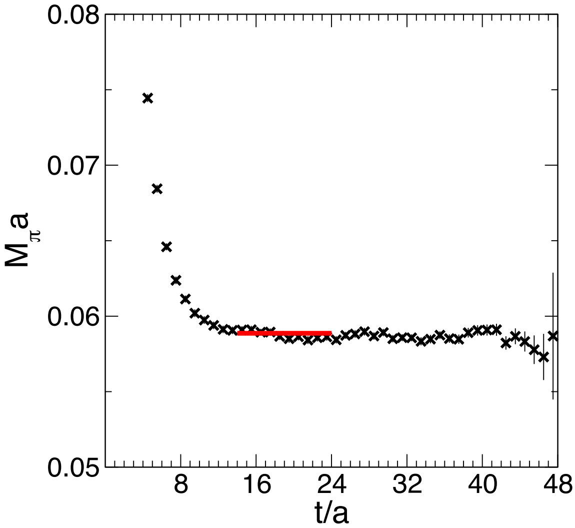
- ✓ Simple 2-pt function
- ✓ No disconnected diagrams
- ✓ Easier renormalization
- ☞ Needs accurate slope at physical point

Our Ensembles




β=3.2
β=3.3
β=3.4
β=3.5
800
600
0
200
400
$(2M_K^2-M_\pi^2)^{1/2}$[MeV]
M_π[MeV]

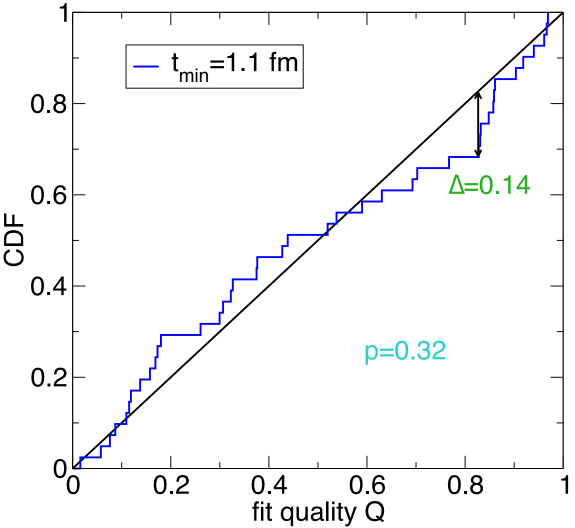
Elimination of excited states



- Fit range is critical
- Exclude excited states
- Determine from data

Conservative method:
Check that fit quality is a flat random distribution in $(0, 1)$

Plateaux range



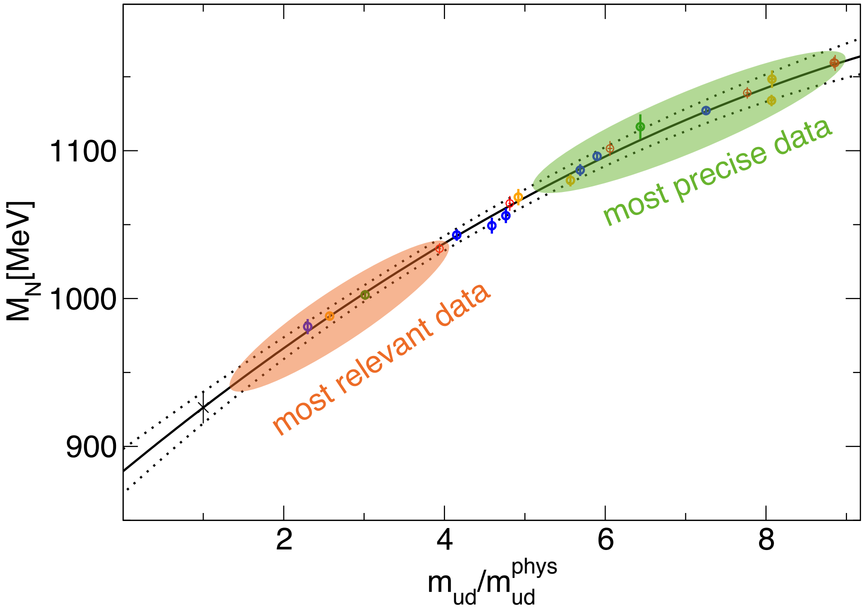
- Need many ensembles
- Plot CDF
- KS test flat distribution

$P(\Delta > \text{observed})$:

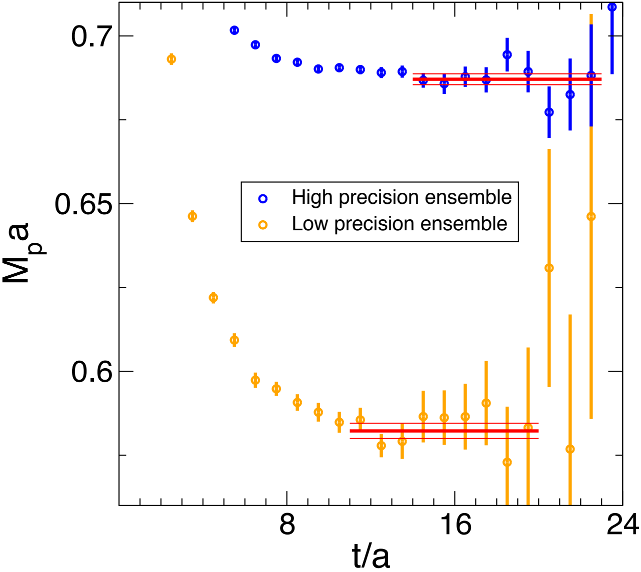
$$p\left(\Delta\left(\sqrt{N} + 0.12 + \frac{0.11}{\sqrt{N}}\right)\right)$$

with

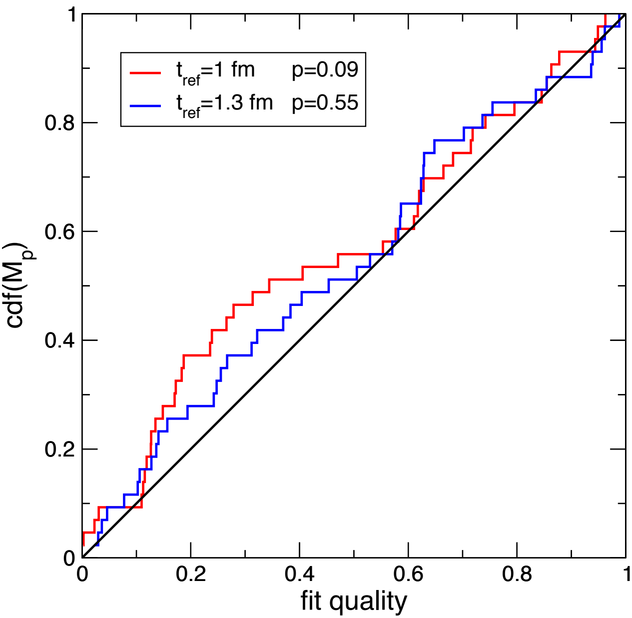
$$p(x) = \sum_j \frac{(-)^{j-1} 2}{e^{-2j^2 x^3}}$$


M_N[MeV]
1100
1000
900
2
4
6
8
m_{ud}/m_{ud}^{phys}
most relevant data
most precise data

The curse of precise data



- Shift plateaux ranges
- Keep constant excited state error (Assume $\Delta M = 500\text{MeV}$)



t_{ref}=1 fm p=0.09
t_{ref}=1.3 fm p=0.55
cdf(M_p)
fit quality
0
0.2
0.4
0.6
0.8
1

Analysis strategy

Problem:

- Determine $m_q = m_{ud}, m_s$ dependence of M_N at physical point

Method:

- Determine physical value of m_{ud}, m_s
  - Fit $m_q(M_\pi, M_K, M_{\Omega/N})$ to physical M_π, M_K and $M_{\Omega/N}$
- Determine physical value of $m_q \frac{\partial M_N}{\partial m_q}$
  - Fit $M_N(m_{ud}, m_s)$ to previously determined physical m_{ud} and m_s
- Perform infinite volume and continuum extrapolation
- Estimate systematic error

Quark mass dependence

Problem:

- To define physical quark masses, we need a renormalization scheme

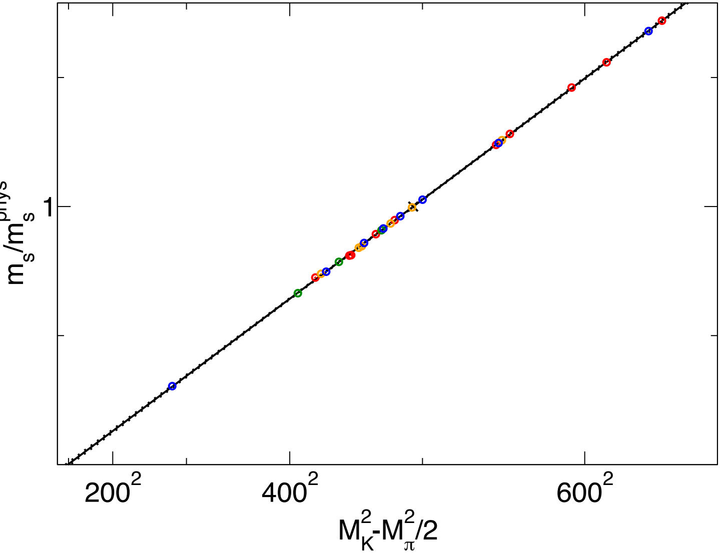
Method:

- Simplest choice on the lattice: $m_s^{\text{phys}} = 1$
  - Equivalent to parameterization

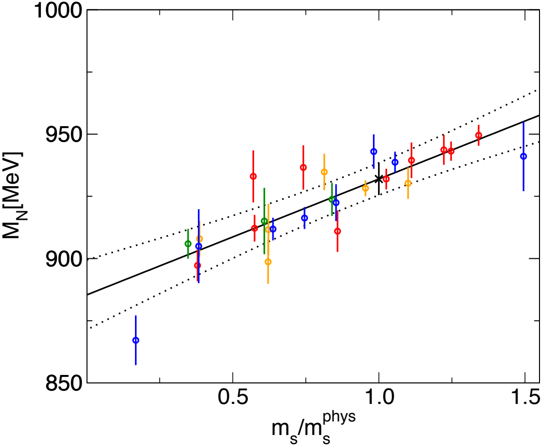
$$c_q \left(\frac{am_q}{aZ_s(\beta)} - m_q^{\text{phys}} \right) \to \tilde{c}_q \left(\frac{am_q}{a\tilde{m}_q^{\text{phys}}(\beta)} - 1 \right)$$

  - Renormalization constants can be computed on the fly
  - Crosscheck with Z_s, Z_P/Z_A where available

Quark mass dependence

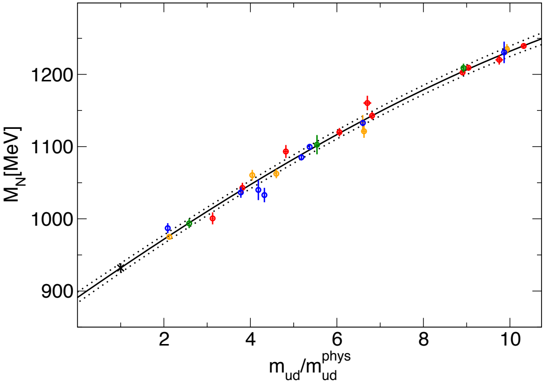


Nucleon fit



- $\frac{M_\pi}{\text{MeV}} < \{290, 340, 400\}$
- Various Polynomial, Padé and χPT ansätze
- Spread into systematic error
- $M_N \propto M_0 + cM_\pi$ misses physical point

Nucleon fit



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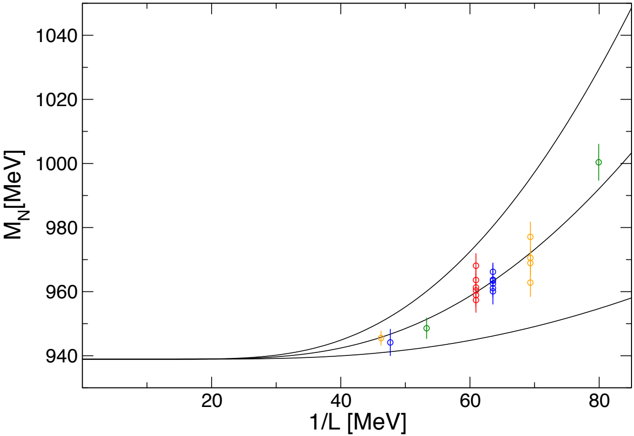
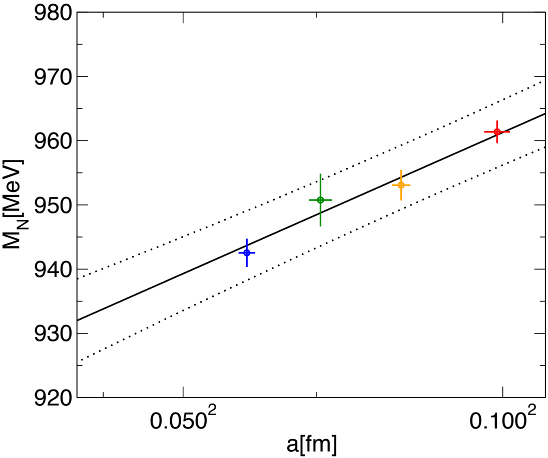
Finite volume effects



- We fit leading effects $\frac{M_X(L)-M_X}{M_X} = cM_\pi^{1/2}L^{-3/2}e^{-M_\pi L}$
- Compatible with χPT expectation (Colangelo et. al., 2010)

Continuum limit



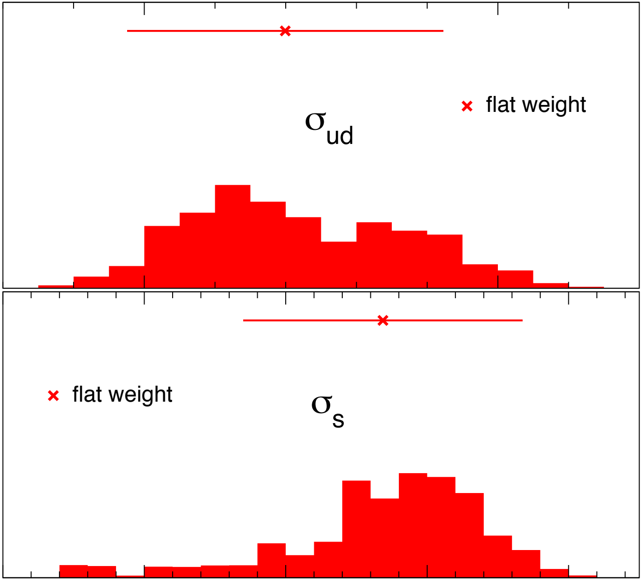
- Formally $O(\alpha a)$
- Subleading $O(a^2)$ can be dominant
- Use both, difference into systematics

Systematic error treatment

One conservative strategy for systematics: (BMWc 2008, BMWc 2014)

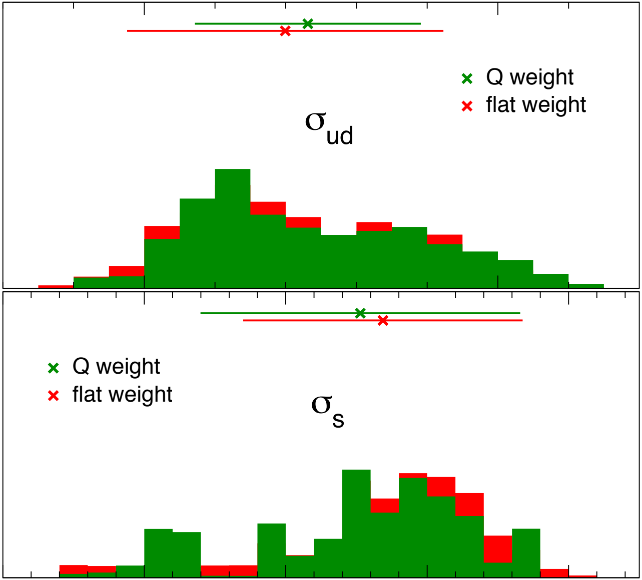
- Identify all higher order effects you have to neglect
- For each of them:
  - Repeat the entire analysis treating this one effect differently
  - Add the spread of results to systematics
- Important:
  - Do not do suboptimal analyses
  - Do not double-count analyses
- Error sources considered:
  - Plateaux range (Excited states)
  - M_π, M_K interpolations/extrapolations
  - Renormalization: NP running mass and matching scale
  - Higher order FV effects
  - Continuum extrapolation

Systematic error



- Perform O(1000) analyses
- Difference: higher order effects
- Draw histogram of results
- Different weights possible
- Crosscheck agreement

Systematic error



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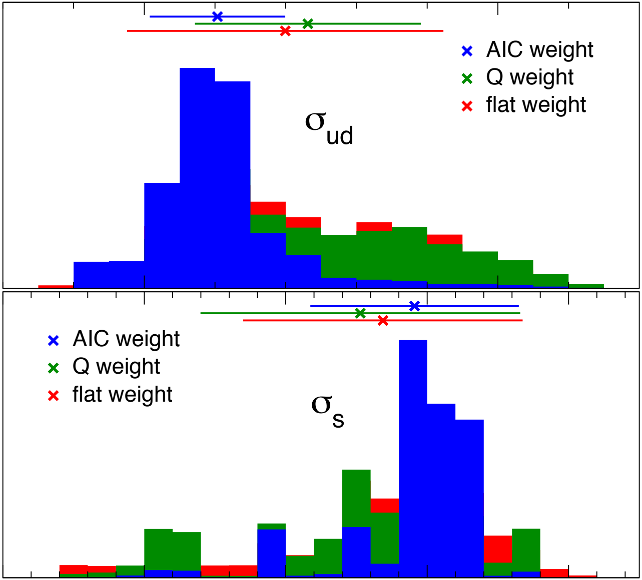

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From the effective Hamiltonean

$$H = H_{\text{iso}} + \frac{\delta m}{2} \int d^3x (\bar{d}d - \bar{u}u)$$

we obtain (with $\delta m = m_d - m_u$)

$$\Delta_{QCD} M_N = \frac{\delta m}{2M_p} \langle p | \bar{u}u - \bar{d}d | p \rangle$$

which, together with

$$f^p_{u/d} = \left(\frac{1}{2} \mp \frac{\delta m}{4m_{ud}} \right) f^p_u d + \left(\frac{1}{4} \mp \frac{m_{ud}}{2\delta m} \right) \frac{\delta m}{2M_p^2} \langle p | \bar{d}d - \bar{u}u | p \rangle$$

gives ($r = m_u / m_d$)

$$f_u^{p/n} = \left(\frac{r}{1+r} \right) f_{ud}^N \pm \frac{1}{2} \left(\frac{r}{1-r} \right) \frac{\Delta_{QCD} M_N}{M_N} + O(\delta m^2, m_{ud} \delta m)$$

$$f_d^{p/n} = \left(\frac{1}{1+r} \right) f_{ud}^N \mp \frac{1}{2} \left(\frac{1}{1-r} \right) \frac{\Delta_{QCD} M_N}{M_N} + O(\delta m^2, m_{ud} \delta m)$$

Results (BMWc, 2015)

Direct results:

$$f_{ud}^N = 0.0405(40)(35) \qquad \sigma_{ud}^N = 38(3)(3)\text{MeV}$$
$$f_s^N = 0.113(45)(40) \qquad \sigma_s^N = 105(41)(37)\text{MeV}$$

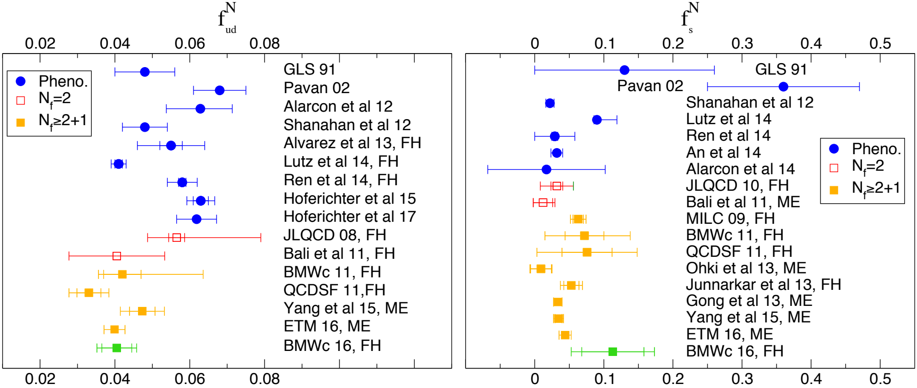
With $\Delta_{QCD} M_N = 2.52(17)(24)\text{MeV}$ from (BMWc 2014)

$$f_u^p = 0.0139(13)(12) \qquad f_d^p = 0.0253(28)(24)$$
$$f_u^n = 0.0116(13)(11) \qquad f_d^n = 0.0302(28)(25)$$

With heavy quark relation

$$f_Q^N = 0.063(4)(1 + O(\alpha_s))$$

PRELIMINARY result on new dataset



Crosschecks:

$$M_N = 929(13)(8)\text{MeV} \qquad m_s/m_{ud} = 27.5(3)(2)$$

Compatible with old results

Tension with Hoferichter et. al. 15,17